

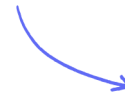
## Structured Questions

# Simple Harmonic Motion

Describing Oscillations / Simple Harmonic Motion (SHM) / Time Period of a Mass-Spring System / Time Period of a Simple Pendulum / Energy Changes in Simple Harmonic Motion (SHM) / Equations for Simple Harmonic Motion (SHM) (HL) / Calculating Energy Changes in SHM (HL) / Phase Angles in Simple Harmonic Motion (SHM) (HL)

Easy (9 questions)	/105
Medium (10 questions)	/113
Hard (8 questions)	/65
<b>Total Marks</b>	<b>/283</b>

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# Easy Questions

1 (a) Complete the table by adding the correct key terms to the definitions.

Definition	Key Term
The time interval for one complete oscillation	
The distance of a point on a wave from its equilibrium position	
The number of oscillations per second	
The repetitive variation with time of the displacement of an object about its equilibrium position	
The maximum value of displacement from the equilibrium position	
The oscillations of an object have a constant period	

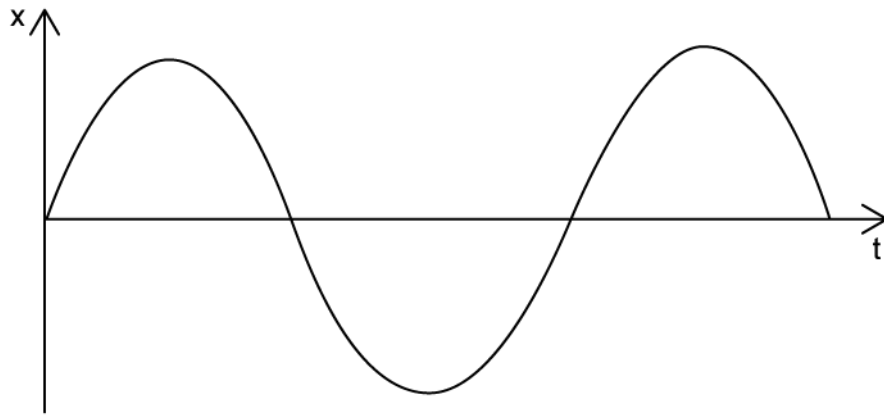
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**(3 marks)**

(b) The graph shows the displacement of an object with time.



On the graph, label the following:

(i) the time period  $T$

[1]

(ii) the amplitude  $x_0$

[1]

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**(2 marks)**

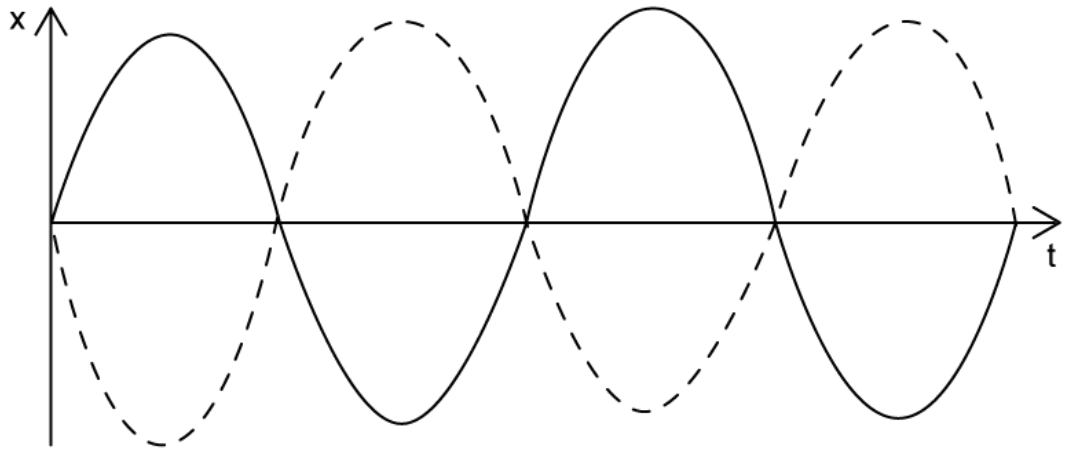
**(c)** An object oscillates isochronously with a frequency of 0.4 Hz.

Calculate the period of the oscillation.

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**(1 mark)**

**(d)** The graph shows the oscillations of two different waves.



For the two oscillations, state:

- (i) The phase difference in terms of wavelength  $\lambda$ , degrees and radians. [3]
- (ii) Whether the oscillations are in phase or in anti-phase. [1]

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**(4 marks)**

2 (a) Fill in the blank spaces with a suitable word or words.

Objects in simple harmonic motion \_\_\_\_\_ about an equilibrium point. The restoring force and \_\_\_\_\_ always act toward the equilibrium and are \_\_\_\_\_ to \_\_\_\_\_, but act in the opposite direction.

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**(3 marks)**

(b) Hooke's law can be used to describe a mass-spring system performing simple harmonic oscillations. Hooke's Law states that;

$$F = -kx$$

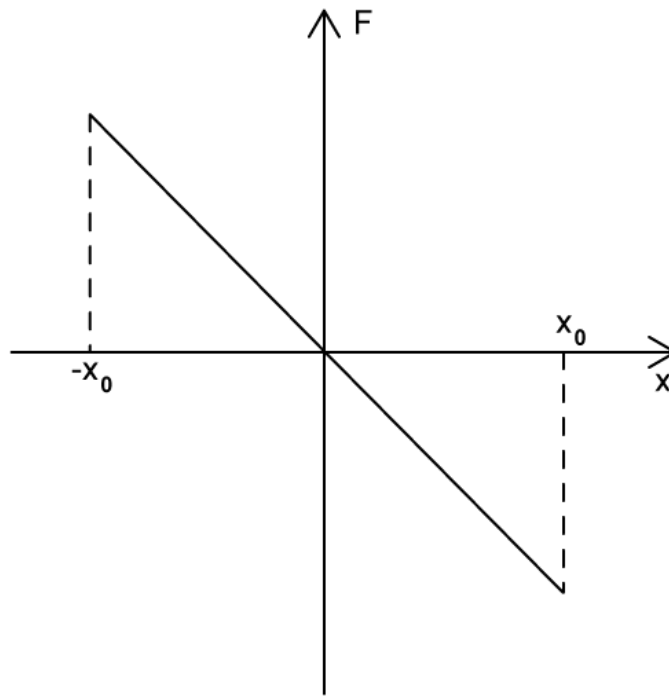
State the definition of the following variables and an appropriate unit for each:

- (i)  $F$  [1]
- (ii)  $k$  [1]
- (iii)  $x$  [1]

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**(3 marks)**

(c) The graph shows the restoring force on a bungee cord.



Identify the quantity given by the gradient, where  $F = -kx$

(1 mark)

(d) For an object in simple harmonic motion:

(i) State the direction of the restoring force in relation to its displacement

[1]

(ii) State the relationship between force and displacement

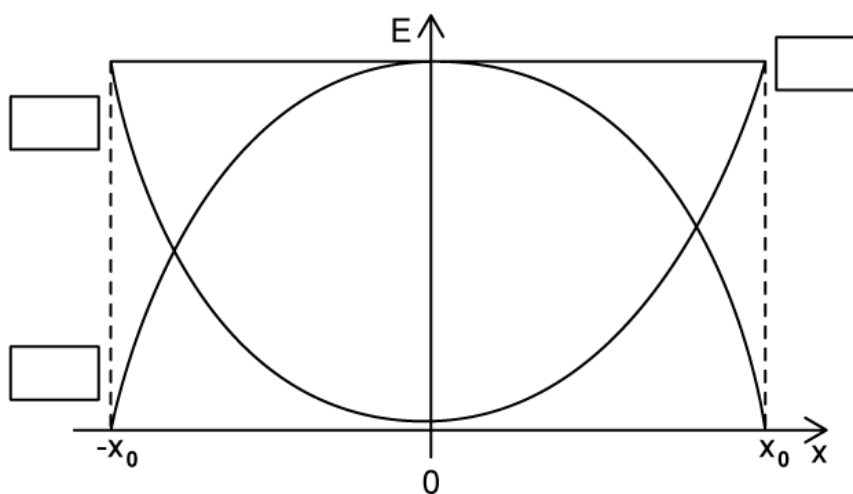
[1]

(2 marks)

3 (a) Define the term 'total energy' for a system oscillating in simple harmonic motion.

(1 mark)

(b) The graph shows the potential energy  $E_p$ , kinetic energy  $E_K$  and total energy  $E_T$  of a system in simple harmonic motion.

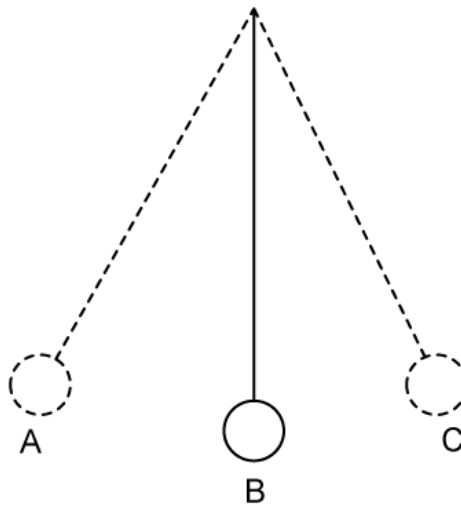


Add the following labels to the correct boxes on the graph:

- $E_T$
- $E_p$
- $E_K$

(3 marks)

(c) The diagram indicates the positions of a simple pendulum in simple harmonic motion.



Identify the position(s) of the pendulum when:

- (i) Kinetic energy is zero [1]
- (ii) Potential energy is at a maximum [1]
- (iii) Kinetic energy is at a maximum [1]
- (iv) Potential energy is zero [1]

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**(4 marks)**

**(d)** The period of the oscillation shown in part (c) is 2.2 s.

Calculate the frequency of the oscillation.

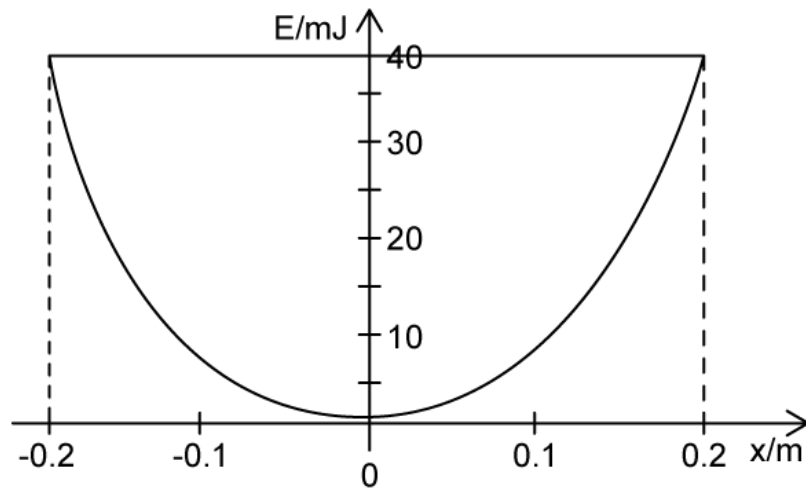
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**(2 marks)**



- 4 (a) A mass-spring system oscillates with simple harmonic motion. The graph shows how the potential energy of the spring changes with displacement.



