HL IB Physics

Your notes

Galilean & Special Relativity

Contents

Your notes

Reference Frames (HL)

Reference Frames

- The term **relative** is used often in Physics to make it clear which point of view we are referring to
  - For example, the velocity of a car **relative** to someone stationary is different to the velocity measured by another car travelling alongside the initial car at the same speed
- A **reference frame**, or a **frame of reference**, refers to the **position** of an object, it is defined as:

A set of coordinates to record the position and time of events

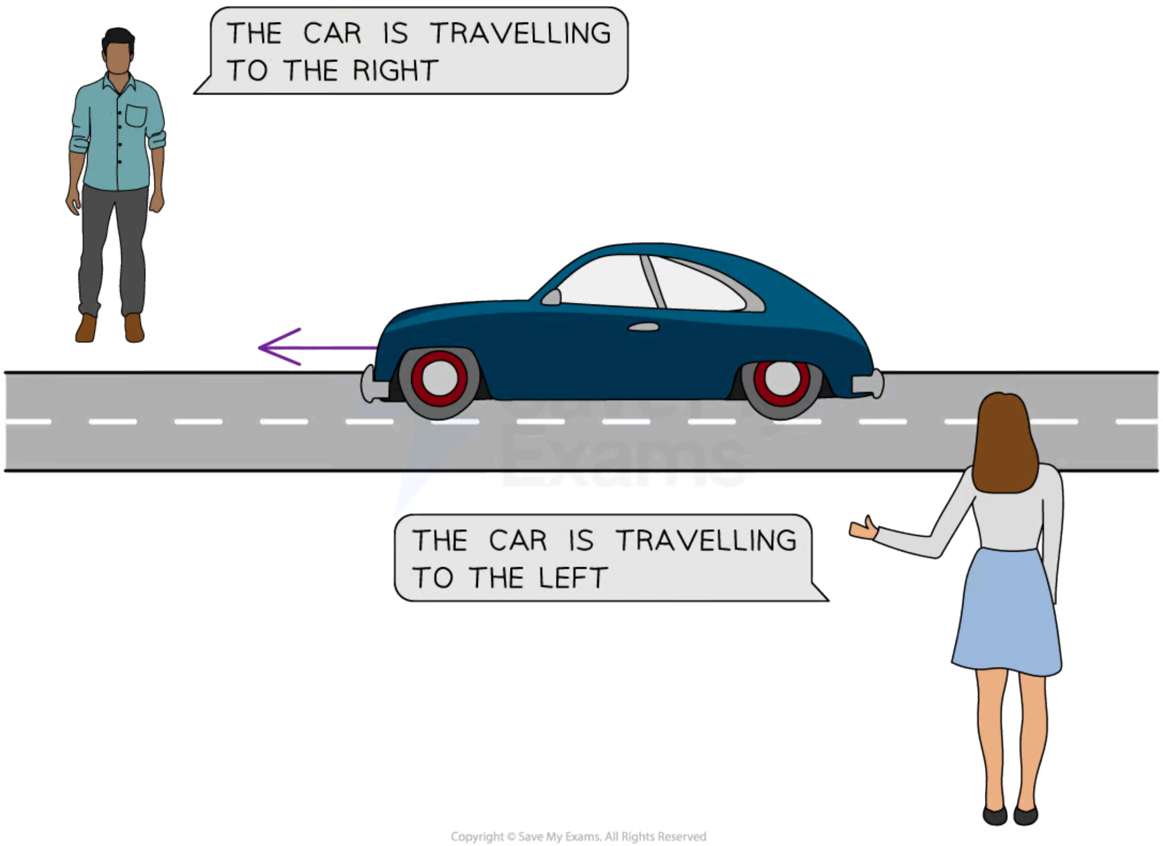
- For example, you currently sitting on your chair at your desk is your current reference frame
  - You feel as if you are stationary, despite the fact the Earth is revolving on its axis and orbiting the sun
- It is the point of view where an object, at a specific co-ordinate, is **at rest**

Examples of Reference Frames

- An everyday example is the **direction** of an object from your point of view in comparison to someone else
- In this example, a car is driving down a road and two people are standing on opposite sides of that road
- Despite the car moving in one direction, each person will view its direction **relative to them** differently
  - The person on one side of the road would say the car is moving to the right, and the person on the other side of the road would say the car is moving to the left
  - Both are correct, but they are viewing the car's motion from **different points of reference**

Diagram showing different points of reference for a moving car

Your notes

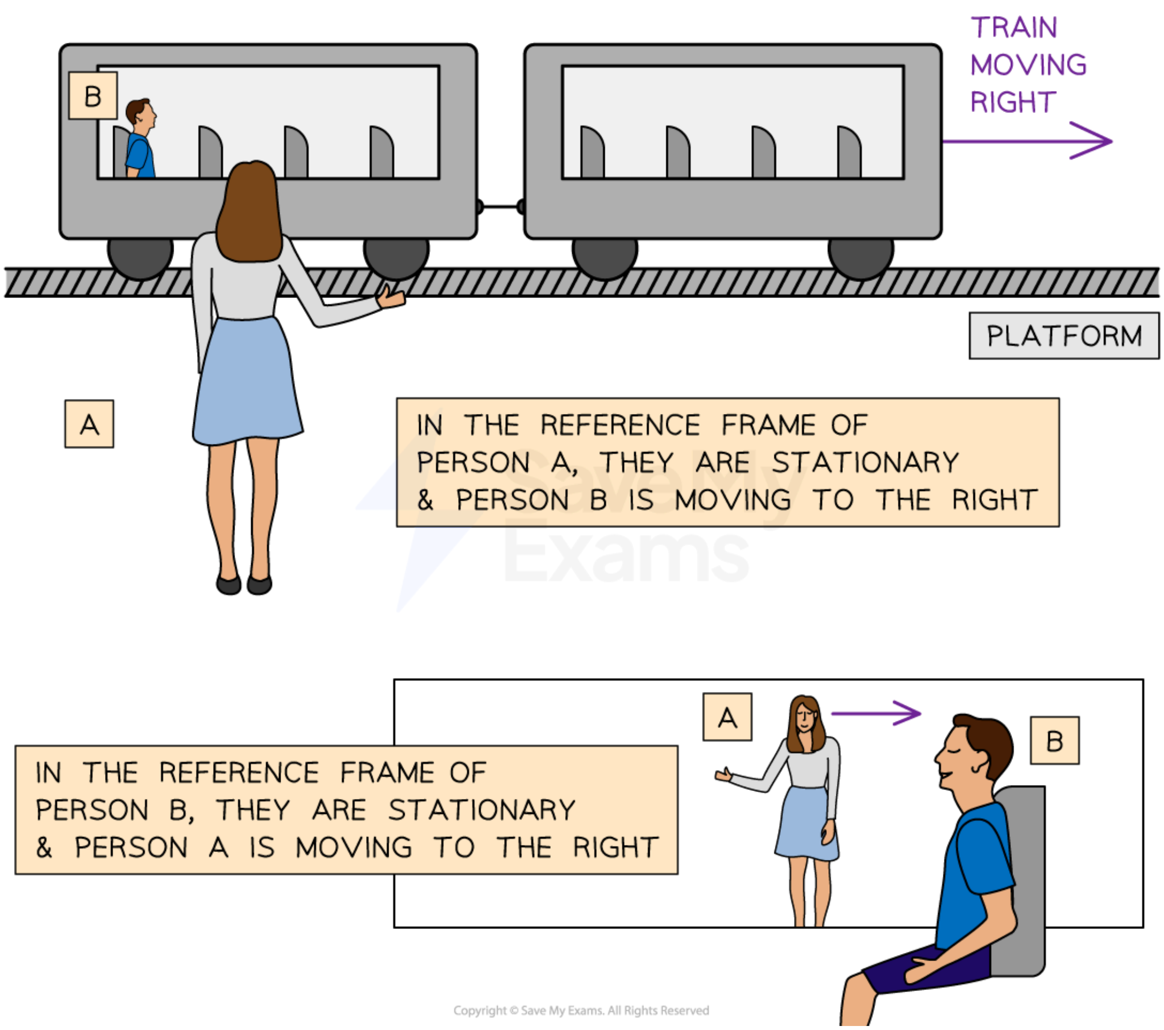


Each person has a different frame of reference, so they interpret the direction of the car differently relative to themselves

- Another common example is of a train pulling out of a station, where Person A is on the platform and Person B is on the train
- As the train begins to move, Person A, on the platform, views Person B, on the train, moving to the right
  - Therefore, according to Person A, they, themselves, are stationary and Person B is moving to the right
- Things look a little different from Person B's perspective
- As the train begins to move (to the left from Person B's perspective), Person B, on the train, views Person A, on the platform, moving to the right
  - Therefore, according to Person B, they, themselves, are stationary (as they cannot feel the train moving to the left) and Person A is moving to the right

Diagram demonstrating different reference frames for a train leaving a station

Your notes



Person A and B are both stationary in their own reference frames and see the other as moving

- Therefore, frames of reference are used to specify the relationship between a moving and stationary object

Inertial Frames of Reference

- An **inertial** reference frame is
  - **A reference frame that is non-accelerating**
- Therefore, all inertial reference frames are moving at **constant velocity** with **respect** to each other
- There is no such thing as an absolute reference frame in our Universe
  - In other words, there is no place in the Universe that is completely stationary
  - Everything is always moving relative to everything else

Worked example

A student is cycling to school with their friend who is also cycling exactly in line. As they cycle past a bus stop, they wave to their aunt who is stationary at the bus stop as she waits for her bus.

The student's aunt estimates the speed of the students to be 5 m s^{-1}.

At what speed would the friend measure the student to be travelling?

A 5 m s^{-1}

B -5 m s^{-1}

C 0 m s^{-1}

D 2 m s^{-1}

Answer:

The correct answer is **C** because:

- We must think about the friend's reference frame for this question, in which they are stationary (according to them)
- Since the friend is cycling **in line** with the student, this means they measure the student to be travelling at **0 m s^{-1}** relative to them

Examiner Tip

In exam questions, look out for terms such as 'for the reference frame of...', 'in the reference frame of...' or 'relative to ...' to know which reference frame is being referred to. You can think of it as 'What do they see from their **point of view?'.** This becomes important when you learn about Galilean relativity and Lorentz transformations.

You will not come across non-inertial reference frames (i.e. ones where a frame is accelerating) in your exam.

