

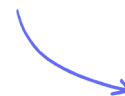
Structured Questions

Rigid Body Mechanics

Torque & Couples (HL) / Rotational Equilibrium (HL) / Angular Displacement, Velocity & Acceleration (HL) / Angular Acceleration Formula (HL) / Moment of Inertia (HL) / Newton's Second Law for Rotation (HL) / Angular Momentum (HL) / Angular Impulse (HL) / Rotational Kinetic Energy (HL)

Easy (5 questions)	/44
Medium (5 questions)	/50
Hard (5 questions)	/54
Total Marks	/148

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Easy Questions

1 (a) A disc rotates from rest with an angular acceleration of 5.0 rad s^{-2} for 3.0 seconds.

Calculate the final angular speed of the disc and state the correct unit.

(3 marks)

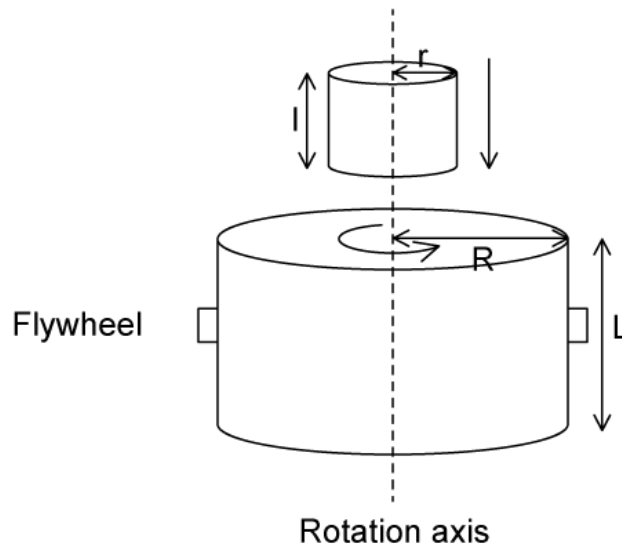
(b) Calculate the angular displacement of the disc after 3.0 s.

(2 marks)

(c) Calculate the number of complete revolutions that the disc has rotated through.

(2 marks)

- 2 (a) A typical flywheel from a van engine is made up of two cylinders modelled as attached point masses about an axle which creates the axis of rotation. The smaller cylinder has a radius r and a mass m . The larger cylinder has a radius R and a mass M .



State the formulae for the moment of inertia of the smaller cylinder, the larger cylinder and the entire system.

(3 marks)

- (b) The cylinders in the flywheel have the following measurements:

- $m = 5 \text{ kg}$
- $r = 10 \text{ cm}$
- $M = 25 \text{ kg}$
- $R = 45 \text{ cm}$

Calculate the total moment of inertia of the flywheel and state the units.

(3 marks)

- (c)** Explain how the moment of inertia of the flywheel can be increased without changing the axis of rotation.

(1 mark)

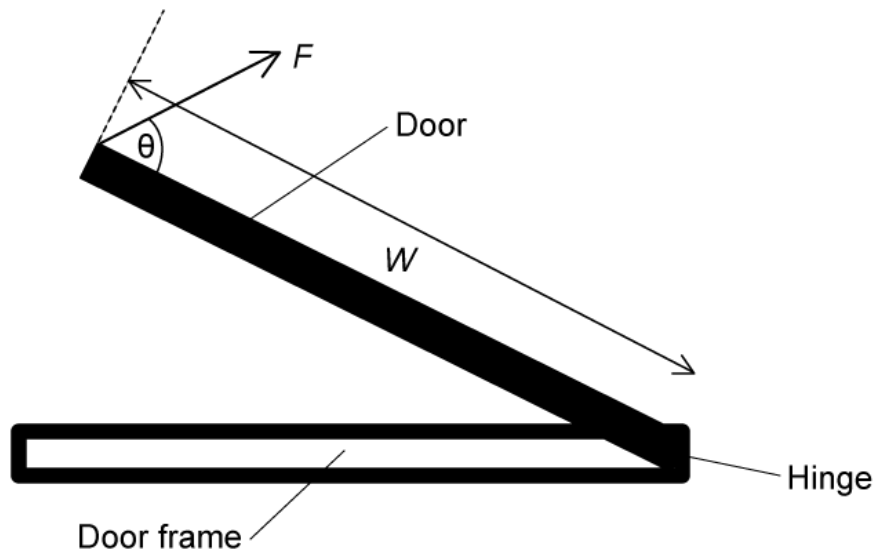
- (d)** The radius of each cylinder is smaller than the length such that:

$$r < l \text{ and } R < L$$

Identify, by drawing on the diagram from part (a), a new axis of rotation that would produce the greatest moment of inertia for the flywheel.

(2 marks)

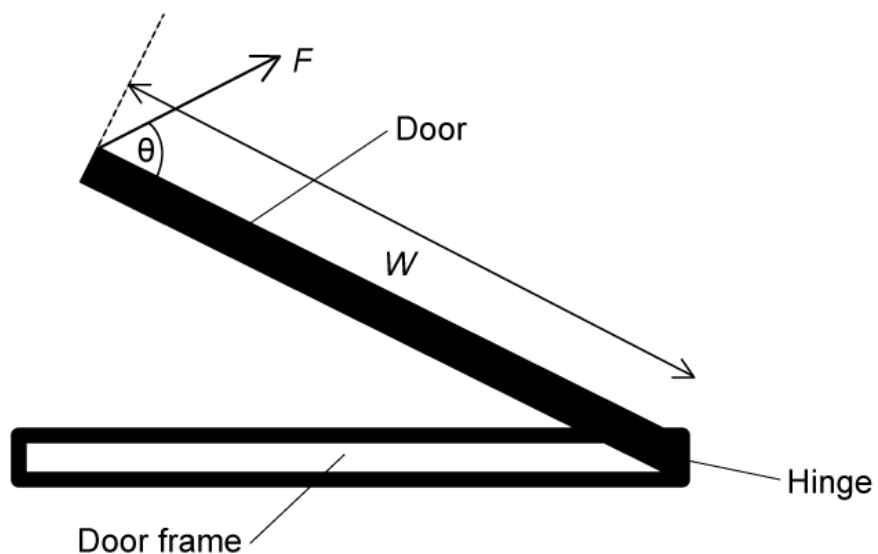
- 3 (a) A force, F is applied at right angles to the end of a door of width W . The door is attached to the door frame by hinges at the other end.



State the equation and units required to calculate the torque applied by the force on the door.

(1 mark)

- (b) The same force, F is now applied at an angle of θ° to the door.



State the new equation for the torque applied by the force and whether this is larger or smaller than the torque applied in part (a).

(3 marks)

- (c)** Force F has a magnitude of 35 N and is now applied at an angle of 50° to a wider door of width 1.5 m.

Calculate the magnitude of the torque applied to the wider door.

(2 marks)

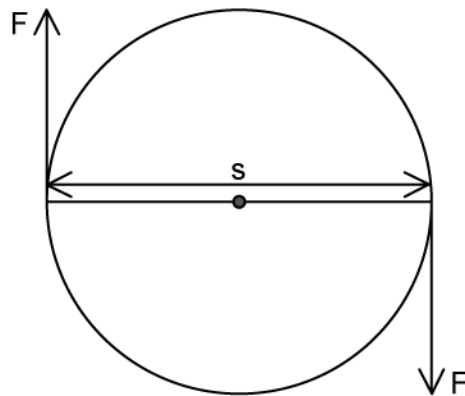
- (d)** Explain whether the torque applied to the wider door from part (c) is greater or smaller than the torque applied to the narrower door in part (b).

(2 marks)

4 (a) Define a couple.

(3 marks)

(b) A pair of forces act on a rotating wheel.



State the equation which can be used to determine the torque.