For the mass-spring system, determine:

- (i) The maximum potential energy [1]
- (ii) The total energy [1]

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(2 marks)

(b) Using the graph in part (a), determine:

- (i) The amplitude  $x_0$  of the oscillation [1]
- (ii) The potential energy in the spring when the displacement  $x = 0.1$  m [1]

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(2 marks)

- (c) The block used in the same mass-spring system has a mass  $m$  of 25 g. The maximum kinetic energy of the block is 40 mJ.

Calculate the maximum velocity of the oscillating block

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**(4 marks)**

- (d) The spring constant  $k$  of the spring used is  $2.1 \text{ N m}^{-1}$

Calculate the restoring force acting on the mass-spring system at amplitude  $x_0$

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**(2 marks)**

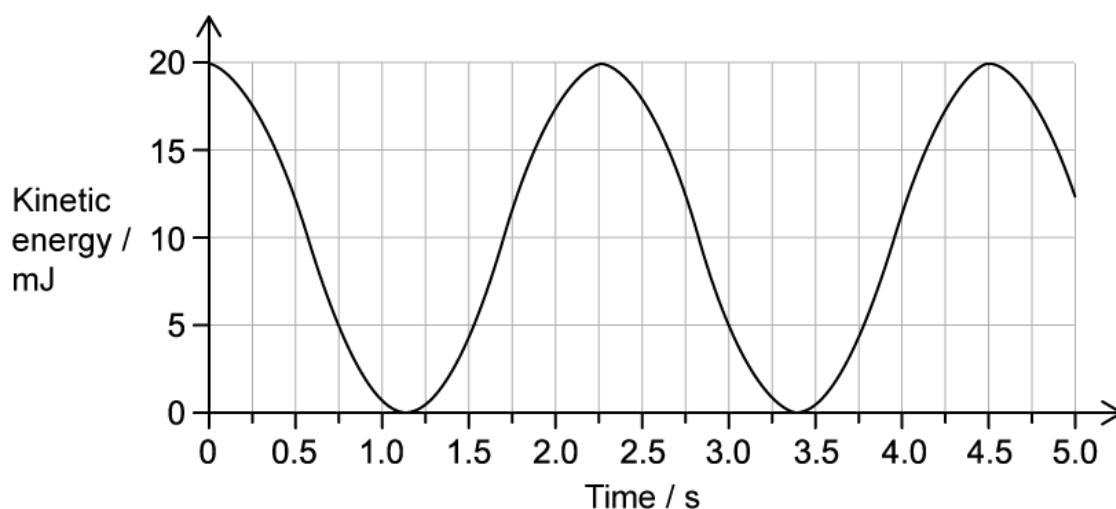
5 (a) State what is meant by the time period of an oscillation.

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(1 mark)

(b) A small metal pendulum bob is suspended from a fixed point by a thread with negligible mass. Air resistance is also negligible.

The pendulum begins to oscillate from rest. Assume that the motion of the system is simple harmonic, and in one vertical plane. The graph shows the variation of kinetic energy of the pendulum bob with time.



Determine the time period of the pendulum.

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(1 mark)

(c) Label a point X on the graph where the pendulum is in equilibrium.

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(1 mark)

(d) The mass of the pendulum bob is  $60 \times 10^{-3}$  kg.

(i) Determine the maximum kinetic energy of the pendulum bob.

[1]

(ii) Hence or otherwise, show that the maximum speed of the bob is about  $0.82 \text{ m s}^{-1}$ .

[3]

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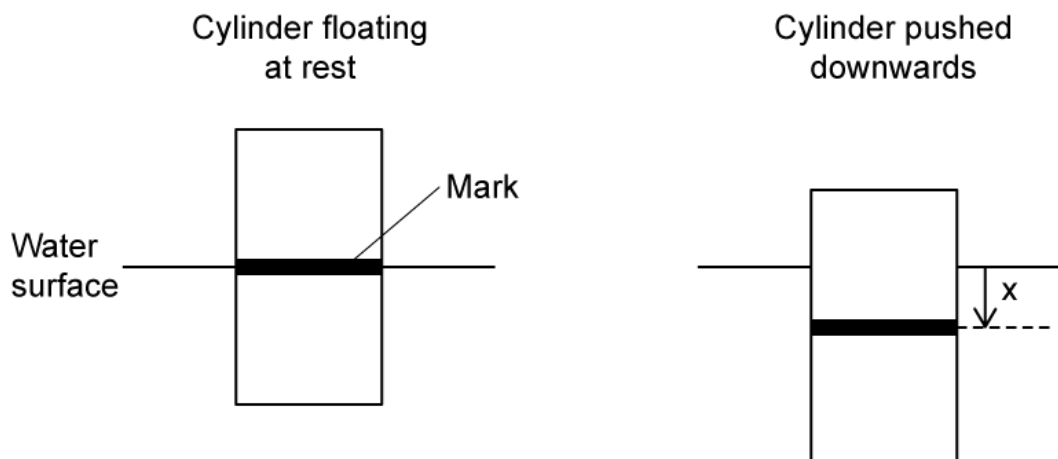
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**(4 marks)**

- 6 (a)** A solid vertical cylinder of uniform cross-sectional area  $A$  floats in water. The cylinder is partially submerged. When the cylinder floats at rest, a mark is aligned with the water surface. The cylinder is pushed vertically downwards so that the mark is a distance  $x$  below the water surface.



The cylinder is released at time  $t = 0$ . The resultant force on the cylinder is related to the displacement  $x$  by:

$$F = kx$$

- (i) Define simple harmonic motion. [2]
- (ii) Outline why the cylinder performs simple harmonic motion when released. [2]

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**(4 marks)**

- (b)** The mass,  $m$ , of the cylinder is 100 kg and the value of  $k$  is  $3000 \text{ kg s}^{-2}$ .

- (i) Define angular frequency.

(ii) Show that the equation from part (a) can be related to an expression for angular frequency to give  $-\omega^2 m = k$ . [1]

(iii) Hence, show that the angular frequency,  $\omega$  of oscillation of the cylinder is  $5.5 \text{ rad s}^{-1}$ . [3]

[2]

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**(6 marks)**

**(c)** The cylinder was initially pushed down by a displacement  $x = 0.15 \text{ m}$ .

(i) Determine the force applied to the cylinder. [1]

(ii) Determine the maximum kinetic energy  $E_{\text{kmax}}$  of the cylinder. [3]

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**(4 marks)**

**(d)** Draw, on the axes below, the graph to show how the kinetic energy of the cylinder varies with time during one period of oscillation  $T$ .



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(2 marks)

**7 (a)** A vibrating guitar string is an example of an object oscillating with simple harmonic motion.

Give three other real-world examples of objects that oscillate with simple harmonic motion.

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**(3 marks)**

**(b)** The guitar string vibrates with simple harmonic oscillations at a frequency of 225 Hz.

Determine the time it takes to perform 15 complete oscillations.

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**(3 marks)**

**(c)** The amplitude of the oscillation is 0.4 mm.

Determine the maximum acceleration of the guitar string.

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**(4 marks)**

**(d)** Calculate the total energy in the string during this oscillation, given that the mass of the string is 3.3 g.

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**(3 marks)**

**8 (a)** The defining equation of simple harmonic motion is:

$$a = -\omega^2x$$

(i) Define each variable and give an appropriate unit for each.

[3]

(ii) State the significance of the minus sign.

[1]

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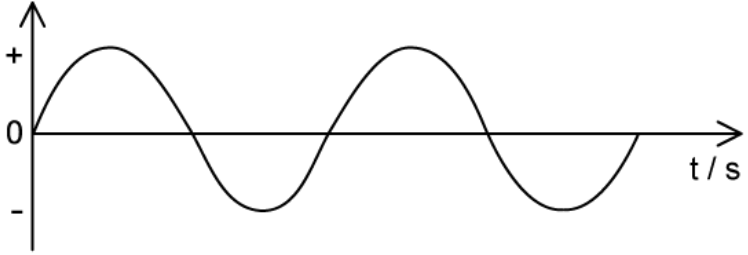
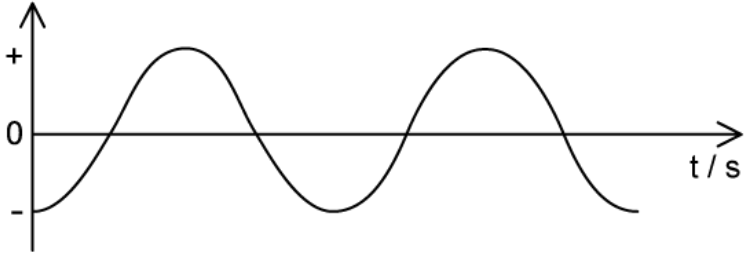
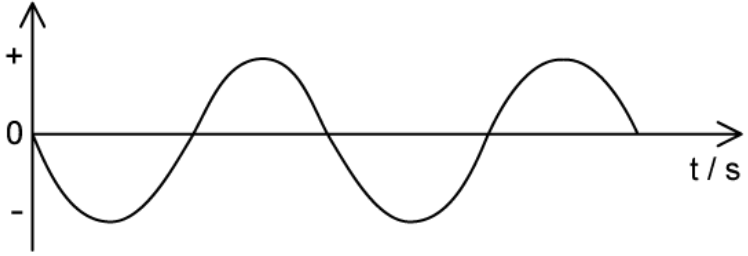
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**(4 marks)**

**(b)** A mass on a spring begins oscillating from its equilibrium position. Time,  $t = 0$  s is measured from where the mass begins moving in the negative direction. The motion of the oscillation is shown in the graphs below.

Graph	Variable on y-axis
	Acceleration
	
	

Complete the table to show the correct variable on the y-axis of each graph.

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**(3 marks)**

- (c) The period of the oscillation  $T = 1.84$  s and the mass is 55 g. The mass-spring system oscillates with an amplitude of 5.2 cm.

Calculate the spring constant of the spring.