Your notes

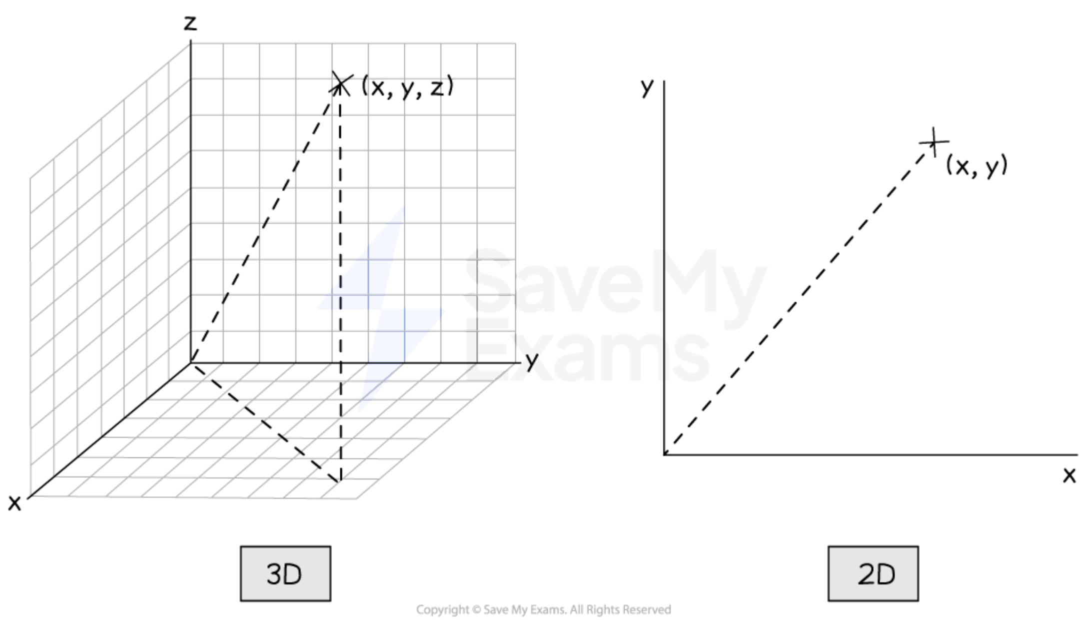
Galilean Relativity (HL)

Your notes

Newton's Postulates of Time & Space

- We use **inertial** reference frames because Newton's laws of motion are the **same** in all of them
  - This is known as **Galilean Relativity**
- For example, an object in an inertial reference frame will continue moving in a straight line with constant velocity unless acted upon by a force
  - This is in accordance with Newton's first law of motion
- This means that the **same** laws of Physics apply, regardless of one's frame of reference relative to another, as long as they are moving in a **straight line** at a **constant velocity**
- For an object moving with constant velocity in one reference frame, it will still have a constant (but **different**) velocity in another reference frame
- The Cartesian coordinate system is generally used for reference frames

Cartesian co-ordinates in 3D and 2D diagram



Cartesian coordinates are used to represent a point in space

- Although there are an infinite number of inertial frames of reference in the Universe, there are ways to move between them

Examiner Tip

Remember, anything that is moving in a curved line is **accelerating**. Therefore, you will not come across reference frames of something moving in a circle or an arc, but only straight lines with no acceleration.

Cartesian co-ordinates are technically used to refer to a point in 3D (x, y, z), although, your exam questions will focus on movement in 2D (x, y) as this is easier to draw diagrams for.

Your notes

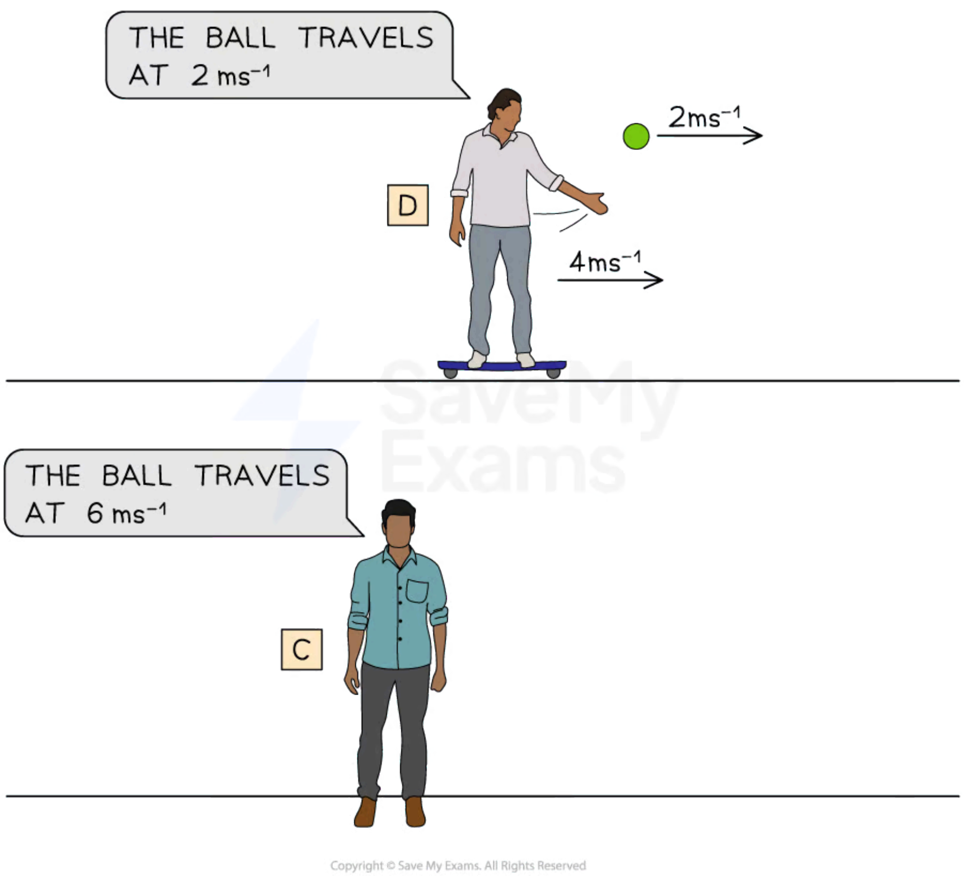
Your notes

Galilean Relativity Equations

Galilean Transformation Equations

- **Galilean transformation equations** are used to **convert** between coordinates of space and time in one frame of reference to another for an event
  - This is because the **velocity**, **position** and **time** of an event appear differently from different reference frames
- For example, Person D is on a skateboard travelling at 4 m s^{-1} when they throw a ball in a straight line at a constant velocity of 2 m s^{-1}. Person C is a stationary observer of the event.
  - In Person D's reference frame, the ball is travelling at 2 m s^{-1}
  - In Person C's reference frame, the ball is travelling at 4 + 2 = 6 m s^{-1}
- So what is the speed of the ball? Well, it depends!

Diagram showing the difference in velocity of an object in two reference frames

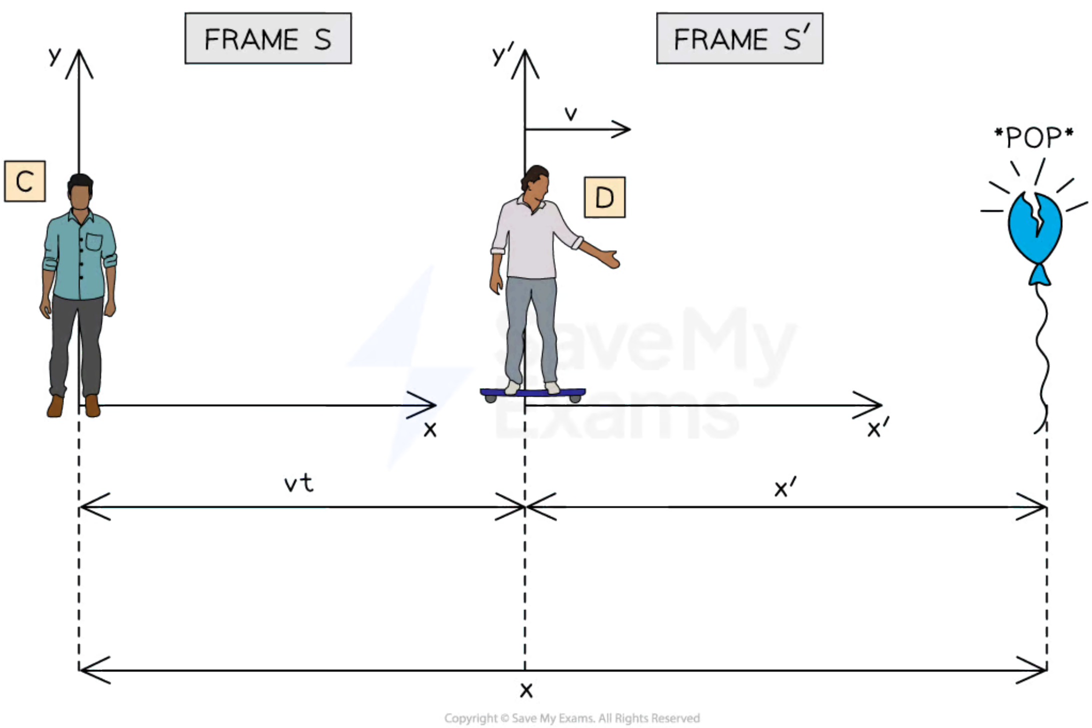


Person C measures the ball to be travelling faster than when measured by Person D

- Mathematically, the position and time of an event in one reference frame, S, is represented by the coordinates (x, y, t)

- The position and time of an event in a **second** reference frame, moving **relative** to the first one, S', is represented by the co-ordinates (*x', y', t'*)
  - Remember from the train example in Reference Frames, both Person A and Person B view the other as moving because they are viewing the event from two different frames of reference
- Now consider another example:
- Person C is stationary whilst Person D is moving away from Person C at velocity *v*
- Both Person C and Person D witness a balloon pop at some distance away
  - Person D, in reference frame S', measures the balloon pop at a distance ***x'*** away
  - Person C, in reference frame S, measures the balloon pop at a distance ***x = x' + vt*** away

Diagram showing a stationary and moving reference frame



Person C and D measure the distance at which a balloon pops differently

- Now let's see the event from Person D's frame of reference (S')
- From Person D's point of view, they are stationary and it is Person C that is travelling **away** from them at speed *v*
  - Since this is the opposite direction to the *v* observed by Person C, Person C's velocity from Person D's perspective is **–*v***
  - Therefore, Person D measures the balloon to pop at a distance ***x' = x – vt*** away
- Remember the *vt* comes from distance (*x*) = speed (*v*) × time (*t*).
- In both frames of reference, the time the balloon pops are still the same, hence ***t = t'***

- In summary:

Table of the Galilean relativity equations for a stationary and moving reference frame

Stationary reference frame	Moving reference frame	Transformation
x	x'	$x' = x - vt$ $x = x' + vt$
y	y'	$y = y'$
z	z'	$z = z'$
t	t'	$t = t'$

- Co-ordinates *y* and *z* are also the same in both reference frames because the relative motion is only in the ***x*** direction

Your notes

Your notes

Worked example

A train travels through a station at a constant velocity of 15 m s^{-1}. One observer is sitting inside the train and another sits on the platform. As they pass each other, they start their stopwatches and watch a child on the train run at a constant speed in the same direction as the motion of the train. The observer on the train measures the speed of the child to be 2 m s^{-1}.