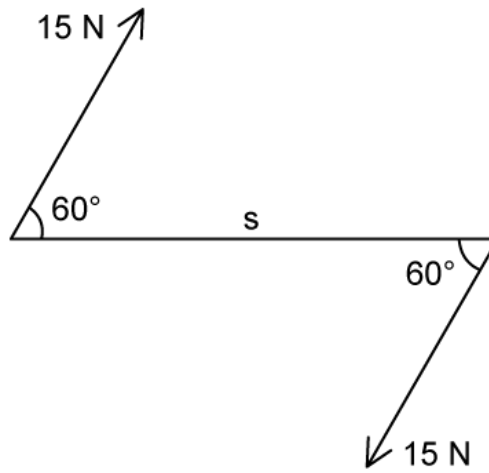
(1 mark)

(c) For the couple in part (b) the two forces, F are both 20 N and the perpendicular distance, s , between them is 0.3 m.

Calculate the torque of this couple.

(2 marks)

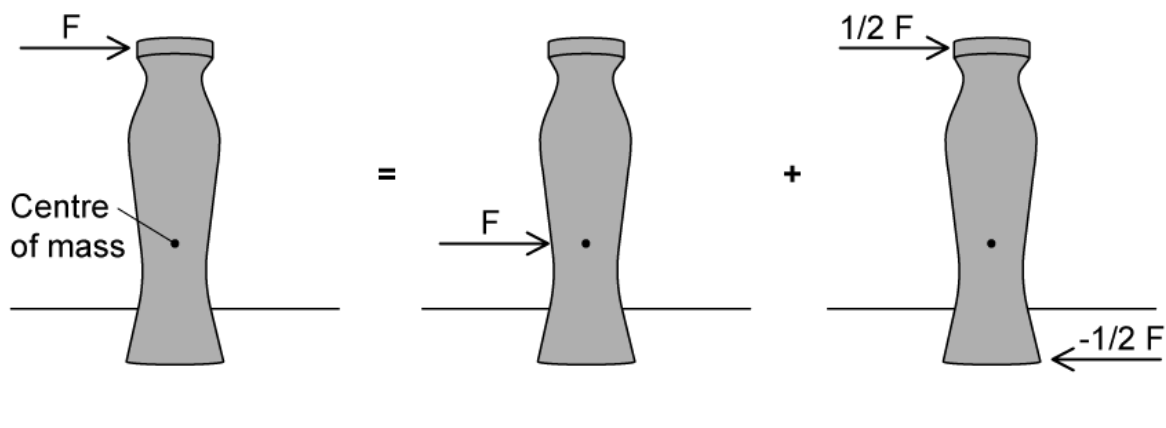
(d) The couple of forces from part (b) are changed so that they now have a magnitude of 15 N and act at an angle of 60° to the horizontal distance between them.



Calculate the new torque on the wheel.

(3 marks)

- 5 (a) A glass vase is left drying upside down on the floor. A force, F is applied at right angles to the base of the vase causing the vase to move. The resultant motion can be broken down into two components of translation and rotation.



Identify the two components by labelling the diagrams as either translation or rotation.

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(2 marks)

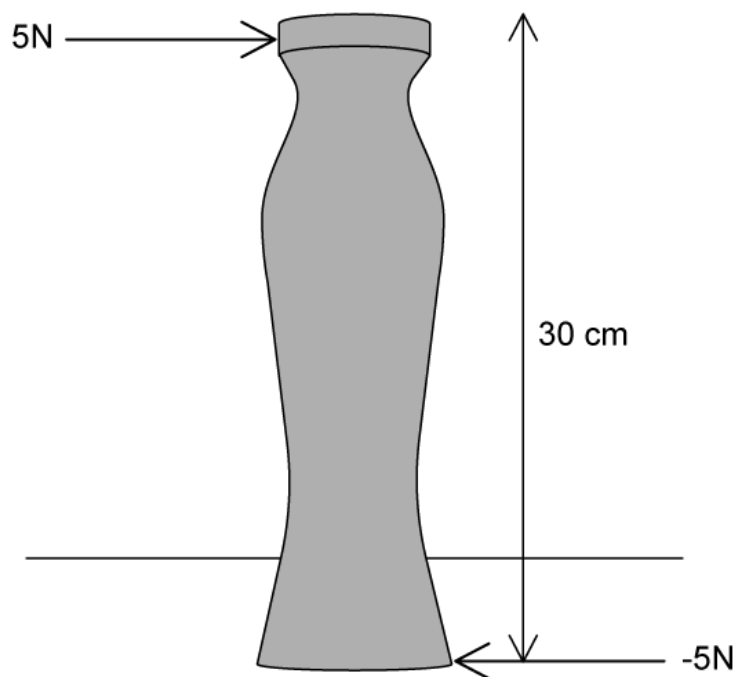
- (b) State the principle of moments and Newton's second law for an object in rotational equilibrium.