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**(3 marks)**

**(d)** Determine the magnitude of the displacement of the mass at  $t = 1.2$  s.

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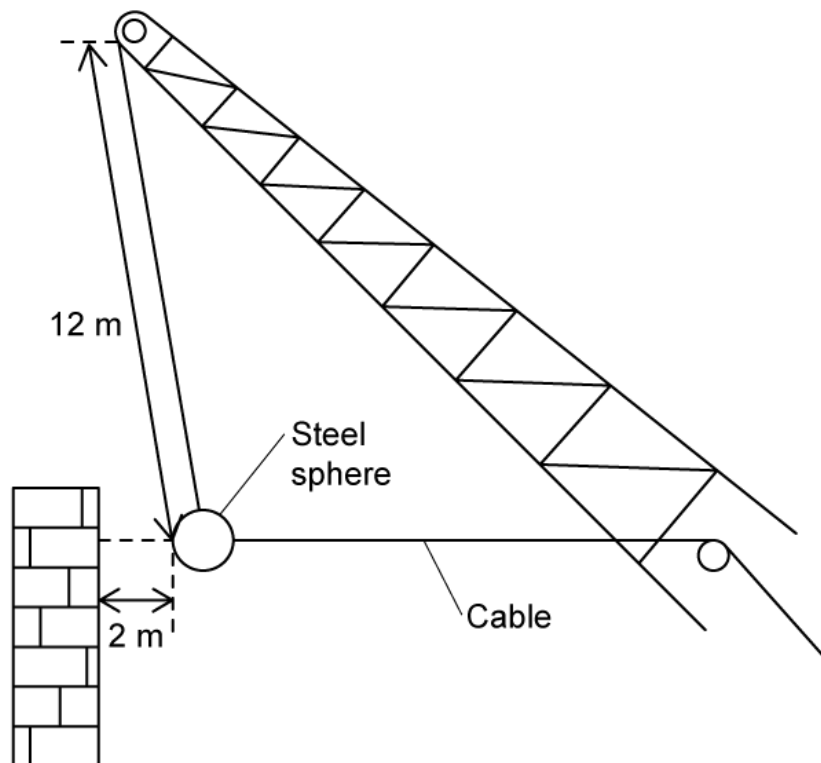
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**(4 marks)**

9 (a) The diagram below shows a system used for demolishing buildings.



A 2750 kg steel sphere is suspended by a steel cable of length 12 m. The steel sphere is pulled 2 m to the side by another cable and then released.

When the wall is not in the way, the system performs simple harmonic motion.

Calculate the frequency of the oscillation.

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(3 marks)

(b) When the steel sphere hits the wall, the suspension cable is vertical.

Calculate the speed of the steel sphere when it hits the wall.

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**(4 marks)**

- (c)** Calculate the kinetic energy of the steel sphere as it hits the wall. Give your answer to an appropriate amount of significant figures.

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**(5 marks)**

- (d)** Complete the following sentences using appropriate words to describe the effect on the kinetic energy of the sphere when doubling its mass and displacement.

Doubling the mass of the steel sphere would cause the kinetic energy to \_\_\_\_\_ by a factor of \_\_\_\_\_. This is because kinetic energy is proportional to \_\_\_\_\_.

Doubling the initial displacement of the steel sphere would cause the kinetic energy to \_\_\_\_\_ by a factor of \_\_\_\_\_. This is because the kinetic energy is proportional to \_\_\_\_\_.

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**(4 marks)**

# Medium Questions

1 (a) A pendulum undergoes small-angle oscillations.

- (i) State the equation and each of the variables that define simple harmonic motion.
- (ii) Explain the meaning of the equation

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(3 marks)

(b) Sketch a graph of displacement against time for one swing of the pendulum. Start the time at zero seconds when the pendulum is at the maximum positive displacement,  $x_0$ , position.

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(2 marks)

(c) The time period of 10 oscillations is found to be 12.0 s.

Determine the frequency when the bob is 1.0 cm from its equilibrium position.

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(2 marks)

(d) The student wants to double the frequency of the pendulum swing. The time period,  $T$  of a simple pendulum is given by the equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where  $L$  is the length of the string and  $g$  is the acceleration due to gravity

Deduce the change which would achieve this.

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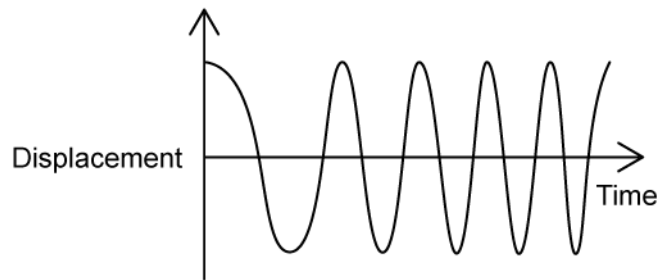
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**(4 marks)**

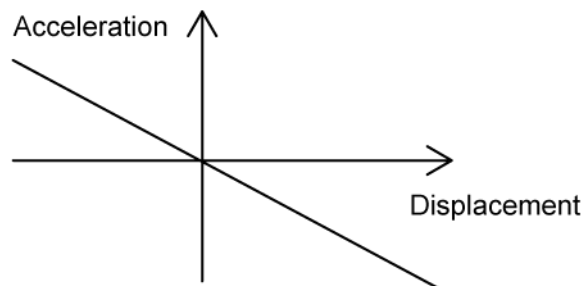


2 (a) State and explain whether the motion of the objects in graphs I, II and III are simple harmonic oscillations

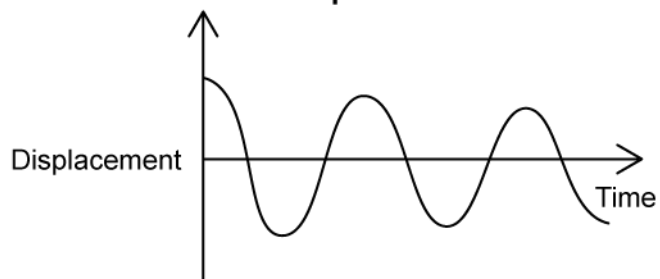
**Graph 1**



**Graph 2**



**Graph 3**



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(6 marks)

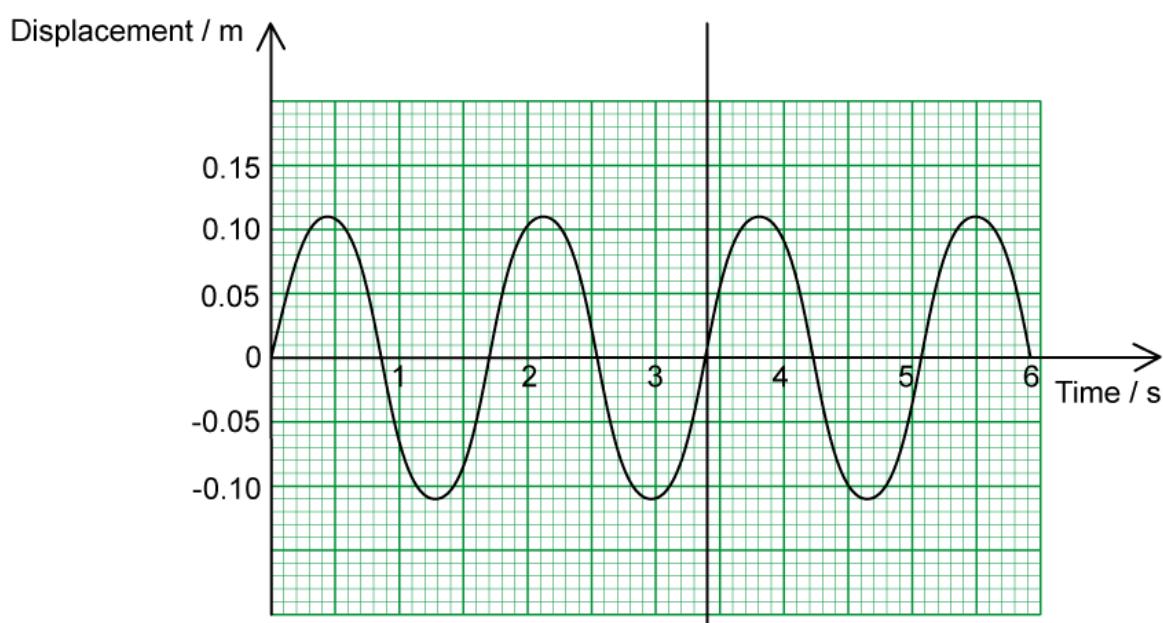
(b) Explain why, in practice, a freely oscillating pendulum cannot maintain a constant amplitude.

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(2 marks)

(c) The motion of an object undergoing SHM is shown in the graph below.



For this oscillator, determine:

- (i) The amplitude,  $A$ . [1]
- (ii) The period,  $T$ . [1]
- (iii) The frequency,  $f$ . [1]

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**(3 marks)**

**(d)** Using the graph from part (c), state a time in seconds when the object performing SHM has:

(i) Maximum positive velocity. [1]

(ii) Maximum negative acceleration. [1]

(iii) Maximum potential energy. [1]

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**(3 marks)**

- 3 (a) A ball of mass 44 g on a 25 cm string oscillating in simple harmonic motion obeys the following equation:

$$a = -\omega^2 x$$

Show mathematically that the graph of this equation is a downward sloping straight line that goes through the origin.

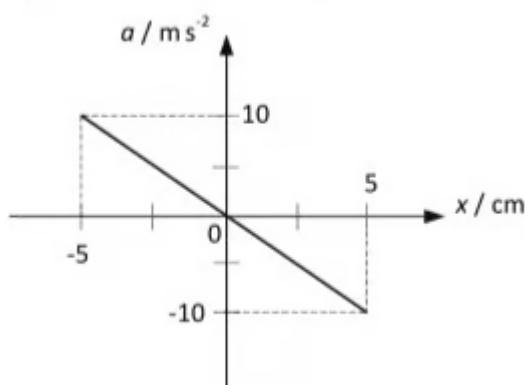
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**(3 marks)**

- (b) The graph below shows the acceleration,  $a$ , as a function of displacement,  $x$ , of the ball on the string.



The angular speed,  $\omega$ , in  $\text{rad s}^{-1}$ , is related to the frequency,  $f$ , of the oscillation by the following equation:

$$\omega = 2\pi f$$

For the ball on the string, determine the period,  $T$ , of the oscillation.

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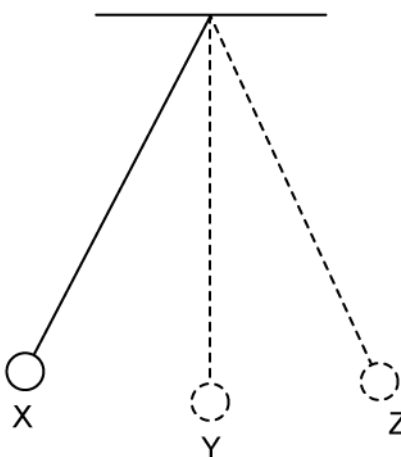
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**(3 marks)**

- (c)** The ball is held in position X and then let go. The ball oscillates in simple harmonic motion.

Explain the change in acceleration as the ball on the string moves through half an oscillation from position X.

You can assume the ball is moving at position X.



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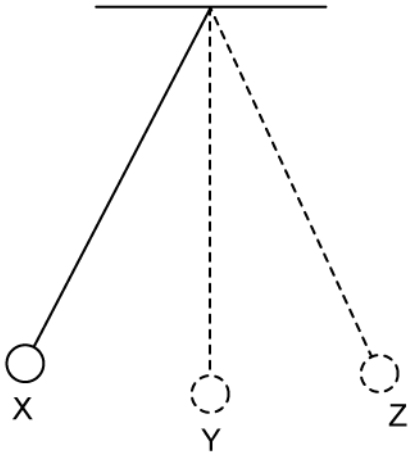
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**(4 marks)**

- (d)** Describe the energy transfers occurring as the ball on the string completes half an oscillation from position X.