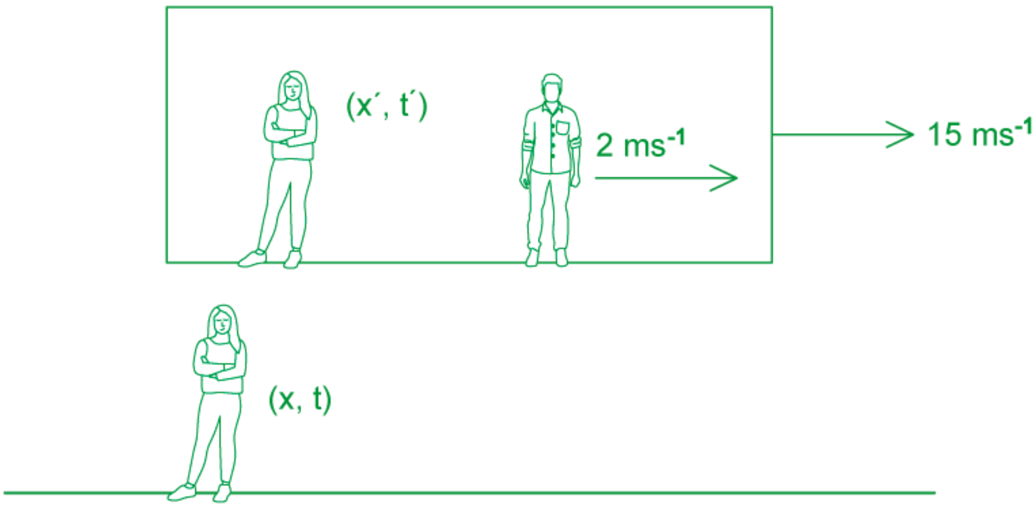
(a) According to the observer on the train, how far has the child moved after 10 s?

(b) According to the observer on the platform, how far has the child moved after 10 s?

Answer:

Draw a quick sketch of the situation

- Label the stationary (x, t) and moving (x', t') reference frames



(a) Calculate the distance the child travels according to the observer on the train

- The observer measures the child to be travelling away at 2 m s^{-1}
- Therefore, using the equation

$$\text{distance } (x) = \text{speed} \times \text{time}$$

- The distance, x' is:

$$x' = 2 \times 10 = 20 \text{ m}$$

(b) Calculate the distance the child travels according to the observer on the platform

- This observer is stationary in their reference frame, so we use the equation

$$x = x' + vt$$

$$x = 20 + (15 \times 10) = 170 \text{ m}$$

- *v* is the speed of the **whole** reference frame

Your notes

Examiner Tip

v is still a **velocity** and hence is a **vector**. Therefore, the **direction** matters i.e. a velocity in the opposite direction is -*v*.

Notice you only need to remember $x = x' + vt$

- $x' = x - vt$ is the same equation but rearranged for *x'* as the subject instead

You must be very careful about **which observer** you are referring to in any exam question.

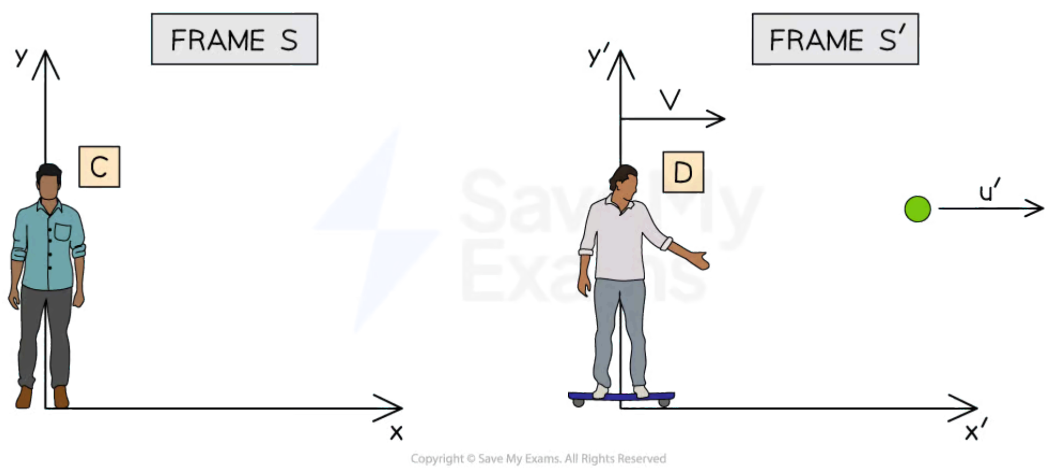
- In part (a) of the worked example, the child is running away from the observer on the train, because the observer on the train assumes that they are stationary
- However, the observer on the platform witnesses the child running **and** the train moving too, so adds both of these velocities to get the total velocity of the child to be much higher

Your notes

Velocity Addition Equation

- Galilean transformations can also be used to transform **velocities**
  - This is known as **velocity addition**
- Velocity addition is used when there are **multiple velocities** in the scenario
- Let's go back to the example of Person D on a skateboard throwing a ball directly in **front** of them in a straight line
- In this example:
  - u is the speed of the ball measured in frame S (by Person C)
  - u' is the speed of the ball measured in frame S' (by Person D)
  - v is the speed of Person D

Diagram showing velocity addition for two objects moving in the same direction

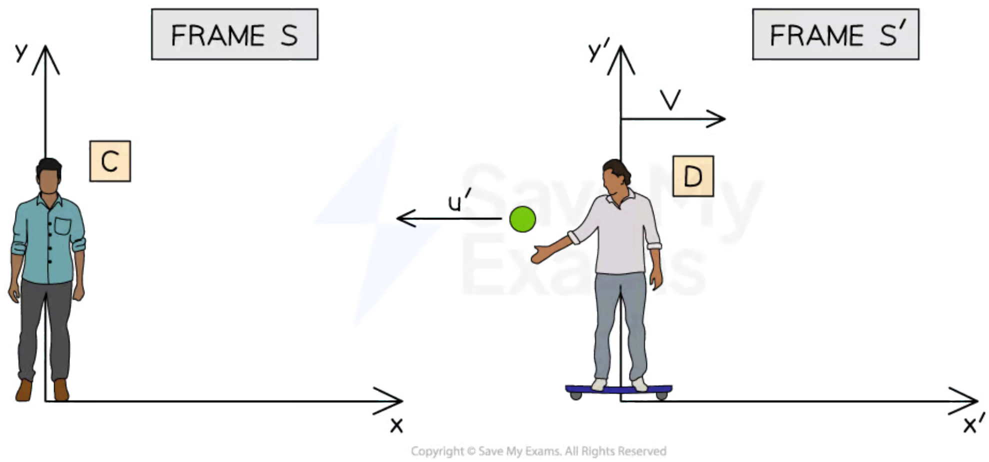


In Person C's reference frame, the ball is travelling at a speed of u' + v

- Therefore:
  - Person D (frame S') measures the velocity of the ball to be $\boldsymbol{u'}$
  - Whilst Person C (frame S) measures the velocity of the ball to be $\boldsymbol{u = v + u'}$
- Hence, the velocity of the ball from Person D's reference frame **in terms of** speeds v and u is
  - $\boldsymbol{u' = u - v}$

- Velocities are vectors, so their direction must be taken into account
- Let's say Person D now throws the ball directly **behind** them in a straight line at constant velocity
  - u, u' and v still refer to the same objects
- This time, since u' is in the **opposite** direction to v, it is now $\boldsymbol{-u'}$
- Therefore:
  - Person D (frame S') measures the velocity of the ball to be $\boldsymbol{-u'}$
  - Whilst Person C (frame S) measures the velocity of the ball to be $\boldsymbol{u = v - u'}$

Diagram showing velocity addition for two objects moving in opposite directions



In Person C's reference frame, the ball is now travelling at a speed of v – u'

- In summary:

Table of the velocity addition equations for a stationary and moving reference frame

Stationary reference frame	Moving reference frame	Transformation
u	u'	$u = u' + v$ $u' = u - v$

Your notes

Your notes

Worked example

Two travellers X and Y walk past each other at an airport. Traveller X walks at 1.5 m s^{-1} whilst Traveller Y walks at 2.1 m s^{-1}.

Calculate the velocity of:

(a) Traveller Y relative to Traveller X
(b) Traveller X relative to Traveller Y

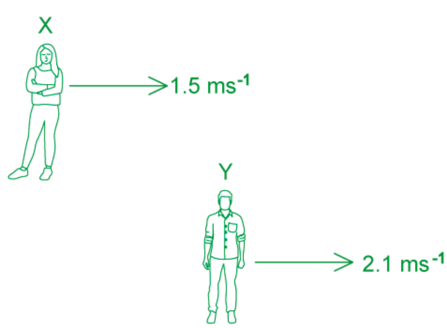
Traveller X now walks onto a travelator walkway in the airport at the same speed. The travelator moves at 0.3 m s^{-1}. Traveller Y decides to take a seat facing the travelator.

Calculate the velocity of:

(c) Traveller X relative to Traveller Y when they are walking in the same direction as the travelator
(d) Traveller X relative to Traveller Y when they are walking in the opposite direction to the travelator (just for fun)

Answer:

(a) The velocity of Traveller Y relative to Traveller X is:



- X is the reference frame at rest
- Y is the reference frame that is moving
- Therefore, from Traveller X's perspective, Traveller Y is moving at:

$$2.1 - 1.5 = 0.6 \text{ m s}^{-1}$$

- From Traveller X's frame of reference, Traveller Y is moving at a speed of 0.6 m s^{-1}
- Traveller Y is moving faster than Traveller X by 0.6 m s^{-1}

(b) The velocity of Traveller X relative to Traveller Y is:

- Y is the reference frame at rest
- X is the reference frame that is moving

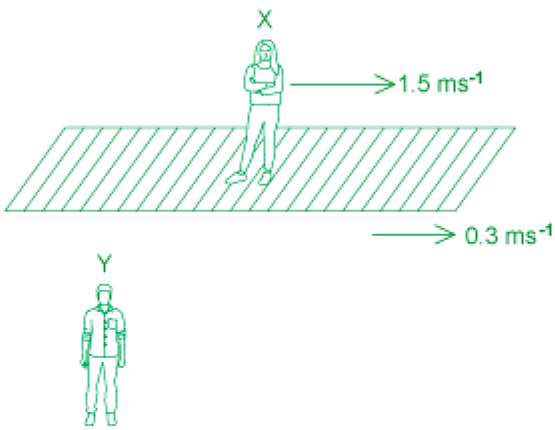
Your notes

- Therefore, from Traveller Y's perspective, Traveller X is moving at:

$$1.5 - 2.1 = -0.6 \text{ m s}^{-1}$$

- From Traveller Y's frame of reference, Traveller X is moving at a speed of -0.6 m s^{-1}
- Traveller X is moving slower than Traveller Y by 0.6 m s^{-1}

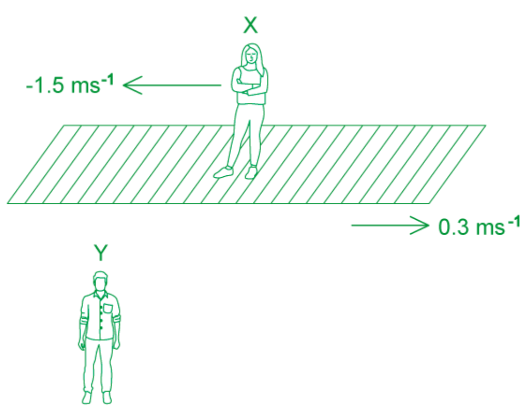
(c) The velocity of Traveller X relative to Traveller Y when walking in the same direction as the travelator



- Y is the reference frame at rest
- X is the reference frame that is moving **with** the travelator
  - u = velocity that Y measures X to be
  - u' = velocity of the travelator
  - v = velocity of X
- Therefore, they are moving at:

$$u = u' + v = 0.3 + 1.5 = 1.8 \text{ m s}^{-1}$$

(d) The velocity of Traveller X relative to Traveller Y when walking in the opposite direction to the travelator



- Y is the reference frame at rest

- X is the reference frame that is moving **against** the travelator
  - u = velocity that Y measures X to be
  - u' = velocity of the travelator
  - $-v$ = velocity of X
- Therefore, they are moving at:

$$u = u' - v = 0.3 - 1.5 = -1.2 \text{ m s}^{-1}$$

Examiner Tip

Always watch out for the **direction** of objects in velocity addition, don't just plug in numbers into the equation!