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(2 marks)

- (c) Two equal and opposite forces of magnitude 5 N are applied to the top and bottom of the vase by a magician. The vase's resultant motion starts from rest and it makes one complete rotation in 1.5 seconds. The vase has a height of 30 cm.



Calculate the angular acceleration of the vase during this motion.

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(3 marks)

(d) Calculate the moment of inertia of the vase.

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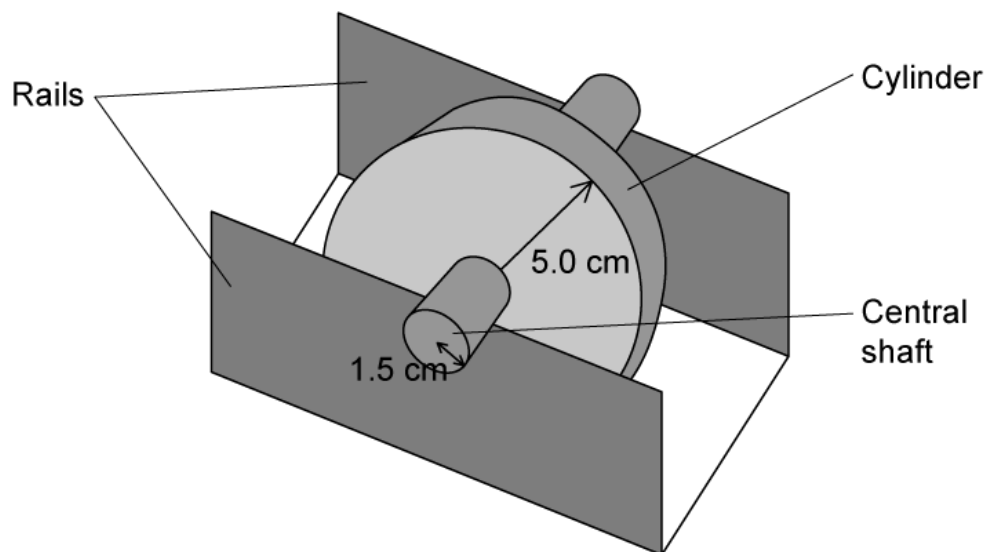
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(4 marks)

Medium Questions

- 1 (a) Part of a delivery system in a factory consists of a freely rotating cylinder attached to a central shaft that sits on top of rails. The cylinder has a mass of 0.60 kg and the central shaft a mass of 0.30 kg. The radius of the central shaft is 1.5 cm and the radius of the cylinder is 5.0 cm. The cylinder and the central shaft have a linear velocity of 10 m s^{-1} .



Calculate the angular velocity of the central shaft and the cylinder.

(2 marks)

- (b) Explain why the central shaft and the cylinder are travelling at the same linear velocity but different angular velocities.

(2 marks)

- (c) The central shaft can be modelled as a cylinder that passes through the hollow centre of the cylinder.

Moment of inertia of a solid cylinder, $I = \frac{1}{2}mR^2$

Moment of inertia of a hollow cylinder, $I = \frac{1}{2}m(A^2 + B^2)$ where A is the radius of the hollow and B is the radius of the outer part.

Calculate the moment of inertia of the central shaft and cylinder.