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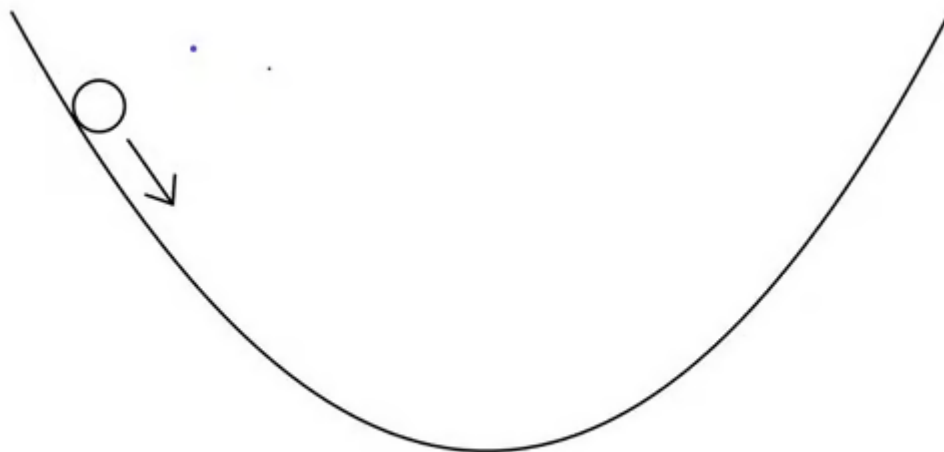
(5 marks)

- 4 (a) A smooth glass marble is held at the edge of a bowl and released. The marble rolls up and down the sides of the bowl with simple harmonic motion.

The magnitude of the restoring force which returns the marble to equilibrium is given by:

$$F = \frac{mgx}{R}$$

Where  $x$  is the displacement at a given time, and  $R$  is the radius of the bowl.



Deduce mathematically why the oscillations of the marble are in simple harmonic motion.

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**(3 marks)**

- (b) Describe the energy changes that take place during the simple harmonic motion of the marble.

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**(3 marks)**

**(c)** As the marble is released it has potential energy of  $15 \mu\text{J}$ . The mass of the marble is  $3 \text{ g}$ .

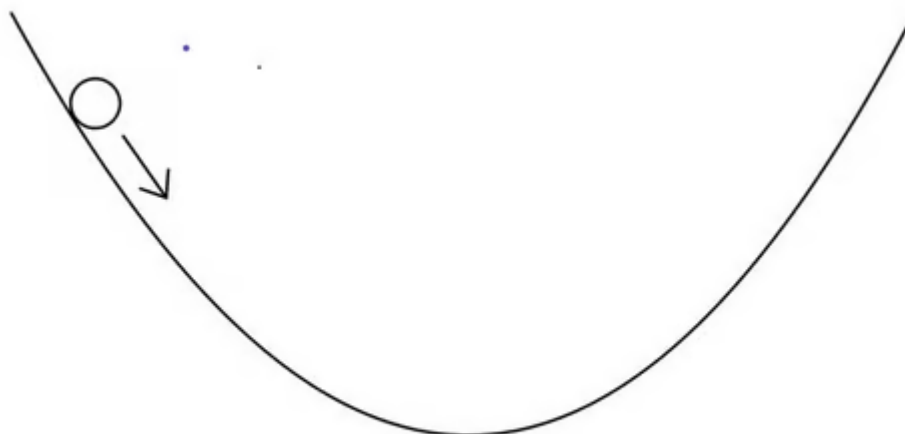
Calculate the velocity of the marble at the equilibrium position.

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**(2 marks)**

**(d)** Sketch a graph to represent the kinetic, potential and total energy of the motion of the marble, assuming no energy is dissipated as heat. Clearly label any important values on the graph.



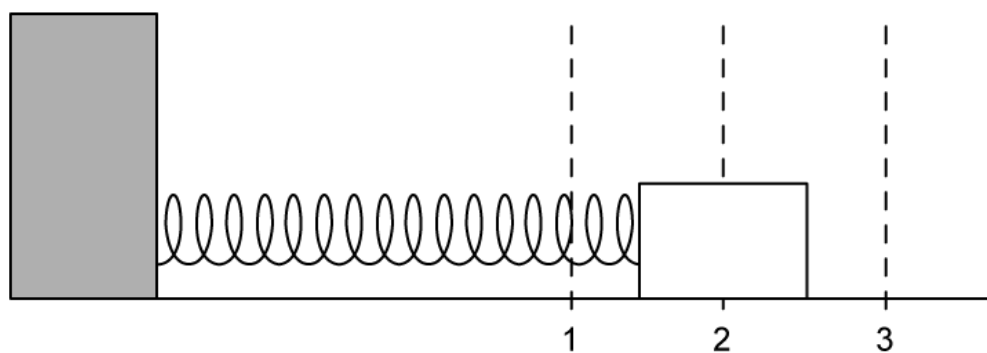
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**(3 marks)**



**5 (a)** An object is attached to a light spring and set on a frictionless surface. It is allowed to oscillate horizontally. Position 2 shows the equilibrium point.



(i) Sketch a graph of acceleration against displacement for this motion.

[2]

(ii) On your graph, mark positions 1, 2 and 3 according to the diagram.

[1]

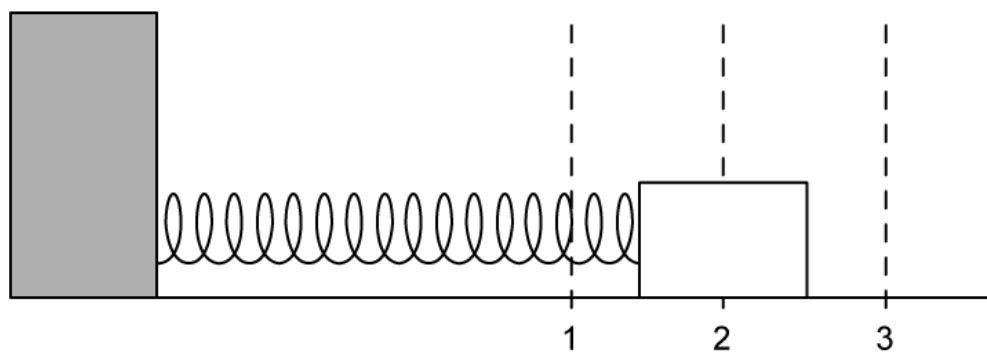
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**(3 marks)**

**(b)** The mass begins its motion from position 1 and completes a full oscillation.



(i) Sketch a graph of velocity against time to show this.

[2]

(ii) On your graph, add labels to show points 1, 2 and 3

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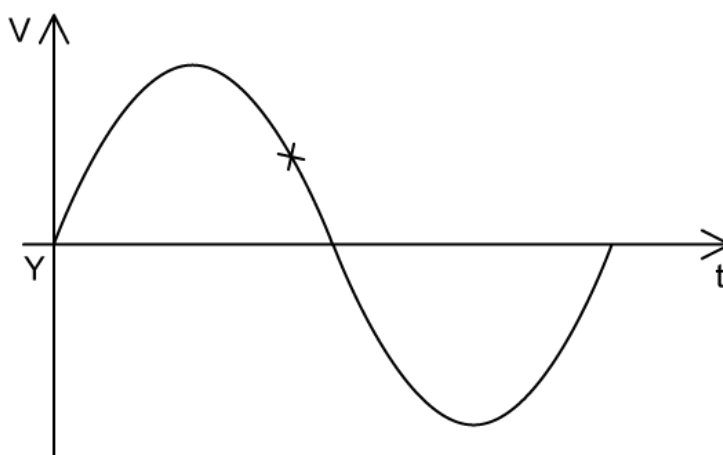
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**(4 marks)**

- (c) At the point marked **Y** on the graph, the potential energy of the block is  $E_p$ . The block has mass  $m$ , and the maximum velocity it achieves is  $v_{max}$ .



Determine an equation for the potential energy at the point marked X.

Give your answer in terms of  $v_{max}$ ,  $v_x$  and  $m$ .

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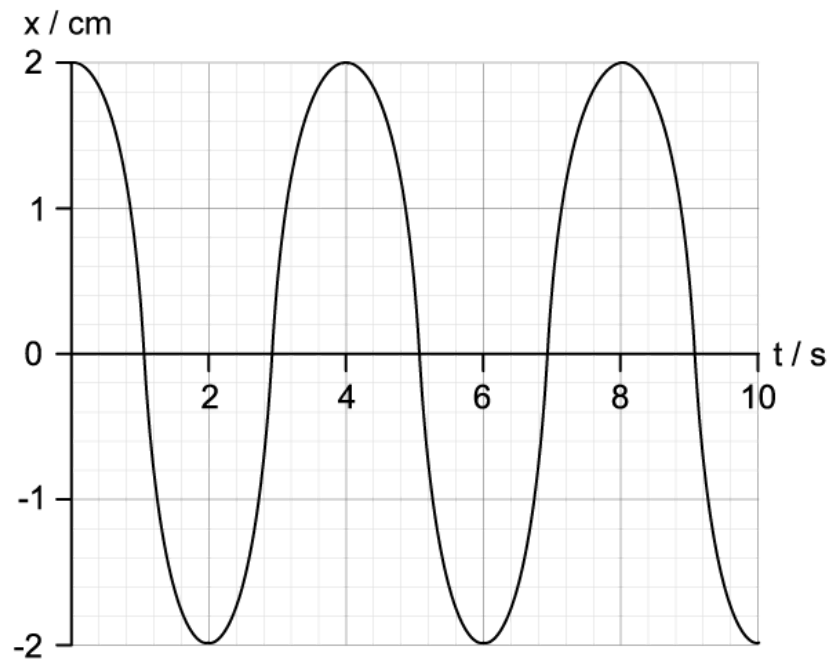
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**(3 marks)**

- (d) The graph shows how the displacement  $x$  of the mass varies with time  $t$ .



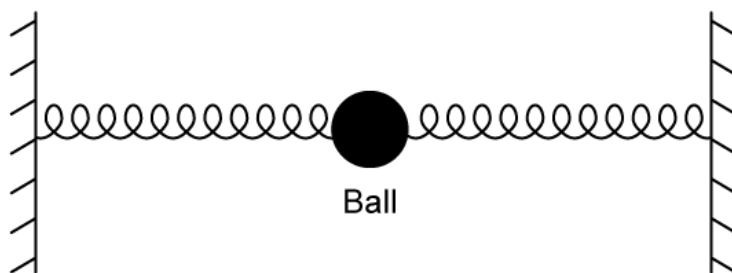
Determine the frequency of the oscillations.

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(2 marks)

**6 (a)** A ball within a ball-spring system oscillates about an equilibrium point.



Describe the conditions for simple harmonic motion.