It helps to draw a quick sketch of the scenario in your exam and label the velocities. It doesn't matter which direction you take as positive, as long as you are consistent throughout your question.

Again, $u = v + u'$ can be rearranged for u' to give $u' = u - v$ i.e. the speed of the object in the other reference frame.

Your notes

Postulates of Special Relativity (HL)

The Postulates of Special Relativity

- Galilean relativity states that Newton's laws of motion are the same in all inertial reference frames
- Newton then treated space and time as fixed and absolute
  - This means the time interval between two events in one frame (x, y) is the same as the time interval between another frame (x', y')
- However, this is **not** what happens when we are close to the speed of light
  - Space and time become **relative**, meaning, the length of an object or a time interval **depends** on the frame of reference
- **Velocity addition** works with speeds much **lower** than the **speed of light (c)**
- It doesn't work for objects travelling closer to the speed of light
  - According to Galilean relativity, if a rocket ship travels at 0.7c and releases a probe directly in front of it at 0.5c, a stationary observer would view this at 0.7c + 0.5c = 1.2c
  - **However**, we know that **nothing** can travel faster than the speed of light, so this is **not** possible
- Einstein's two **postulates of special relativity** are:

First Postulate

The laws of physics are the same in all inertial frames of reference

- In our own reference frame, we are always **stationary**
- This means in practice, we should **not** be able to tell whether we are moving or not
  - Someone conducting a physics experiment on a moving train versus on a stationary platform should produce the **exact same** results

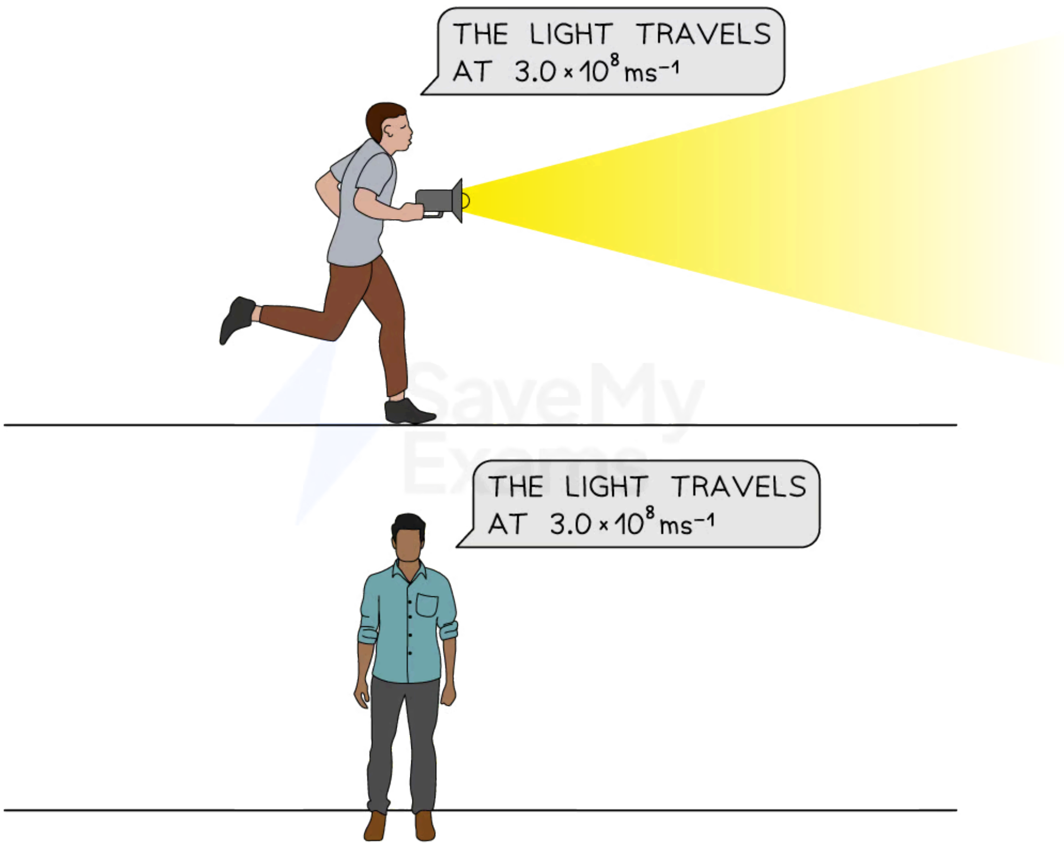
Second Postulate

The speed of light, c, in a vacuum, is the same in all inertial frames of reference

- Two different observers will always measure the speed of light to be the same value, c in their reference frame
  - It makes no difference whether they are travelling or not. If it did, you would know whether you are moving, which counteracts the first postulate
- For example, a runner holding a flashlight in front of them will measure the speed of the light as **c**
  - However, someone stationary observing the runner will **also** see the speed of light as c and **not** c + the velocity of the runner
- This only works for the **speed of light**, not any other speed

Diagram demonstrating Einstein's postulate of special relativity

Your notes



Both a moving and stationary observer would measure the same speed of light from the torch. They are happy to finally agree on something

Examiner Tip

You must remember these two postulates, as they play an important part conceptually and mathematically in further equations in special relativity.

Lorentz Transformations (HL)

Your notes

Lorentz Transformation Equations

- To relate measurements from one reference frame to another in Galilean relativity, we used Galilean transformations
- However, Galilean transformations can no longer be used to describe distances, times and speeds for objects travelling close to the speed of light
- Einstein's postulates of special relativity lead to the **Lorentz transformation** equations for the coordinates of an event in two inertial reference frames

The Lorentz Factor

- Lorentz transformations are a **correction** of the Galilean transformations for speeds close to the speed of light, by multiplying by a scaling factor called the **Lorentz factor**, γ

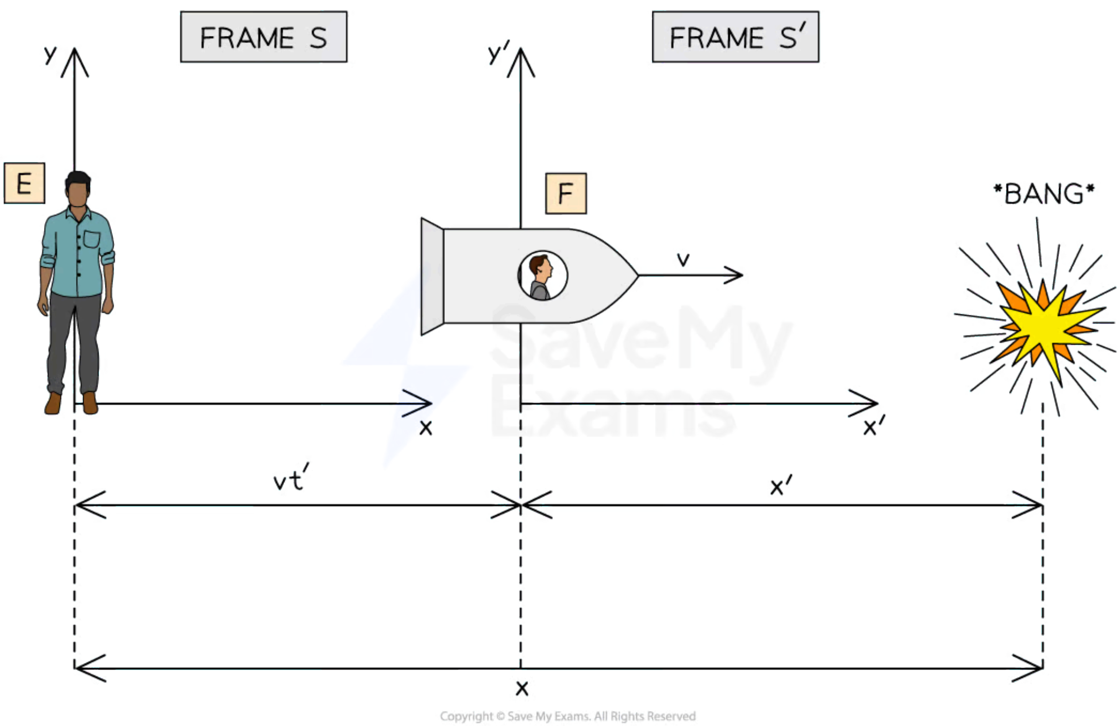
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- As *v* will always be less than c (since nothing can travel faster than the speed of light), this means that γ will always be **greater than 1**
- This is especially important for time dilation and length contraction

Lorentz Transformation Equations

- Again we have the reference frame S measuring with co-ordinates (*x*, *y*, *t*), and S' with co-ordinates (*x*', *y*', *t*')
- Person F is moving **away** from Person E in their rocket ship at speed *v* which is close to the speed of light c
  - Person E is a stationary observer on Earth
- Both Person E and Person F witness a loud bang some distance away
  - Person F measures the loud bang to be a distance **x'** away
  - Person E measures the loud bang to be a distance $x = \gamma(x' + vt')$ away

Your notes



People E measures the loud bang to be at a different distance to person F

- However, motion is relative
- According to Person F, they are stationary and Person E is moving away from them at a velocity *v* in the **opposite** direction
- So from Person F's point of view:
  - Person E's velocity is **–*v***
  - Therefore, Person F measures the bang to happen at a distance ***x*' = γ(*x* – *vt*)** away
- But, the time the bang happens is **not** the same in both frames of reference! ***t* ≠ *t*'**
- These are the exact same equations as the Galilean transformation equations, just with the **added** Lorentz factor
- In summary, the Lorentz equations from frame S → S' are:

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

- Where:
  - (*x*, *y*, *z*, *t*) = the co-ordinates measured from one reference frame
  - (*x*', *y*', *z*', *t*') = the co-ordinates measured from another reference frame **moving at speed *v* relative to it**

Your notes

Galilean vs. Lorentz Transformations

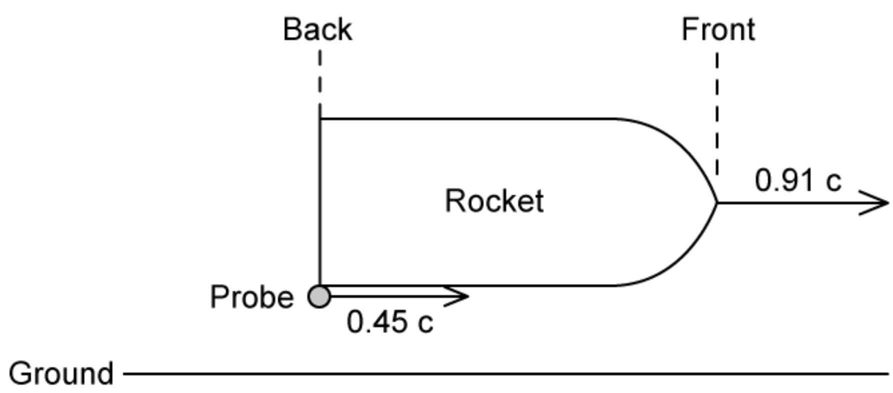
Lorentz ($v \approx c$)	Galilean ($v << c$)
$x' = \gamma(x - vt)$	$x' = x - vt$
$x = \gamma(x' + vt')$	$x = x' + vt$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$	$t' = t$
$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$	$t = t'$
$y = y'$	$y = y'$
$z = z'$	$z = z'$

- Notice that **time** is different between reference frames *t* and *t'* for objects travelling close to the speed of light, whilst in Galilean transformations, time was absolute (it doesn't change) between reference frames

Your notes

Worked example

A rocket of proper length 150 m moves to the right with speed 0.91c relative to the ground.