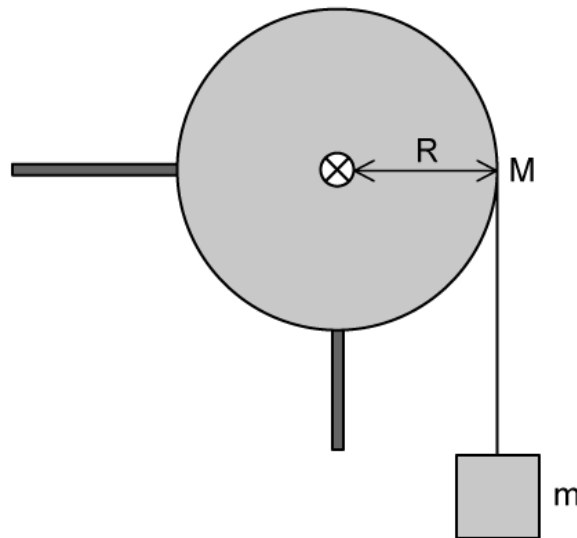
(3 marks)

(d) Calculate the angular momentum of the cylinder and the central shaft.

(2 marks)

- 2 (a)** A block of mass 10 g is attached to a light inextensible string hooked around a cylindrical pulley of mass 5.0 kg and radius 0.30 m.

Moment of inertia of a solid cylinder, $I = \frac{1}{2}mR^2$



When the block is released the pulley begins to turn as the block falls.

Calculate the moment of inertia of the cylindrical pulley.

(2 marks)

- (b)** The linear acceleration of the block as it begins to fall is 2.4 m s^{-2} .

Calculate the angular acceleration of the pulley as it begins to turn.

(2 marks)

- (c)** Determine the value of the tension in the string.

(4 marks)

(d) Calculate the angular velocity of the pulley after it has rotated 180° .

(2 marks)

- 3 (a)** The moment of inertia of a hollow sphere is $I = \frac{2}{3}mr^2$ where m is the mass of the sphere and r is the radius.

Show that the total kinetic energy of the sphere E_k when it rolls without slipping at an angular velocity ω is $E_k = \frac{5}{6}mr^2\omega^2$.

(4 marks)

- (b)** The hollow sphere has a mass of 0.80 kg and a radius of 30 cm. It rolls without slipping for 20 seconds making 50 rotations.

Calculate the angular velocity.

(2 marks)

- (c)** After making 50 rotations the sphere encounters a ramp in its path and starts rolling without slipping up its smooth inclined surface.

Determine the maximum height the sphere reaches as it travels up the ramp.

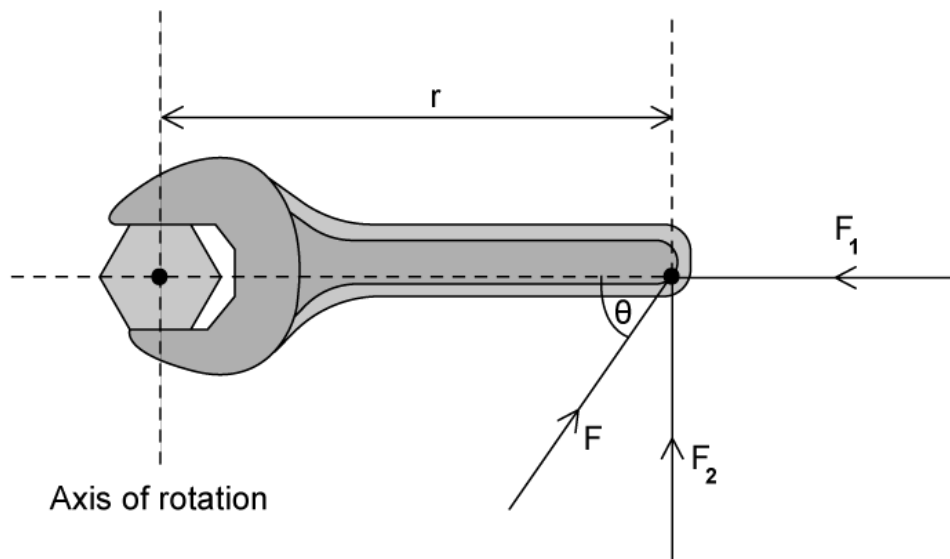
(4 marks)

(d) It takes the sphere 15 seconds to reach its maximum height on the ramp.