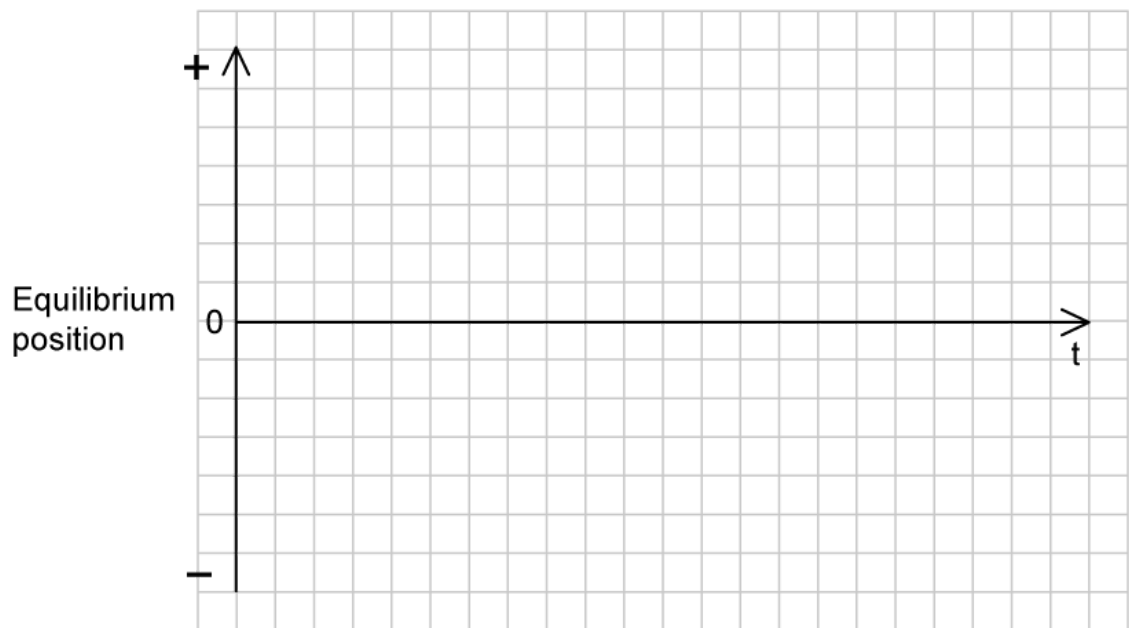
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**(2 marks)**

**(b)** The ball oscillates with simple harmonic motion. On the axes provided, taking the oscillation as beginning at the positive amplitude when  $t = 0$ , sketch for the ball's motion:

- (i) The variation of displacement with time, label this graph  $x$  [1]
- (ii) The variation of velocity with time, label this graph  $v$  [1]
- (iii) The variation of acceleration with time, label this graph  $a$  [1]



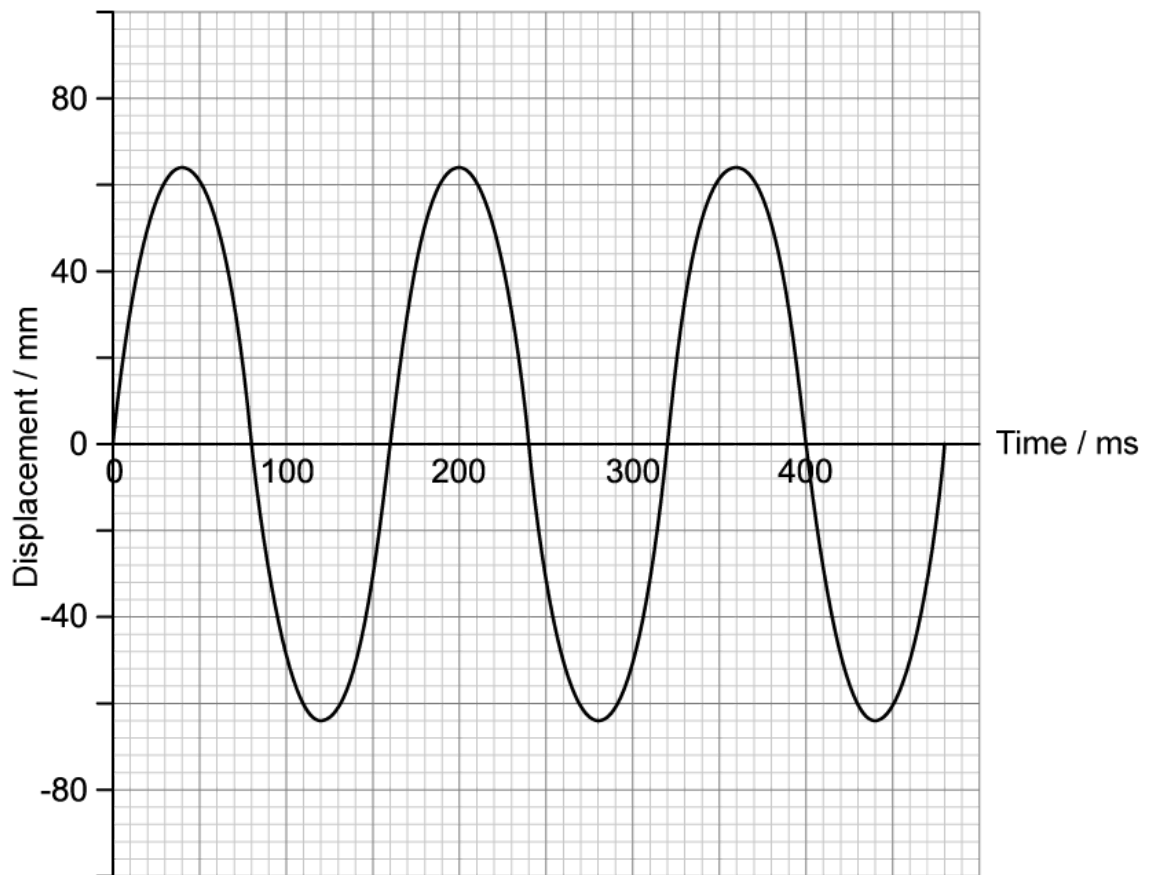
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**(3 marks)**

- (c) In a separate experiment, the ball is oscillating from the equilibrium position. The graph shows the displacement of the ball over time.



Determine the maximum velocity of the ball.

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**(2 marks)**

**(d)** Using the graph from part (c) for the motion of the ball

(i) Show that the acceleration at 90 ms is  $43 \text{ m s}^{-2}$ . [2]

(ii) On the graph, mark an X at a point where the resultant force acting on the ball is zero. [1]

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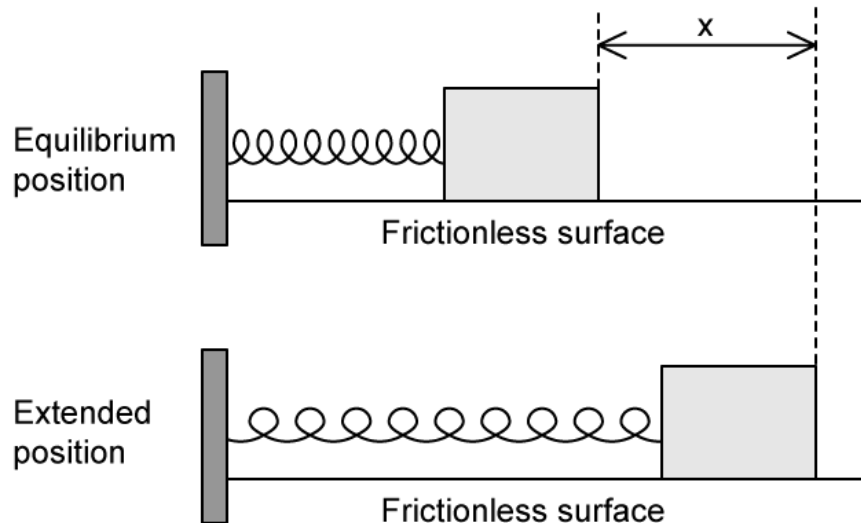
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(3 marks)

A steel block with a mass of 45 g on a spring undergoes simple harmonic motion with a period of 0.84 s.

7 (a)

The steel block is removed. Now, a wooden block attached to the same spring undergoes simple harmonic motion with a period of 0.64 s. The wooden block is displaced horizontally by 3.6 cm from the equilibrium position on a frictionless surface.



Determine the total energy in the oscillation of the wooden block.

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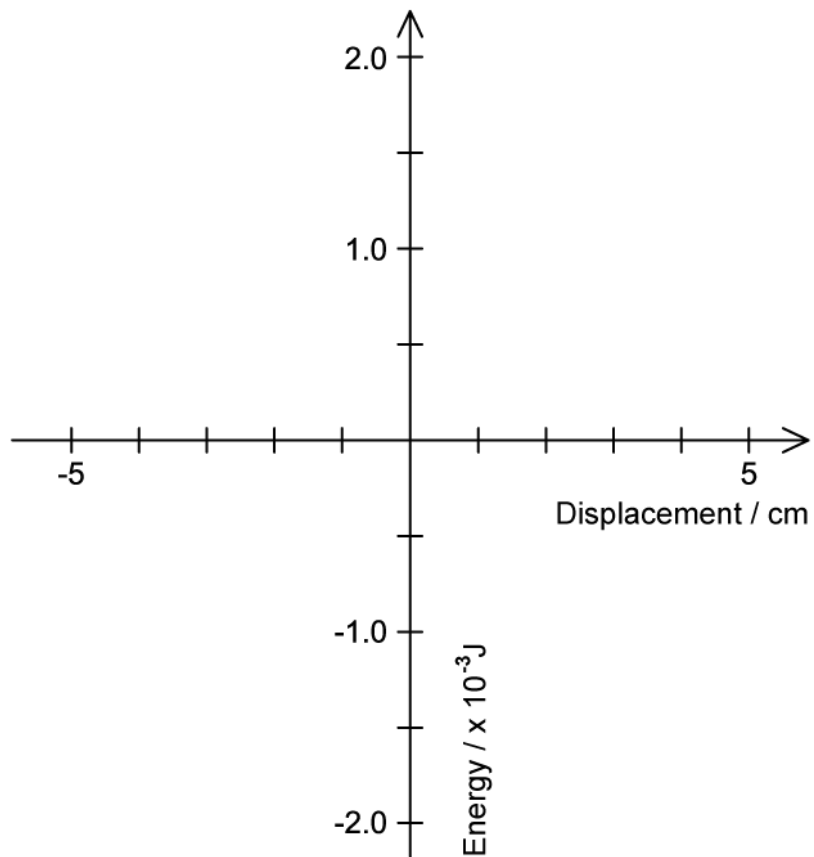
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(4 marks)

(b) Using the information from part (a), sketch on the axes the kinetic, potential, and total energies of the oscillating wooden block as they vary with displacement.