A probe is released from the back of the rocket at speed 0.45c relative to the rocket.

Determine the time it takes the probe to reach the front of the rocket according to an observer

(a) At rest in the rocket.

(b) At rest on the ground.

Answer:

(a)

Step 1: List the known quantities

- Length of the rocket, *l* = 150 m
- Speed of the probe, *v'* = 0.40c

Step 2: Analyse the situation

- In the reference frame of an observer at rest in the rocket, they are **stationary**
- Therefore, the probe travels at a constant speed 0.45c across the full length of the rocket of 150 m

$$t' = \frac{l}{v'} = \frac{150}{0.45c} = \frac{150}{0.45 \times (3 \times 10^8)}$$

$$t' = 1.11 \times 10^{-6}\ \text{s}$$

- t' and v' are used because they are the times and velocity of the **moving** object **in the reference frame** of the observer at **rest**

Your notes

(b)

Step 1: List the known quantities

- Length of the rocket, *l* = 150 m
- Speed of the rocket, *v* = 0.91c
- Time taken for the probe to reach the front of the rocket, $t' = 1.11 \times 10^{-6}$ s

Step 2: Analyse the situation

- In reference to an observer at rest on the ground, they will see the probe taking **longer** to reach the front of the ship
- Since object in question, the probe, is **moving** in both reference frames, we need to use a Lorentz transformation

Step 3: Calculate the gamma factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.91c)^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - 0.91^2}} = 2.412$$

Step 3: Substitute values into the Lorentz transformation

- Since the observer is at **rest**, the Lorentz equation for time *t* must be used

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$t = 2.412\left((1.11 \times 10^{-6}) + \frac{(0.91c)(150)}{c^2}\right) = 2.412\left((1.11 \times 10^{-6}) + \frac{(0.91)(150)}{3 \times 10^8}\right)$$

$$t = 3.8 \times 10^{-6} \text{ s}$$

Your notes

Examiner Tip

Always check that your value of γ is greater than 1.

You will often be given a speed *v* **in terms of c** e.g. *v* = 0.90c etc. When you put this value into the gamma factor, this is **squared**. Therefore, you do not need to put in 3.0×10^8 at all into your calculator, as the c^2 will cancel.

$$\text{E.g. } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.90c)^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.90)^2(c)^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.90)^2\cancel{(c)^2}}{\cancel{c^2}}}} = \frac{1}{\sqrt{1 - (0.90)^2}}$$

The equations for *x*', *t*' and γ are given in your data booklet, but you must remember the sign change if you want to calculate *x* or *t* (from the rest frame) instead!

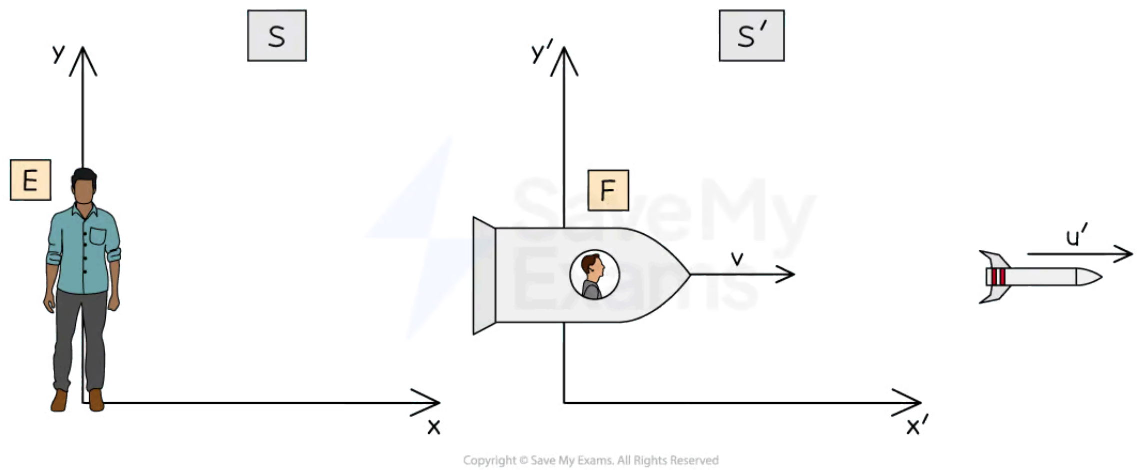
Some textbooks may go further into this for your understanding, you will **not** be expected to derive these equations in your exam. You will only be assessed on how to use them.

Your notes

Velocity Addition Transformations (HL)

Velocity Addition Transformations

- Similar to velocity addition to Galilean transformations, the Lorentz transformation equations lead to relativistic velocity addition equations
- These are again used when there are **multiple** velocities in the scenario but now some are **close to the speed of light**
- Let's go back to the example of Person F in the rocket ship. They now release a missile in front of them
- In this example:
  - *u* is the speed of the missile measured in frame S (by Person E)
  - *u'* is the speed of the missile measured in frame S' (by Person F)
  - *v* is the speed of frame S' (Person F)



Person F releases a missile in front of them. Both observers will view the missile travelling at different speeds

- In **Galilean** velocity addition, when *v* << c, these were:
  - The speed of the missile as measured by Person E: $u = u' + v$
  - Or, $u' = u - v$
- If *v* and *u'* are close to the speed of light, we have to use **Lorentz** velocity addition transformations instead
- These equations are:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$u = \frac{u' + v}{1 + \frac{u' v}{c^2}}$$

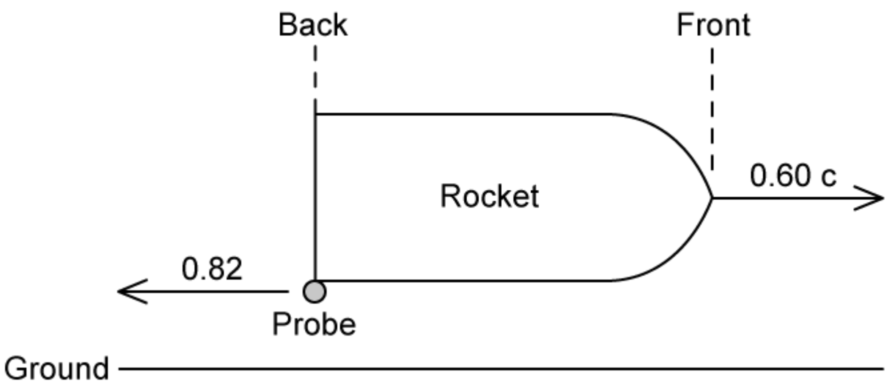
Your notes

- Where:
  - u = the velocity of an object measured from the **stationary** reference frame
  - u' = the velocity of an object measured from a **moving** reference frame
  - v = the velocity of the moving reference frame
  - c = the speed of light

Your notes

Worked example

A rocket moves to the right with speed 0.60c relative to the ground.



A probe is released from the back of the rocket at speed 0.82c relative to the rocket.

Calculate the speed of the probe relative to the ground.

Answer:

Step 1: List the known quantities

- Speed of the rocket, $v = 0.60c$
- Speed of the probe relative to the rocket, $u' = 0.82c$

Step 2: Analyse the situation

- We have multiple velocities in this scenario in terms of c, so we need to use the **Lorentz** velocity addition equations
- The probe is travelling in the **opposite** direction to the rocket, so its velocity is –0.82c
- We want the speed relative to the ground, which is a reference frame **at rest**, so this is u

Step 3: Substitute values into the equation

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

$$u = \frac{-0.82c + 0.60c}{1 + \frac{(-0.82c)(0.60c)}{c^2}} = \frac{(-0.82 + 0.60)c}{1 + (-0.82)(0.60)}$$

$$u = -0.43c$$

Your notes

Examiner Tip

Be very careful which reference frame you are asked to calculate the velocity from, as this determines whether you find *u* or *u*'. Notice the equations are very similar, except one is with – and the other +. However, the signs will match on the numerator and denominator.

The equation for *u*' is given in your data booklet.

Anytime you see the word 'relativistic' in physics such as 'relativistic speeds' it just means 'close to the speed of light'. Physics gets a bit weird at this point!

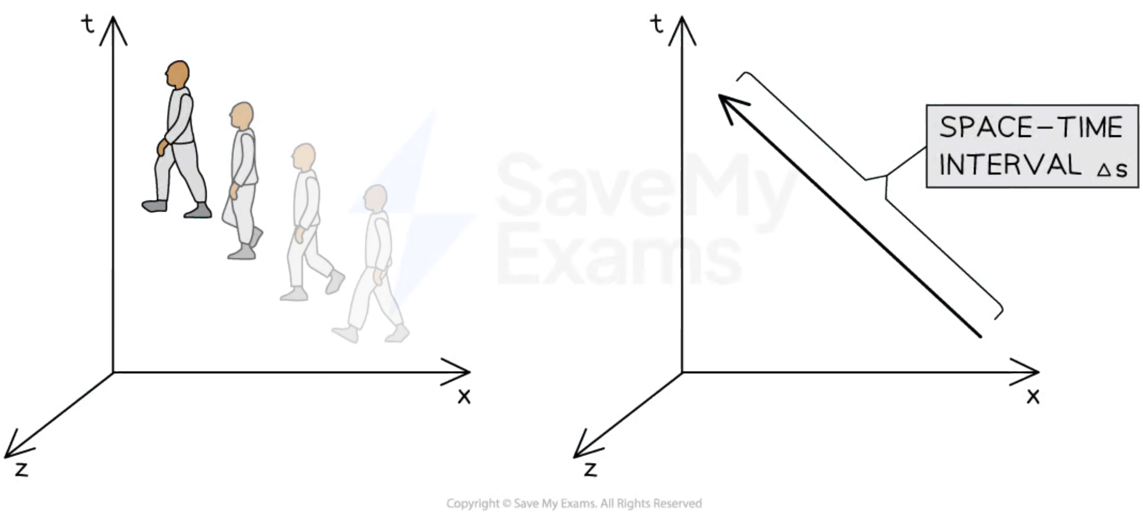
It is fine, and often encouraged, to give your final answers for relativistic velocities in terms of c. In the denominator of the velocity addition equations, the c^2 will cancel out if two velocities *u* and *v* are given in terms of c.