Calculate the angular acceleration of the sphere as it rolls up the ramp.

(3 marks)

4 (a) A force of 50.0 N is applied at an angle of 43° to a spanner of length 25 cm.



Calculate the torque provided by this force on the spanner.

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(3 marks)

(b) The person applying the force to the spanner changes their grip to obtain a bigger torque. The magnitude of the force applied remains the same.

State and give a reason for the angle the force should be applied to the spanner to obtain its maximum possible value.

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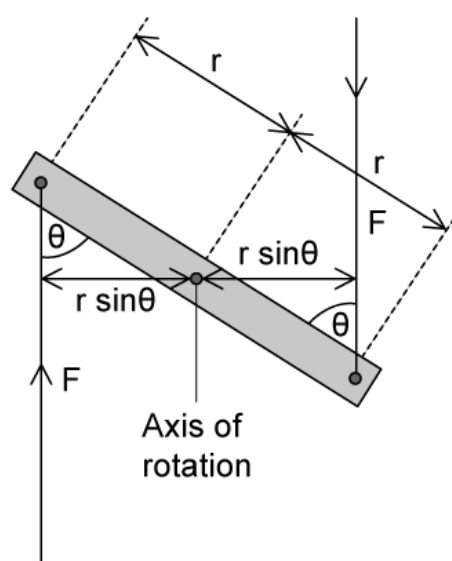
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(3 marks)

(c) Give two examples of couples.

(2 marks)

- (d) Turning a uniform metal bar of length 23.5 cm rotates the mechanism that opens and closes a mains water tap. A couple of forces of magnitude 175 N each are applied at an angle of 75.0° to the bar.



Calculate the torque provided by the couple.

(2 marks)

- 5 (a) A 3.5 kg mass hangs vertically from a string fixed 2.8 m above. The mass is free to swing from side to side.

Calculate the moment of inertia of the mass.

(2 marks)

- (b) State the assumptions made in your calculation of the moment of inertia in part (a).

(2 marks)

- (c) The mass is set oscillating and it makes one complete oscillation of $\frac{2}{3}\pi$ radians in 2.4 seconds.

Determine the angular velocity of the mass.

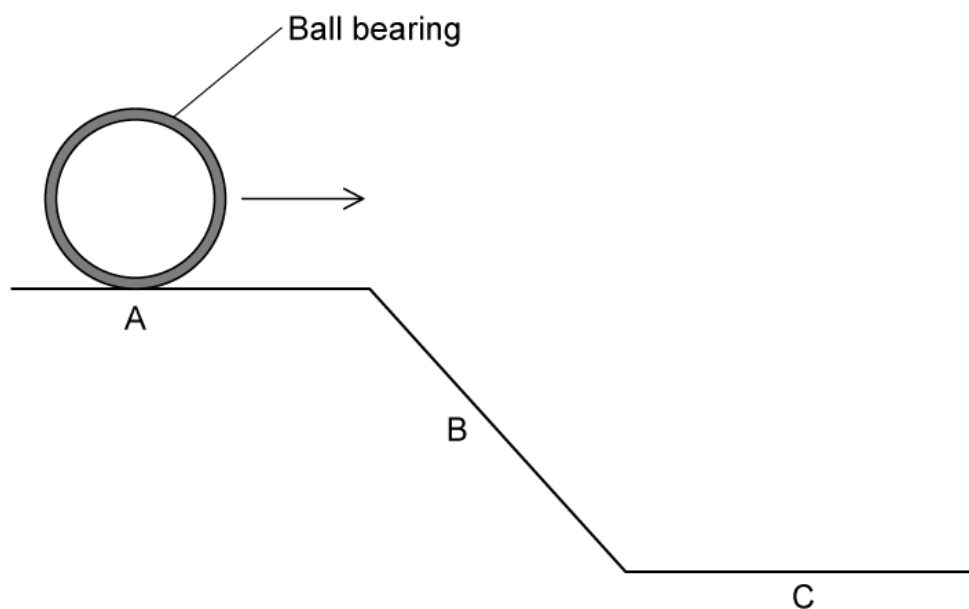
(2 marks)

- (d) Calculate the angular momentum of the mass.