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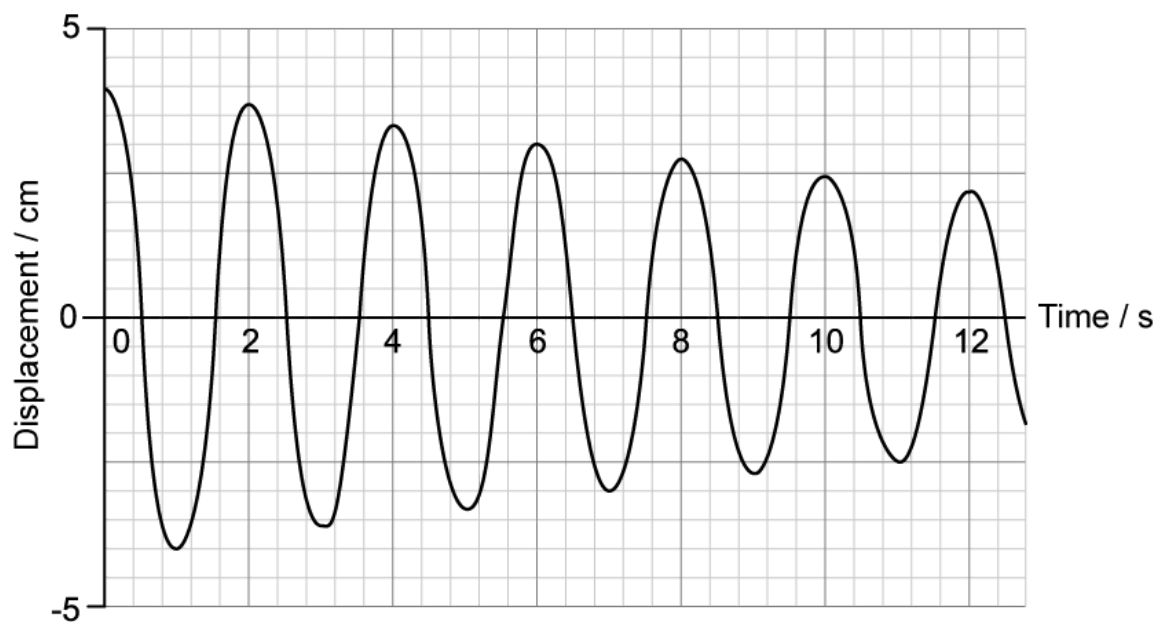
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**(3 marks)**

- (c) In a separate experiment, another mass-spring system is set to oscillate in simple harmonic motion. The mass has a mass of 45 g.

The graph shows the variation of displacement with time  $t$  of the mass on the spring.



For the new experiment on the mass-spring system

(i) Describe the motion of the mass on the spring.

[1]

(ii) Determine the initial energy of the mass-spring system.

[2]

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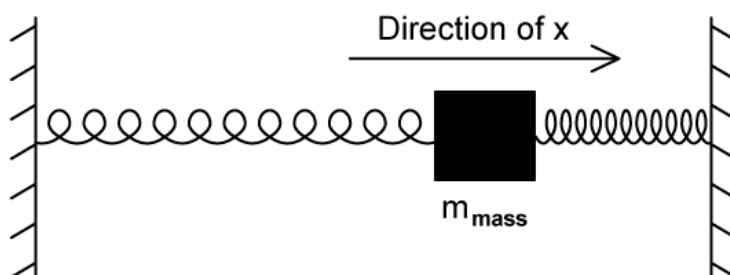


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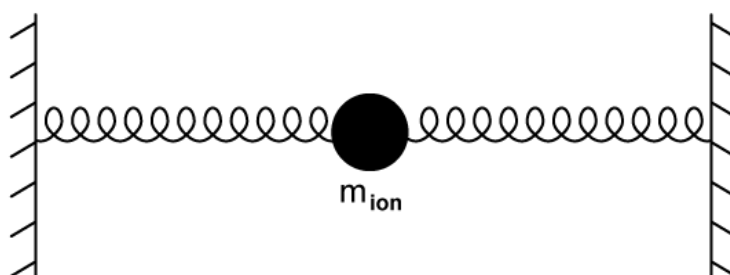
**(3 marks)**

- 8 (a)** A mass of 75 g is connected between two identical springs. The mass-spring system rests on a frictionless surface. A force of 0.025 N is needed to compress or extend the spring by 1.0 mm.

The mass is pulled from its equilibrium position to the right by 0.055 m and then released. The mass oscillates about the equilibrium position in simple harmonic motion.



The mass-spring system can be used to model the motion of an ion in a crystal lattice structure.



The frequency of the oscillation of the ion is  $8 \times 10^{12}$  Hz and the mass of the ion is  $6 \times 10^{-26}$  kg. The amplitude of the vibration of the ion is  $2 \times 10^{-11}$  m.

For the oscillations of the mass-spring system:

- (i) Calculate the acceleration of the mass at the moment of release [2]
- (ii) Estimate the maximum kinetic energy of the ion. [2]

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(4 marks)

(b) For the mass-spring system

(i) Calculate the total energy of the system.

[1]

(ii) Use the axes to sketch a graph showing the variation over time of the kinetic energy of the mass and the potential energy of the springs.

You should include appropriate values, and show the oscillation over one full period.



[2]

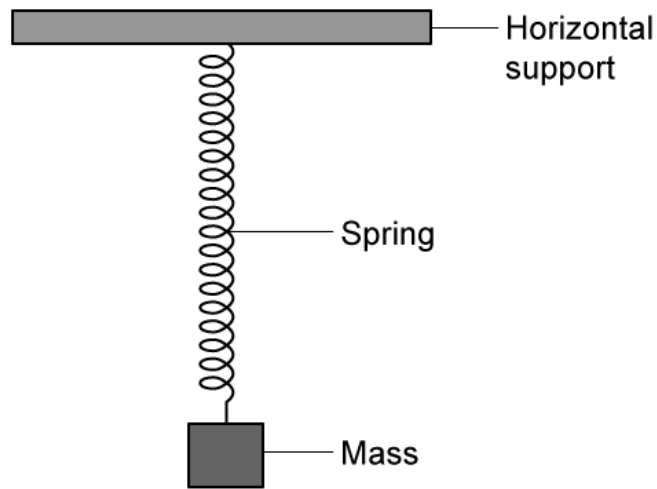
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(3 marks)

(c) The same mass and a single spring from part (a) are attached to a rigid horizontal support.



The length of the spring with the mass attached is 64 mm. The mass is pulled downwards until the length of the spring is 76 mm. The mass is released, and the vertical mass-spring system performs simple harmonic motion.

For the new mass-spring system

(i) Determine the velocity of the mass 2 seconds after its release. [1]

(ii) Determine the kinetic energy of the mass at this point. [2]

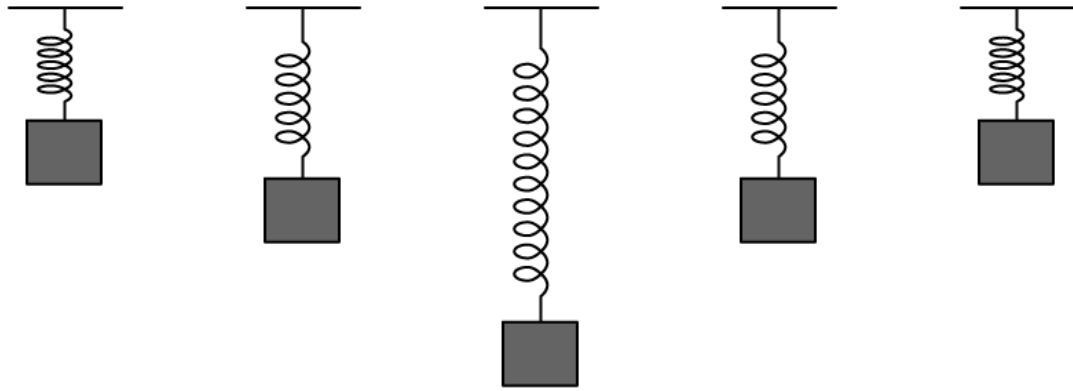
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**(3 marks)**

(d) The diagram shows the vertical spring-mass system as it moves through one period.



Annotate the diagram to show when:

$$E_p = \text{max}$$

$$E_k = \text{max}$$

$$v = 0$$

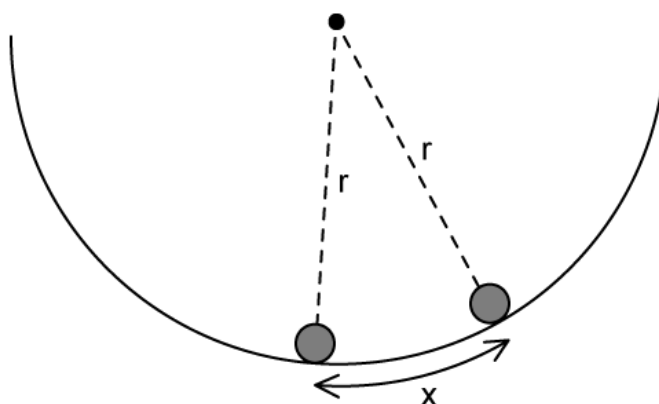
$$v = \text{max}$$

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(2 marks)

- 9 (a) A ball is displaced through a small distance  $x$  from the bottom of a bowl and then released.



The frequency of the resulting oscillation is  $1.5 \text{ Hz}$  and the maximum velocity reaches  $0.36 \text{ m s}^{-1}$ .  $r$  is the radius of the bowl.

For the oscillating ball:

- (i) Show that the radius of the bowl in which it oscillates is approximately  $11 \text{ cm}$ . [2]
- (ii) Calculate the amplitude of oscillation. [2]

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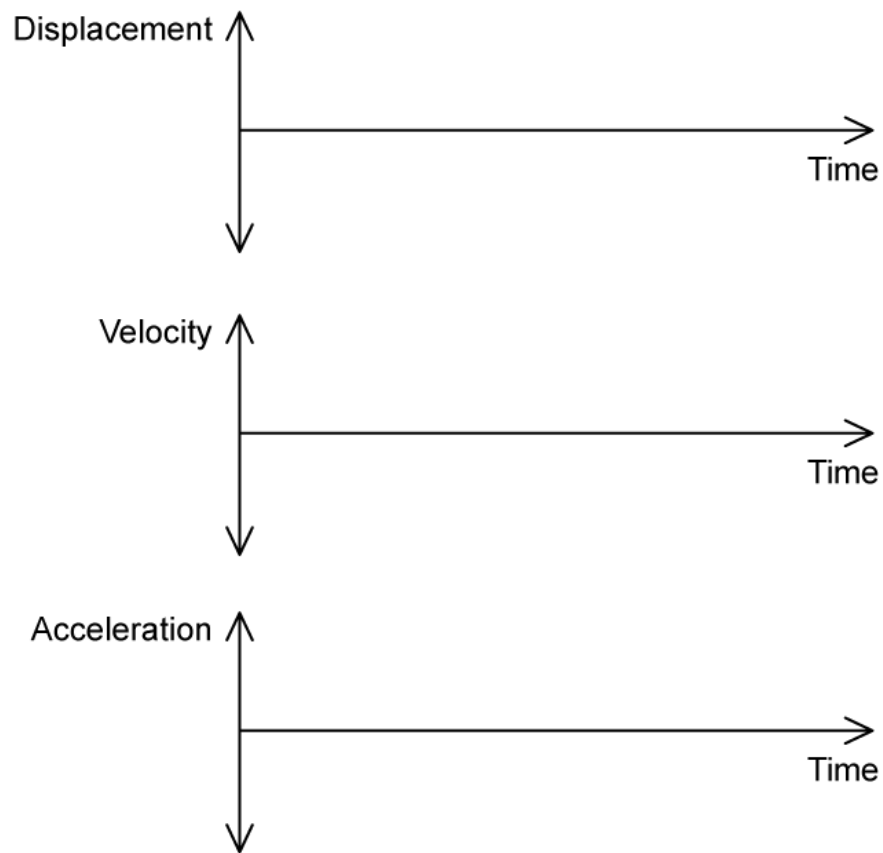
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**(4 marks)**

- (b) Sketch the graphs showing how the displacement, velocity and acceleration of the ball vary with time. You should include any relevant values.