Your notes

Space-Time Interval (HL)

Space-Time Interval

- Einstein discovered that time and distance **changes** when moving from one inertial reference frame to another when travelling at speeds close to the speed of light
  - In other words, these reference frames are **not absolute**
- However, some quantities **are** the same in all inertial frames. These are called **invariant**
- These are:
  - Proper time, t_0
  - Proper length, L_0
  - Space-time interval, Δs
- These are a product of Einstein's second postulate
- In Galilean relativity:
  - Space and time are the **same** in all reference frames, i.e. $\Delta t = \Delta t'$ and $\Delta x = \Delta x'$
- In special relativity:
  - These are replaced with a **space-time** interval, as space and time are connected together as 4 coordinates (*x, y, z, t*) for an event
  - Motion can be represented as spanning both space **and** time using this coordinate system
- The diagram below shows a person moving in both space *x* and *z* **and** in time *t*
  - They can also move in the *y* direction, but 4 dimensions are not possible to draw accurately here (in 3-dimensional space)



Motion in space-time. The length of the arrow for the space-time interval is the same for all inertial reference frames

- An interval in space-time is an **invariant** quantity in all inertial reference frames and is defined as:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

- Where:
  - Δt = time interval / separation (s)
  - c = the speed of light
  - Δx = spacial separation (m)
  - Δs = space-time interval (m)
- This means that in two inertial reference frames, although Δt and Δx will be different in both frames, Δs will be the **same**
- These will be used in space-time diagrams

Your notes

Worked example

An inertial reference frame S' moves relative to S with a speed close to the speed of light. When clocks in both frames show zero the origins of the two frames coincide.

An event P has coordinates *x* = 2 m and *ct* = 0 in frame S, and *x* = 2.3 m in frame S'. Show that the time coordinate of event P in frame S' is –1.1 m.

Answer:

Step 1: List the known quantities:

- Spacial separation in frame S, Δx = 2 m
- Time separation in frame S, $c\Delta t$ = 0
- Spacial separation in frame S', $\Delta x'$ = 2.3 m

Step 2: Calculate the space-time interval in frame S

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$(\Delta s)^2 = (0) - (2)^2 = -4$$

Step 3: Substitute values into the space-time interval for S'

- Δs is the same (invariant) in both reference frames

$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$(c\Delta t')^2 = (\Delta s)^2 + (\Delta x')^2$$

$$(c\Delta t')^2 = -4 + (2.3)^2 = 1.29$$

$$c\Delta t' = \sqrt{1.29} = \pm 1.14$$

Examiner Tip

The units still work out on both sides of the equation. Remember, $c\Delta t$ is a **speed** × **time** which is a **distance** in metres, so is Δx so Δs is in metres.

Whether ct' in the worked example is + or – will come in later with space-time diagrams.

Your notes

Your notes

Proper Time & Length

- In special relativity, we have found that distances and times are **relative**
- This means the length of an object and the time interval of an event **changes** when observed in frames that are **moving relative** to that **object**
  - More on this is explored in length contraction and time dilation
- We need to define the difference mathematically between the time and lengths measured between each frame, to know which one is being referred to

What is Proper Time and Proper Length?

- **Proper time interval**, Δt_0 is defined as:

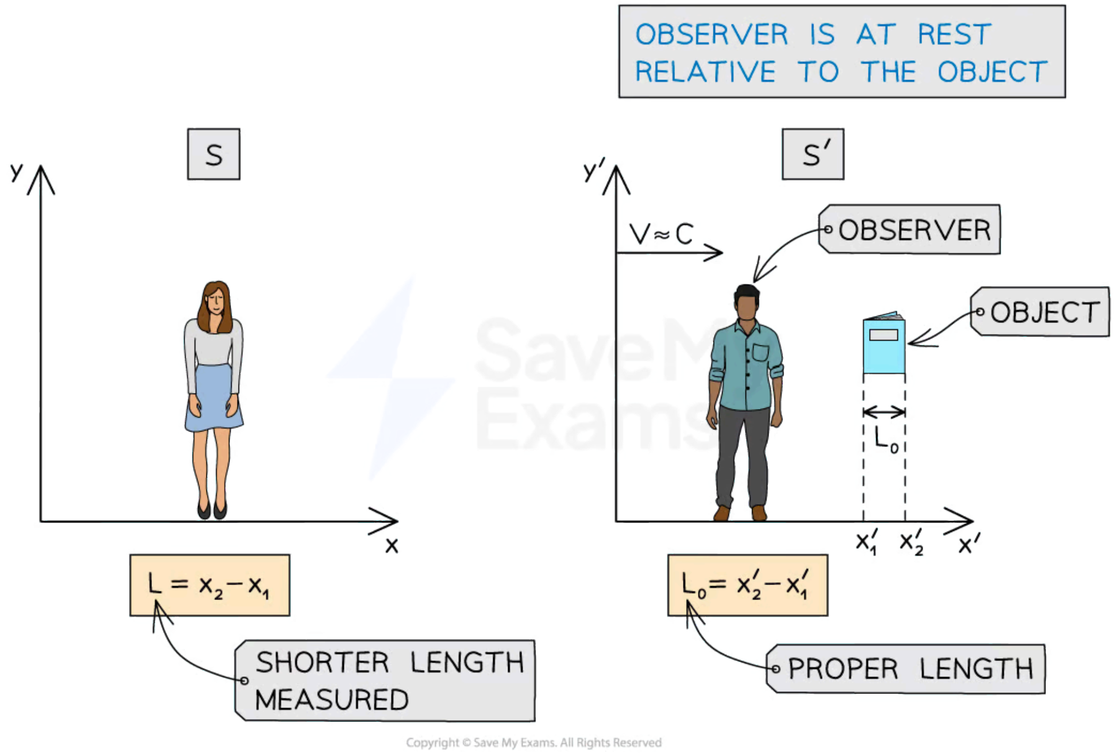
  The time interval between two events measured from within the reference frame in which the two events occur at the same place

- **Proper length**, L_0 is defined as:

  The length measured in a reference frame where the object is at rest (relative to the observer)

- These can be measured either in the moving frame S' **or** a rest frame S
  - This depends on the reference frame you are calculating the length and time from
- For example, if a person in moving frame S' (e.g. on a train) measures the length of a book, they are **at rest relative** to the book
  - They measure the distance between points x_2' and x_1'
  - This is the proper length, L_0 although they are technically moving (but they don't know this - otherwise it would go against Einstein's first postulate)
- However, for an observer in frame S, at rest (e.g. on a platform
  - They will measure the distance between points x_2 and x_1
  - This a shortened length, L

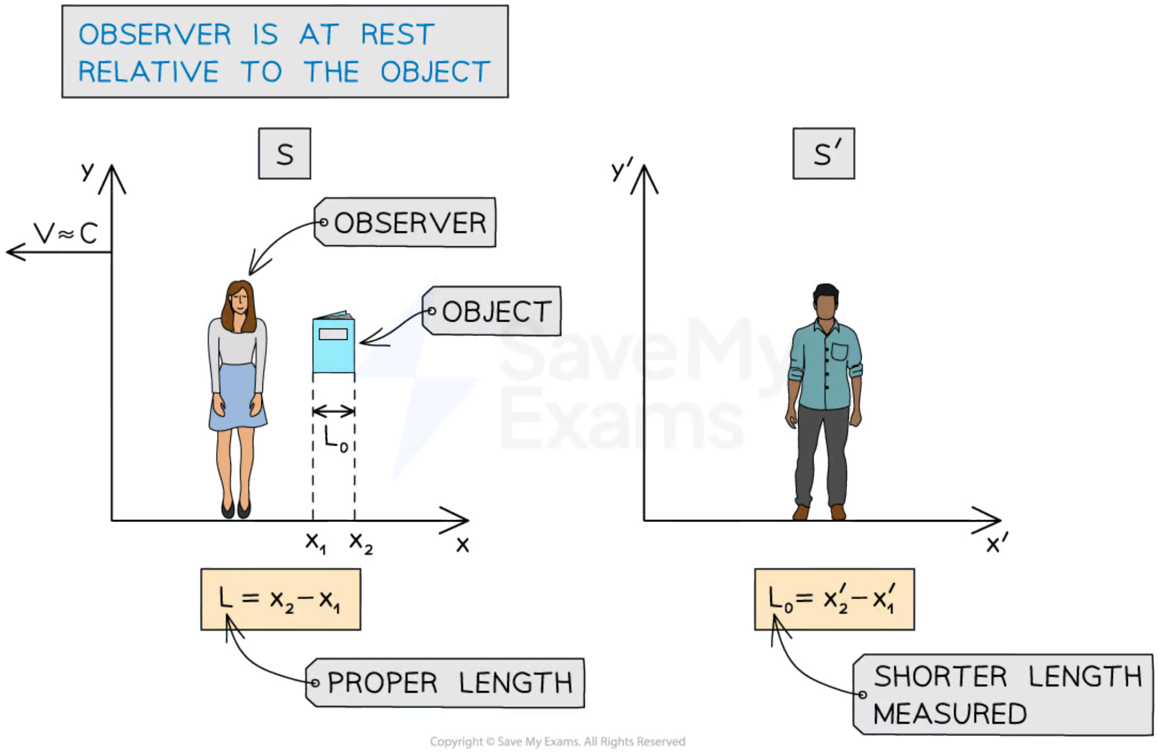
Your notes



The observer in S' is measuring the proper length, because they are at rest relative to the object

- For example, if a person in the stationary frame S (e.g. on the platform) measures the length of a book, **they** are now **at rest relative** to the book
  - They measure the distance between points x_2 and x_1
  - This is the proper length, L_0
- However, an observer in frame S', which is moving (e.g. on a train) sees **themselves** at rest but instead sees frame S as moving (remember, motion is **relative**)
  - They will measure the distance between points x_2' and x_1'
  - This a shortened length, L

Your notes



The observer in S is measuring the proper length because they are at rest relative to the object

- The same rules apply for the proper time, t_0, except that the time measured from a frame **moving** relative to an event will be measured **longer**

Examiner Tip

Do not misinterpret this as the time or length measured in the stationary reference frame! This is not the case. It could be the time and length measured in the **moving** frame too, because it is a length and time measured by an observer **at rest relative to the object**