(2 marks)

Hard Questions

- 1 (a) A ball bearing, modelled as a uniform solid sphere, travels with constant angular velocity along a smooth horizontal surface before rolling down a uniform inclined plane. It moves with a constant angular velocity again when it reaches a second smooth horizontal surface.



Sketch a graph to show the variation with time of the angular velocity during its journey.

(2 marks)

- (b) The ball bearing is now placed at the top of the slope and released from rest before it rolls down the slope to the bottom.

Assuming no energy is lost, state an equation that represents the total energy of the ball bearing at the bottom of the slope.

(1 mark)

- (c) The ball bearing has a diameter of 5.0 mm and a mass of 4.0 g. At the bottom of the slope its linear velocity is 12 m s^{-1} .

Moment of inertia of a solid sphere: $I = \frac{2}{5}mr^2$

Calculate the rotational kinetic energy of the ball bearing at the bottom of the slope.

(3 marks)

- (d) Calculate the height of the slope.

(2 marks)

- 2 (a)** A hollow sphere of radius 650 mm rolls down an inclined plane of length 5.0 m in 4.3 seconds. Assume that the sphere does not slip.

Moment of inertia of hollow sphere, $I = \frac{2}{3}mR^2$

Calculate the linear velocity of the sphere as it reaches the bottom of the plane to three significant figures.

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(2 marks)

- (b)** Show that the total kinetic energy, E_{ktotal} of the sphere at the bottom of the plane is given by the equation:

$$E_{ktotal} = \frac{5}{6}m(\omega R)^2$$

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(3 marks)

- (c)** Determine the angle of incline of the plane to an appropriate degree of accuracy.

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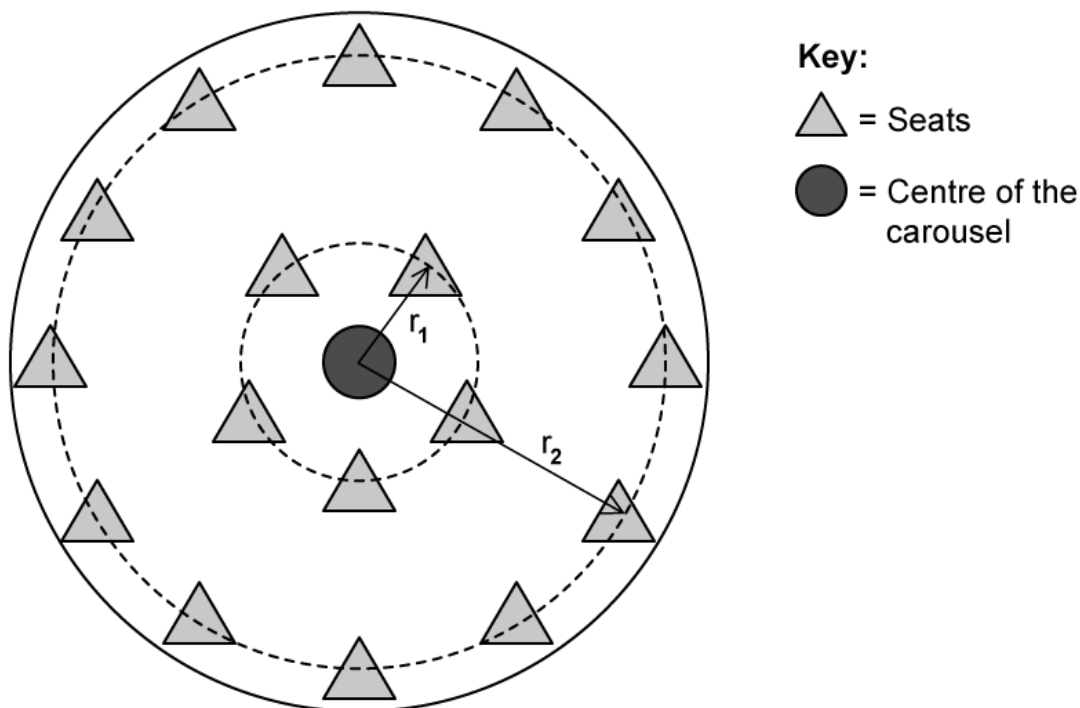
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(4 marks)

- (d)** Comment on whether the assumption that the sphere does not slip is valid in this situation.