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**(4 marks)**

**(c)** The ball was replaced by a ball of the same size, but with a greater mass.

Outline what effect this would have on the period of the oscillation.

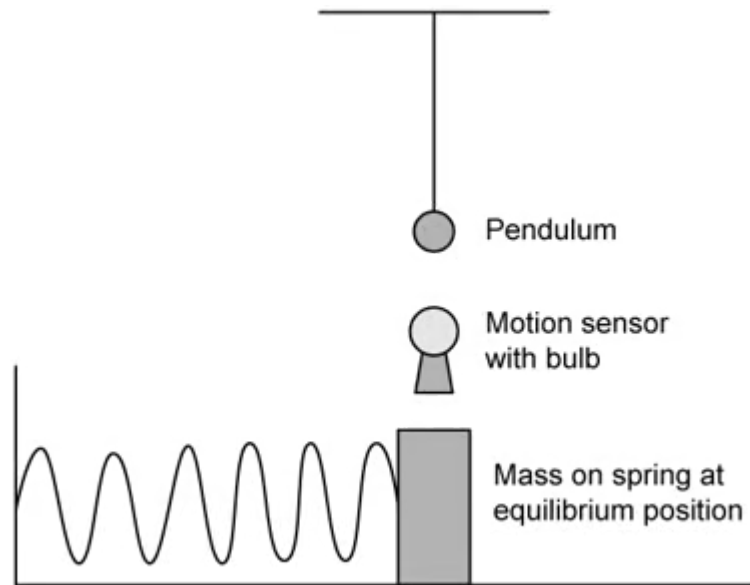
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**(2 marks)**



- 10 (a)** An experiment is carried out on Planet Z using a simple pendulum and a mass-spring system. The block moves horizontally on a frictionless surface. A motion sensor is placed above the equilibrium position of the block which lights up every time the block passes it.



The pendulum and the block are displaced from their equilibrium positions and oscillate with simple harmonic motion. The pendulum bob completes 150 full oscillations in seven minutes and the bulb lights up once every 0.70 seconds. The block has a mass of 349 g.

Show that the value of the spring constant  $k$  is approximately  $7 \text{ N m}^{-1}$ .

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**(2 marks)**

- (b)** The volume of Planet Z is the same as the volume of Earth, but Planet Z is twice as dense.

For the experiment on Planet Z

- (i) Considering the motion of both the spring and the pendulum, show that the length of the pendulum,  $l = \frac{4mg}{k}$
- [2]
- (ii) Calculate the value of  $l$ .

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**(4 marks)**

- (c)** Compare and contrast how performing the experiment on Planet Z, rather than on Earth, affects the period of the oscillations of the pendulum and the mass-spring system.

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**(2 marks)**

# Hard Questions

- 1 (a) A mass-spring system has been set up horizontally on the lab bench, so that the mass can oscillate.

The time period of the mass is given by the equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- (i) Calculate the spring constant of a spring attached to a mass of 0.7 kg and time period 1.4 s.

[1]

- (ii) Outline the condition under which the equation can be applied.

[1]

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(2 marks)

- (b) Sketch a velocity-displacement graph of the motion of the block as it undergoes simple harmonic motion.

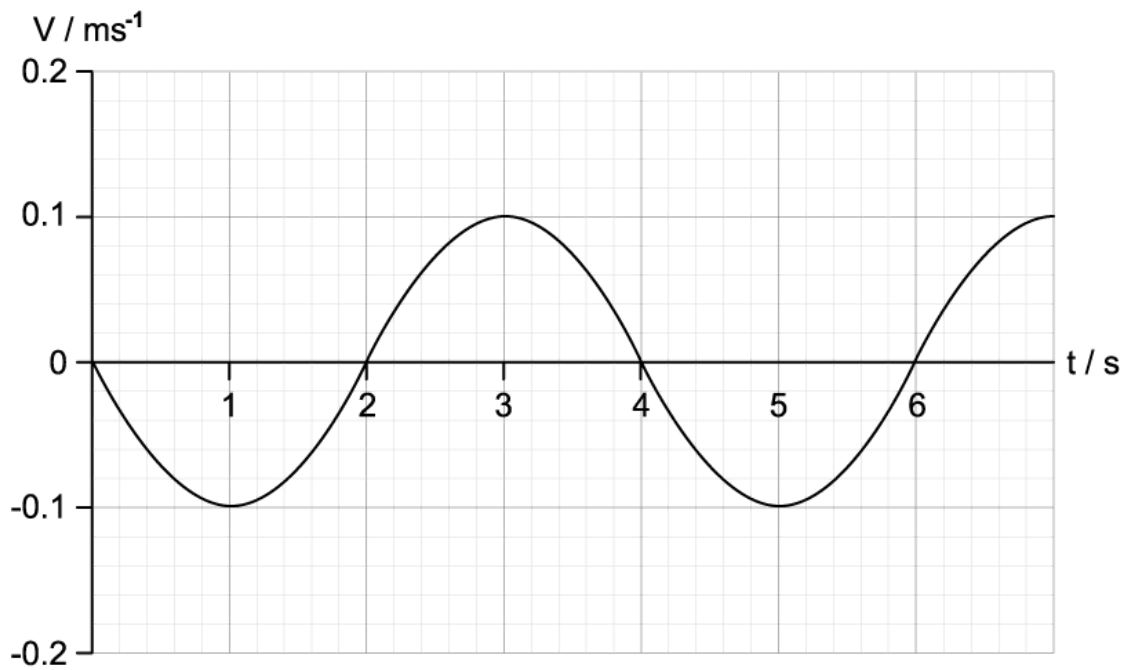
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(2 marks)

- (c) A new mass of  $m = 50$  g replaces the 0.7 kg mass and is now attached to the mass-spring system.

The graph shows the variation with time of the velocity of the block.



Determine the total energy of the system with this new mass.

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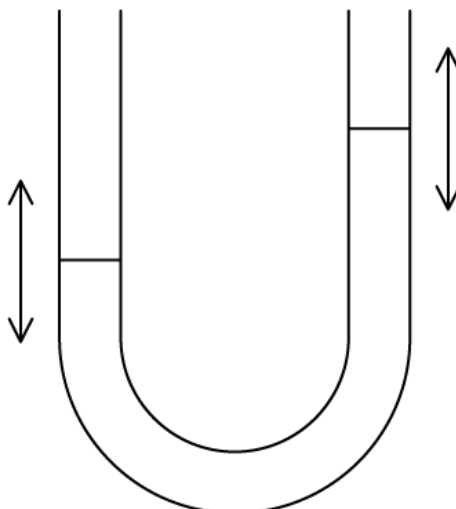
**(2 marks)**

**(d)** Determine the potential energy of the system when 6 seconds have passed.

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**(1 mark)**

2 (a) A volume of water in a U-shaped tube performs simple harmonic motion.



State and explain the phase difference between the displacement and the acceleration of the upper surface of the water.

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(2 marks)

(b) The U-tube is tipped and then set upright, to start the water oscillating. Over a period of a few minutes, a motion sensor attached to a data logger records the change in velocity from the moment the U-tube is tipped. Assume there is no friction in the tube.

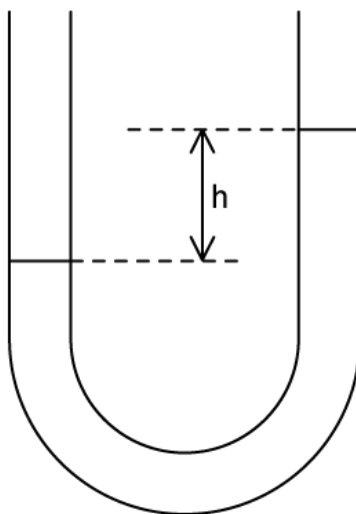
Sketch the graph the data logger would produce.

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(2 marks)

(c) The height difference between the two arms of the tube is  $h$ , and the density of the water is  $\rho$ .



Construct an equation to find the restoring force,  $F$ , for the motion and simplify as much as possible.

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**(3 marks)**

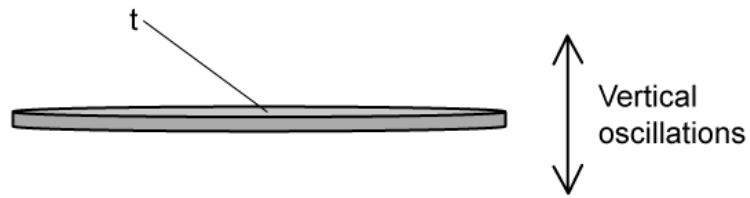
- (d) The time period of the oscillating water is given by  $T = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the height of the water column at equilibrium and  $g$  is the acceleration due to gravity.

If  $L$  is 15 cm, determine the frequency of the oscillations.

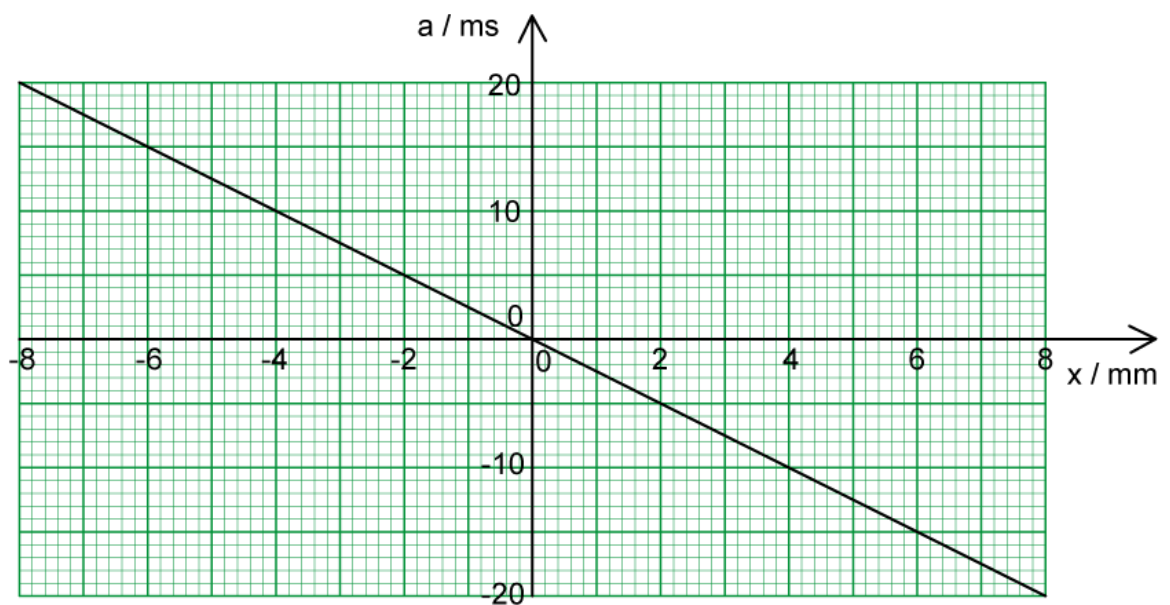
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**(1 mark)**

**3 (a)** The diagram shows a flat metal disk placed horizontally, that oscillates in the vertical plane.



The graph shows how the disk's acceleration,  $a$ , varies with displacement,  $x$ .