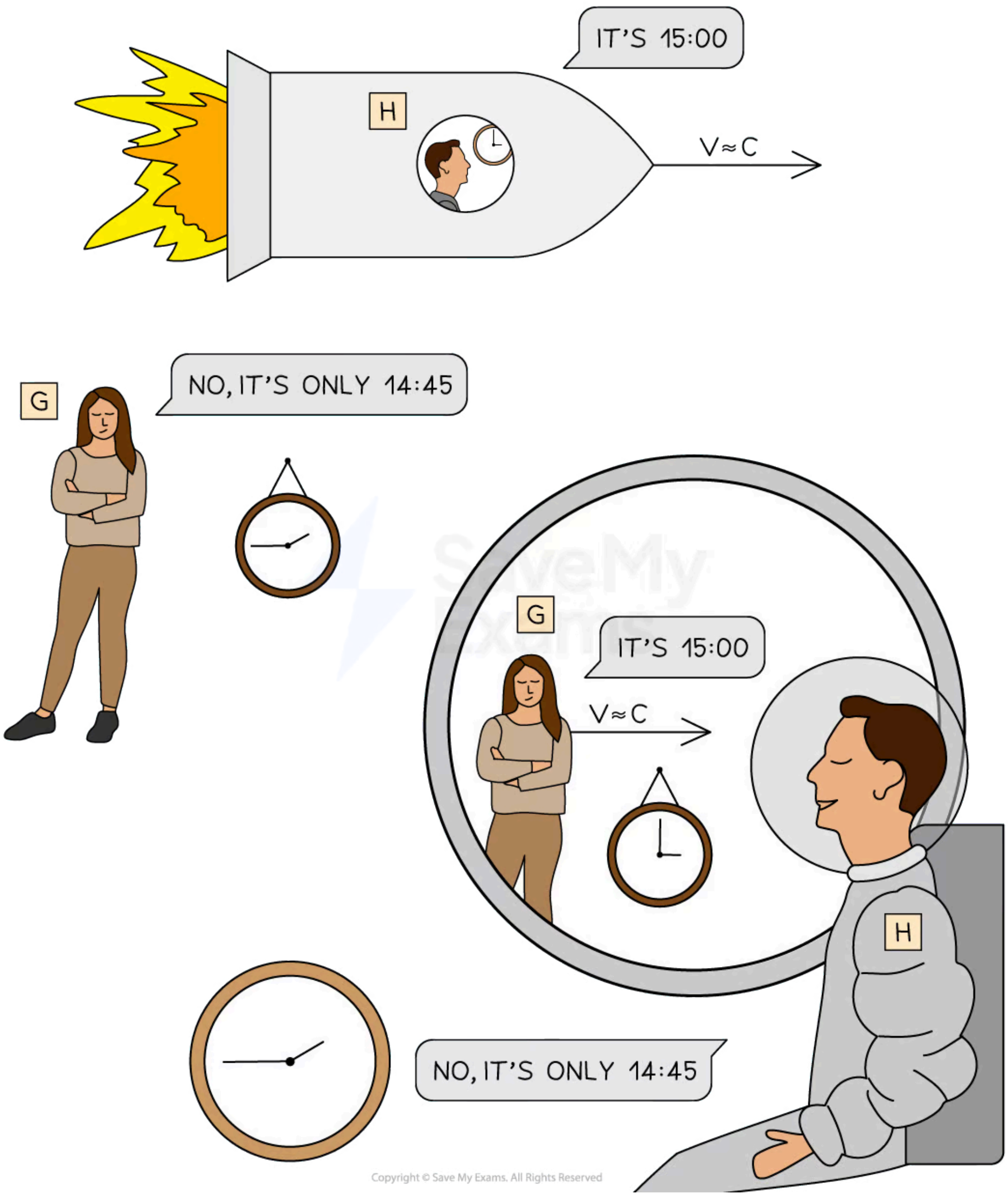
Again, this only works for objects moving close to the speed of light. You will never encounter this in everyday life!

Time Dilation (HL)

Your notes

Time Dilation

- When objects travel close to the speed of light, to an observer **moving relative** to that object, it looks as if the object has **slowed down**
  - This is best demonstrated by **clocks**
- Observer H is in a rocket moving close to the speed of light
  - They will see their clock ticking at a regular pace, say, it is reading 15:00
- Observer G at rest on Earth, with remarkable eyesight, will measure the clock as ticking **slower**
  - They will observe that time has **slowed down** in the spaceship from **their** reference frame i.e. they may see the time as 14:45 instead of 15:00
- However, the **same** occurs the **other way around**
- For observer H on the rocket, it is observer G that is **moving relative** to them
  - Therefore, observer H will measure observer G's clock as ticking **slower** i.e. they see time **slow down** on Earth from **their** reference frame



A stationary observer in their own reference frame views clocks as running slower in the moving reference frame. We're back to disagreeing

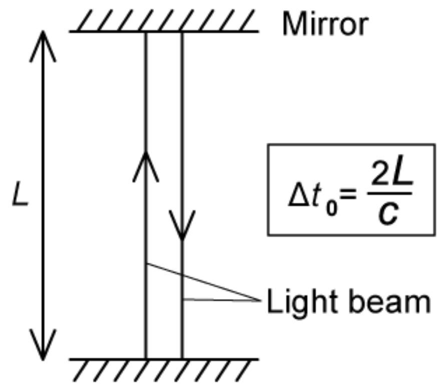
Time Dilation Equation

- Consider a light clock. This consists of two mirrors facing each other with a beam of light travelling up and down between them

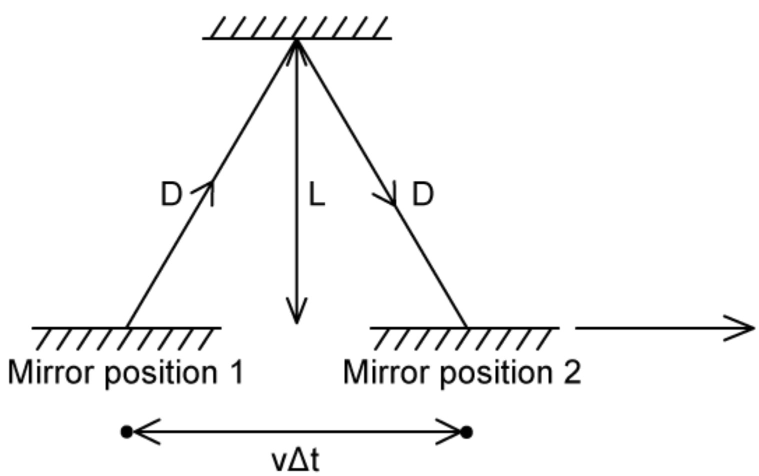
Your notes

- An observer is at **rest relative** to the clock on a train platform and watches the light reflect between the mirrors
- The distance between the mirrors is *L* and the light travels at a speed c
- Therefore, the time interval for the light to travel from the top mirror back down to the bottom is:

$$\Delta t_0 = \frac{2L}{c}$$



- Another light clock is on a moving train, **relative** to the initial observer on the platform, travelling at constant velocity *v*
- The stationary observer on the platform sees the light clock on the train and watches the reflection of the rays between the mirrors
- It appears that the light rays travel to the right at an angle to the direction of motion



- The observer on the train platform sees the mirror at:
  - Position 1, when the light leaves the bottom mirror
  - Position 2, when the light returns to it

Your notes

- The length of the light path as seen by the stationary observer is not *L*, they see the longer path, *D*
- The distance travelled by the light ray is now 2D, and the time observed between the reflections is now

$$\Delta t = \frac{2D}{c}$$

- The apparent distance horizontally travelled by the mirror is $v\Delta t$ where *v* is the speed of the train
- Notice that this is part of a right-angled triangle, so using Pythagoras' theorem we can see that:

$$D^2 = L^2 + \left(\frac{v\Delta t}{2}\right)^2$$

- Where: $D = \frac{c\Delta t}{2}$
- We want to find Δt, the time taken for the light ray to travel up and down in the reference frame of the stationary observer on the train platform, who is moving **relative** to the light clock on the train
  - Remember, although it is the train that is moving, in the reference frame of an observer on the train it is the observer on the **platform** that is moving

$$\Delta t = \frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Remember the gamma factor $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is from the Lorentz transformation equations
- Therefore:

$$\Delta t = \gamma \Delta t_0$$

- Where:
  - Δt = the time interval measured from an observer **moving relative** to the time interval being measured (s)
  - Δt_0 = the **proper** time interval (s)
- As γ > 1, this means that the $\Delta t > \Delta t_0$
  - In other words, a clock observed from a reference frame moving **relative** to it will be measured to tick slower than a clock that is at rest in its frame of reference
- The observer on the **platform** will view the **train** clock as moving **slower**
- The observer on the **train** will view the **platform** as moving **slower**
- You may be wondering why it's the time that slows down for the light beam, and not the light beam just speeding up to hit each mirror at the same frequency

Your notes

    - This is due to Einstein's second postulate
    - Both Observers G and H **must** measure the speed of light to be c, so it doesn't slow down or speed up according to either reference frame
- It is important to note that the time has been measured at the same **position**
    - In other words, the time interval is the position at which the light leaves the first mirror and at which it returns to the second mirror in the reference frame of the mirror

Your notes

Worked example

Alex's spacecraft is on a journey to a star travelling at 0.7 c. Emma is on a space station on Earth at rest. According to Emma, the distance from the base station to the star is 14.2 ly.

Show that Alex measures the time taken for her to travel from the base station to the star to be about 14.5 years.

Answer:

Step 1: List the known quantities:

- Distance of space station according to Emma = 14.2 ly
- Speed of Alex's spacecraft, v = 0.7c

Step 2: Analyse the situation

- We are trying to find the time that Alex measures for **her** travel i.e. the time she would measure on her own clock in the spaceship which **she is stationary relative to**
- This is the proper time, Δt_0

Step 3: Calculate the time taken according to Emma, Δt

- 1 ly (light year) is the **distance**, s, light (at speed c) travels in a year
- Therefore it takes the light 14.2 years (time) to travel the distance at speed c

$$s = speed \times time = 14.2c \text{ m}$$

- Therefore, the time taken according to Emma is:

$$\Delta t = \frac{s}{v} = \frac{14.2c}{0.7c} = \frac{14.2}{0.7} = 20.29 \text{ years}$$

Step 4: Substitute values into the time dilation equation

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma}$$

$$\Delta t_0 = \frac{20.29}{\frac{1}{\sqrt{1 - \frac{(0.7c)^2}{c^2}}}} = \frac{20.29}{\frac{1}{\sqrt{1 - (0.7)^2}}} = 14.5 \text{ years}$$

Step 5: Check whether your answer makes sense

- Since Emma (who is stationary) is viewing Alex's clock (which is moving) she would measure a **longer** time for Alex to reach the star than Alex will
- As Emma records 20.29 years, but Alex only records 14.5 years, this time makes sense

Your notes

Examiner Tip

A nice way to remember this is 'moving clocks run slower'. The caveat is what is considered 'moving' **depends on the reference frame.**

You will **not** be expected to remember this derivation, but it's helpful to know where all the factors have come from. The time dilation equation is given on your data sheet.

The notion of 'proper time' is incredibly important here, as it depends on the reference frame the time interval is being measured from.

The maths for the derivation is only using $speed = \frac{distance}{time}$ and Pythagoras' theorem.