(2 marks)

- 3 (a) A carousel ride consists of two rings of seats at different distances r_1 and r_2 from the centre of the carousel.



After starting from rest the carousel accelerates to a certain angular velocity when the torque of the motor that rotates the carousel is equal to the torque of the forces opposing the motion. The carousel then makes 3.0 revolutions in 10 minutes.

Obtain an expression for the linear velocity of the passengers sitting in ring 2.

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(3 marks)

- (b) It takes 30 seconds for the carousel to accelerate to its constant angular velocity. If $r_1 = \frac{1}{3}r_2$, show that the linear acceleration of passengers in ring 1 compared to ring 2 is given by the expression $a_2 = \frac{r_1 \pi}{1000}$.

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(2 marks)

- (c)** After 10 minutes on the carousel some of the passengers sitting in ring 1 move to ring 2.

Explain how this could affect the overall angular velocity of the carousel.

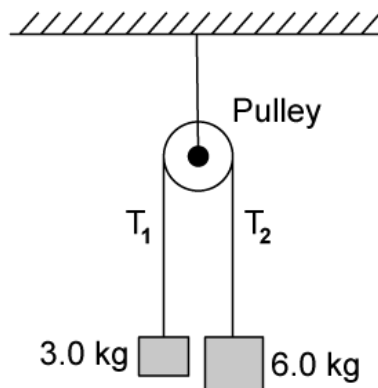
(3 marks)

- (d)** The movement of the passengers from r_1 to r_2 takes 20 seconds and creates an overall rate of change of moment of inertia of $9 \text{ kg m}^2 \text{ s}^{-1}$ on the carousel.

Calculate the new angular velocity of the carousel after the passengers have moved.

(3 marks)

- 4 (a) In a counterweight system, two wooden blocks of mass 3.0 kg and 6.0 kg are connected with a light inextensible string that passes over a smooth pulley. The pulley has a diameter of 40 cm and a mass of 20 kg.



The masses are released from rest and tensions T_1 and T_2 are created on the strings attached to the masses of 3.0 kg and 6.0 kg respectively. The acceleration of the 6.0 kg mass is 1.1 m s^{-2} .

Show that the pulley is in translational but not rotational equilibrium.

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(4 marks)

- (b) The system starts from rest and the angular velocity after 5.0 seconds is 3.5 rad s^{-1} .

Calculate the number of revolutions made by the pulley in this time.

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(3 marks)

(c) Sketch a graph and indicate the known values to show the variation of torque τ on the pulley against time for the motion described in part (b).

(3 marks)

(d) Calculate the angular impulse of the pulley from $t = 0$ s to $t = 5$ s.

(2 marks)

- 5 (a)** Students are performing tricks with pool balls and cues. A ball of diameter 7.5 cm is hit horizontally with a cue and rolls without slipping along the smooth horizontal pool table before rolling up a ramp. The pool ball has a mass of 0.21 kg and a moment of inertia of $\frac{2}{5}mr^2$. The average acceleration of the pool ball during the impact is 163 m s^{-2} .

Calculate the average net torque applied to the pool ball during the impact.

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(3 marks)

- (b)** The pool ball is in contact with the cue for 3.2 ms.

Calculate the total kinetic energy of the pool ball after impact.

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(4 marks)

- (c)** Immediately following the impact the pool ball starts to roll up a smooth uniform ramp placed directly in its line of motion. Air resistance is negligible. Determine the maximum vertical height reached by the pool ball.

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(2 marks)

- (d)** After reaching its maximum height the ball rolls back down the ramp.

Determine the angular velocity of the ball at the bottom of the ramp.

(3 marks)