Show that the oscillations of the disk are an example of simple harmonic motion.

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**(3 marks)**

(b) Some grains of salt are placed onto the disk.

The amplitude of the oscillation is increased gradually from zero.

At amplitude  $A_z$ , the grains of salt are seen to lose contact with the metal disk.

(i) Determine and explain the acceleration of the disk when the grains of salt first lose contact with it. [3]

(ii) Deduce the value of amplitude  $A_z$ . [1]

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**(4 marks)**

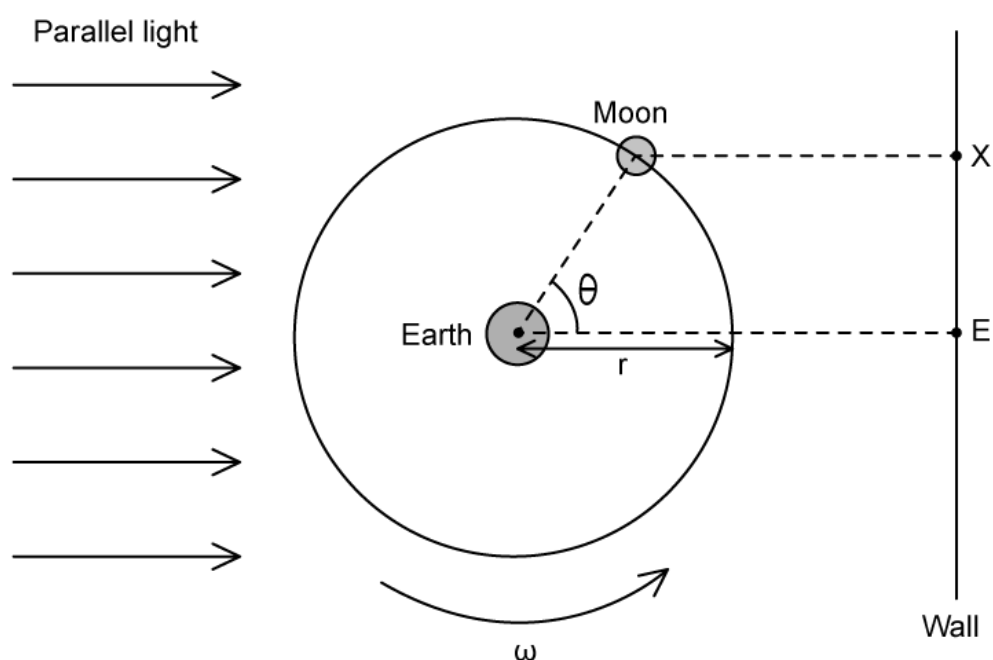


- 4 (a) For a homework project, some students constructed a model of the Moon orbiting the Earth to show the phases of the Moon.

The model was built upon a turntable with radius  $r$ , that rotates uniformly with an angular speed  $\omega$ .

The students positioned LED lights to provide parallel incident light that represented light from the Sun.

The diagram shows the model as viewed from above.



The students noticed that the shadow of the model Moon could be seen on the wall.

At time  $t = 0$ ,  $\theta = 0$  and the shadow of the model Moon could not be seen at position E as it passed through the shadow of the model Earth.

Some time later, the shadow of the model Moon could be seen at position X

For this model Moon and Earth

- (i) Construct an expression for  $\theta$  in terms of  $\omega$  and  $t$  [1]
- (ii) Derive an expression for the distance EX in terms of  $r$ ,  $\omega$  and  $t$  [1]

(iii) Describe the motion of the shadow of the Moon on the wall

[1]

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**(3 marks)**

**(b)** The diameter,  $d$ , of the turntable is 50 cm and it rotates with an angular speed,  $\omega$ , of  $2.3 \text{ rad s}^{-1}$ .

For the motion of the shadow of the model Moon, calculate:

(i) The amplitude,  $A$ .

[1]

(ii) The period,  $T$ .

[1]

(iii) The speed as the shadow passes through position E.

[2]

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**(4 marks)**

**(c)** The defining equation of SHM links acceleration,  $a$ , angular speed,  $\omega$ , and displacement,  $x$ .

$$a = -\omega^2 x$$

For the shadow of the model Moon:

(i)

Determine the magnitude of the acceleration when the shadow is instantaneously at rest.

[2]

- (ii) Without the use of a calculator, predict the change in the maximum acceleration if the angular speed was reduced by a factor of 4 and the diameter of the turntable was half of its original length.

[1]

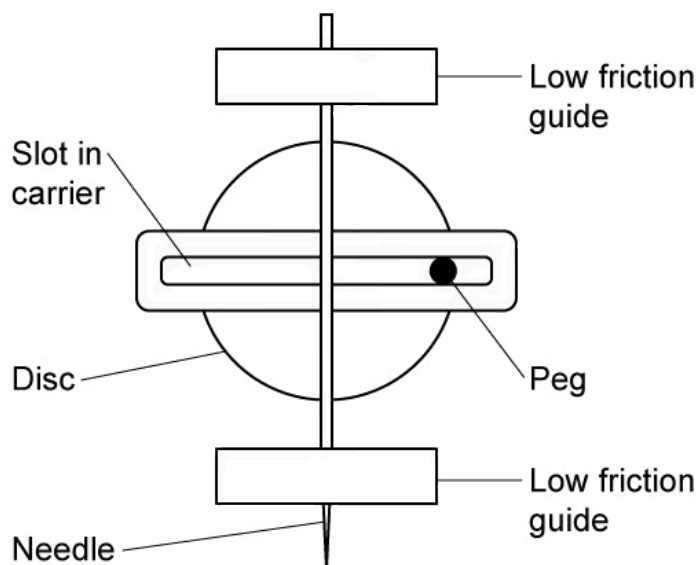
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**(3 marks)**

- 5 (a) The needle carrier of a sewing machine moves with simple harmonic motion. The needle carrier is constrained to move on a vertical line by low friction guides, whilst the disk and peg rotate in a circle. As the disk completes one oscillation, the needle completes one stitch.



The sewing machine completes 840 stitches in one minute. Calculate the angular speed of the peg.

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(2 marks)

- (b) The needle carrier has a mass of 23.9 g, and the needle has a mass of 0.7 g. The needle moves a distance of 36 mm between its extremities of movement.

Assuming that the fabric requires a negligible force for the needle to penetrate it:

- (i) Calculate the maximum speed of the needle. [1]
- (ii) Determine the kinetic energy of the needle carrier system, at this point. [1]

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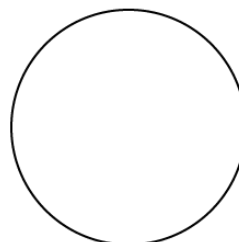


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(2 marks)

(c) For the needle-carrier system:

- (i) Label, on the diagram, the positions of the peg at the point of maximum velocity, and the points of maximum contact force of the peg on the slot.



Disc front view

- (ii) Calculate the maximum force acting on the peg by the slot.

[2]

[1]

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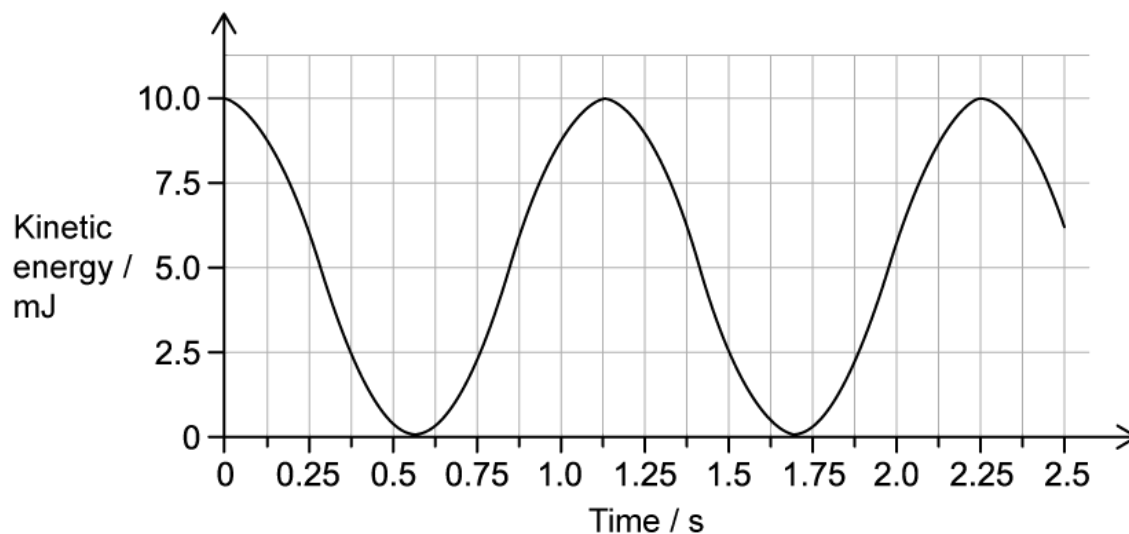
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(3 marks)

- 6 (a) A metal pendulum bob of mass 7.5 g is suspended from a fixed point by a length of thread with negligible mass. The pendulum is set in motion and oscillates with simple harmonic motion.

The graph shows the kinetic energy of the bob as a function of time.



Calculate the length of the thread.

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**(2 marks)**

(b) For the simple pendulum:

- (i) Label the graph from part a with an A at the point where the restoring force is acting at a maximum. [1]
- (ii) Label the graph from part a with a B at the point where the speed of the pendulum is half of its initial speed. [1]

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**(2 marks)**

(c) Show that the amplitude of the oscillation is around 0.6 m.

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**(2 marks)**

- 7 (a)** A steel spring with an unstretched length of 33 cm is attached to a fixed point and a mass of 35 g is attached and gently lowered until equilibrium is reached and the spring has a length of 37.5 cm. The spring is then stretched elastically to a length of 42 cm and released.

Design a plan to investigate if the oscillation is simple harmonic motion.

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**(5 marks)**

- (b)** For the stretching of the spring:

- (i) Calculate the gravitational potential energy lost by the mass. [1]
- (ii) Determine the elastic potential energy gained by the spring. [2]
- (iii) Explain why the two answers are different. [1]

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**(4 marks)**



(c) For the simple harmonic oscillation:

(i) Determine the resultant force acting on the load at the lowest point of its movement.

[2]

(ii) Calculate the maximum speed of the mass.

[2]

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**(4 marks)**

**8 (a)** A student with mass 68 kg hangs from a bungee cord with a spring constant,  $k = 270 \text{ N m}^{-1}$ . The student is pulled down to a point where the cord is 4.0 m longer than its unstretched length, and then released. The student oscillates with SHM.

For the student:

- (i) Determine their position 15.7 s after being released. [3]
- (ii) Calculate their velocity 15.7 s after being released. [1]
- (iii) Explain where in the oscillation the student is at 15.7 s after being released. You may want to include a sketch diagram to aid your explanation. [2]

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**(6 marks)**

**(b)** A second student wants to do the bungee jump, however, they would like a greater number of bounces in their five minute session.

Evaluate the possibilities for facilitating the student's wishes.

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**(1 mark)**