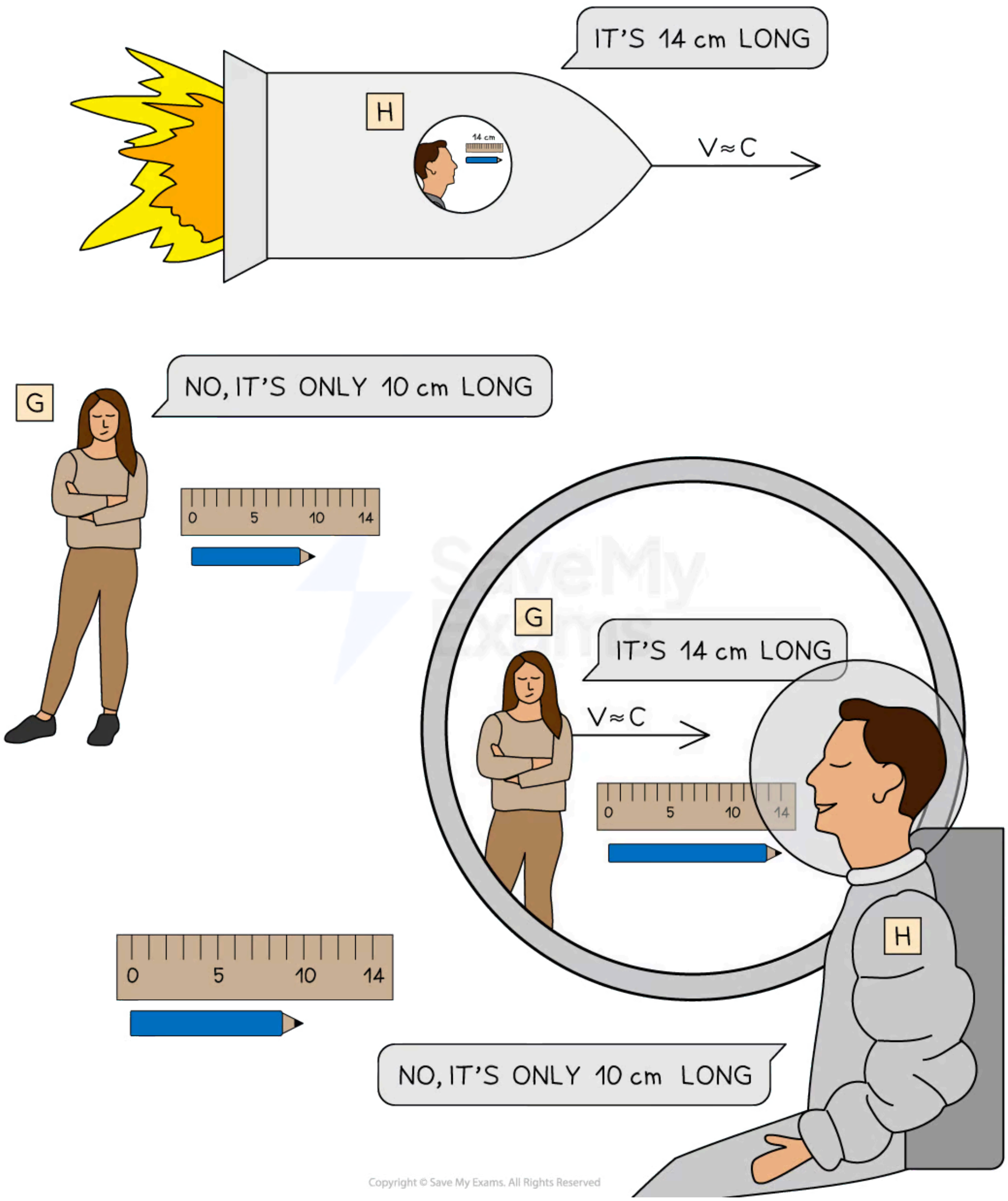
Length Contraction (HL)

Your notes

Length Contraction

- When objects travel close to the speed of light, to an observer **moving relative** to that object, it looks as if the object has **become shorter**
  - This is best demonstrated using **rulers**
- Observer H, in their rocket moving close to the speed of light, measures the length of their pencil to be 14 cm
- Observer G, at rest on Earth, would measure (with remarkable eyesight) the length of the pencil to be **shorter**
  - They will see lengths **contracted** in the spaceship from **their** reference frame, e.g. the length may appear to be 10 cm instead of 14 cm
- However, the **same** occurs the **other way around**
- For observer H on the rocket, it is observer G that is **moving relative** to them
  - Therefore, observer H would measure the length of observer G's pencil as **shorter** i.e observer H, on the rocket, sees lengths **contracted** on Earth from **their** reference frame

Your notes

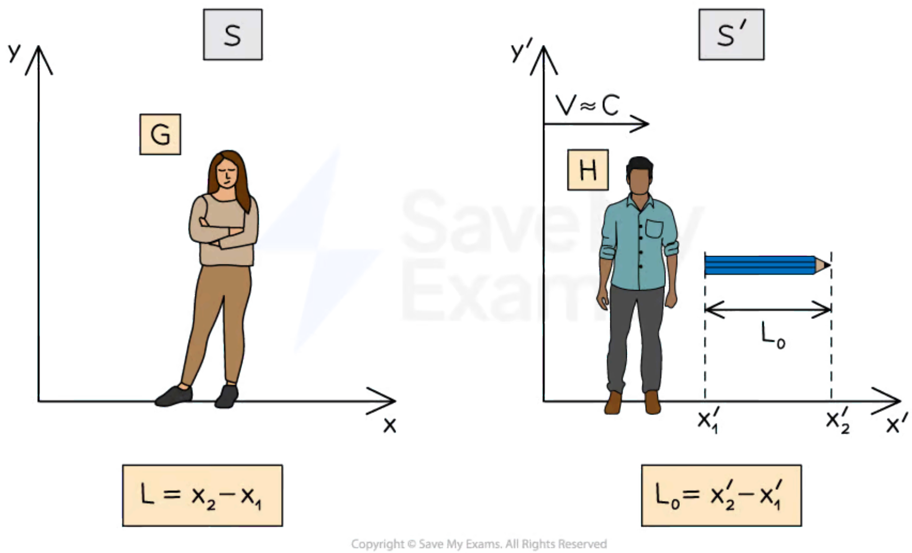


A stationary observer in their own reference frame views lengths as shorter in the moving reference frame

Length Contraction Equation

- The length of an object is the difference in the position of its ends

- Consider the observers G and H measuring the length of a pencil, which is stationary in reference frame S' (for observer H)



Observer G measures the length of the pencil to be different to observer H

- As observer H is in the moving frame S', they measure the length of the ruler as:

$$\Delta x' = x_2' - x_1' = L_0$$

- This is the **proper length, L_0** as the pencil is **not** moving relative to observer H
  - Both the pencil and observer H are, however, moving relative to observer G
- Observer G needs to measure the length of the pencil by measuring the position of its ends at the same time (just like observer H did)
- They measure the length of the ruler to be:

$$\Delta x = x_2 - x_1 = L$$

- This is the **observed length, *L*** as the pencil **is** moving relative to observer G
  - **Lorentz** transformations tell us how the *x* and *x*' are related
- We want to find Δx, the length measured in the reference frame of the stationary observer on Earth (G), who is moving **relative** to the observer on the rocket (H)
- Transforming these distances gives:

$$x_1' = \gamma(x_1 - vt)$$

$$x_2' = \gamma(x_2 - vt)$$

- These are then substituted into the equation for the proper length, L_0:

$$x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$$L_0 = \gamma L$$

- Therefore:

$$L = \frac{L_0}{\gamma}$$

- Where:
  - L = the length measured by an observer **moving relative** to the length being measured (m)
  - L_0 = the **proper** length (m)
- As γ > 1, this means that the $L < L_0$
  - In other words, lengths measured from a reference frame moving **relative** to the object will be measured as shorter than the lengths measured at rest from within their frame of reference
- Similar to time dilation, length contraction is also due to Einstein's second postulate
  - Both observers G and H **must** measure the speed of light to be c
  - Since the time for observer H will run slower, according to observer G (i.e. *t* increases), then for c to stay the same, the length of the object, *L* must **decrease**
- It is important to note that the length has been measured at the same **time**
  - This length is the difference between the ends of the pencil, with both ends measured at the same time
- The ruler used in both reference frames is **stationary** in their **own** reference frame
  - Otherwise, observer G would see the ruler on observer H's rocket contracting as well and wouldn't measure any difference in length

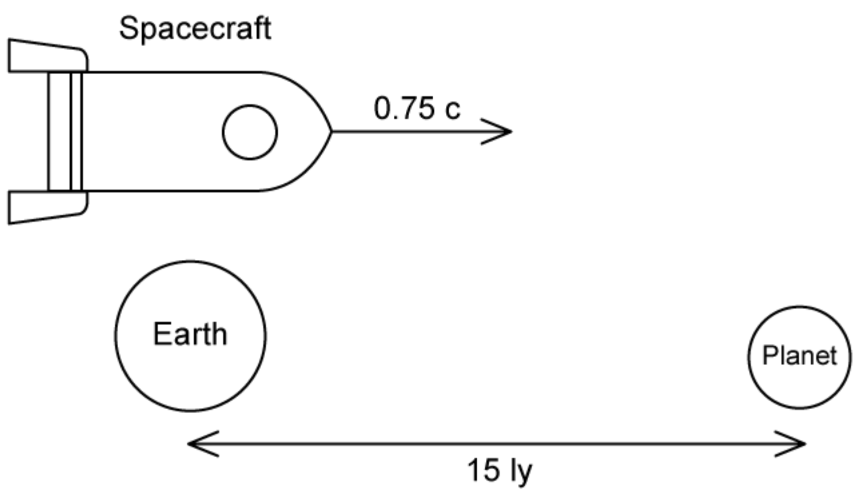
Your notes

Your notes

Worked example

A spacecraft leaves Earth and moves towards a planet.

The spacecraft moves at a speed of 0.75c relative to the Earth. The planet is a distance of 15 ly away according to the observer on Earth.



The spacecraft passes a space station that is at rest relative to the Earth. The proper length of the space station is 482 m.

Calculate the length of the space station according to the observer in the spacecraft.

Answer:

Step 1: List the known quantities

- Speed of the spacecraft, v = 0.75c
- Proper length of the space station, L_0 = 482 m

Step 2: Analyse the situation

- We are trying to find the length of the space station in the reference frame of the observer in the **spacecraft**
- In this observer's reference frame, it is the **space station** that is moving away from them at 0.75c
- Therefore, we a measuring a length in the moving reference frame (relative to the spacecraft) - this is the length, ***L***

Step 3: Substitute values into the length contraction equation

Your notes

$$L = \frac{L_0}{\gamma} = \frac{L_0}{\dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}}$$

$$L = \frac{482}{\dfrac{1}{\sqrt{1 - \dfrac{(0.75c)^2}{c^2}}}} = \frac{482}{\dfrac{1}{\sqrt{1 - (0.75)^2}}} = 319 \text{ m}$$

Step 4: Check whether your answer makes sense

- As the observer in the spacecraft is stationary, the length of the space station they measure should be **shorter** than the proper length
- As the length recorded from the spacecraft is 319 years, and the proper length is 482 m, this length makes sense

Examiner Tip

You will **not** be expected to remember this derivation, but it's helpful to know where all the factors have come from. The time dilation equation is given on your data sheet.

The notion of 'proper length' is incredibly important here, as it depends on the reference frame the length is being measured from.

You will find in some exam questions you can use time dilation **or** length contraction, you will receive marks for either way.

Your notes

Simultaneity in Special Relativity (HL)

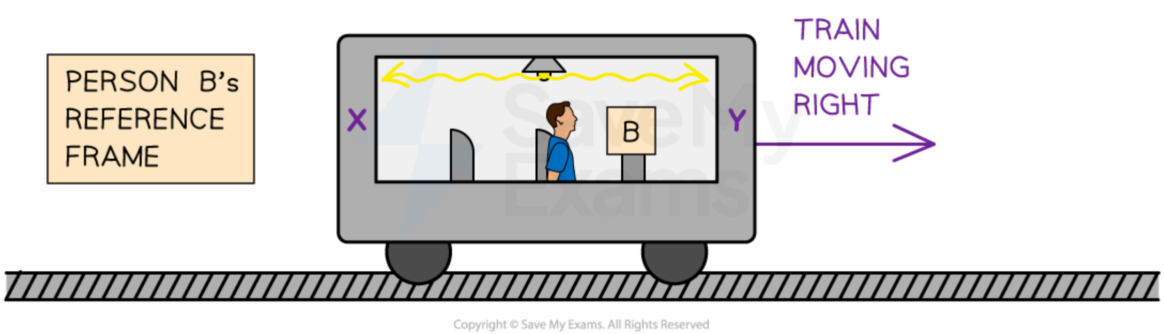
Simultaneity in Special Relativity

- The term 'simultaneous' means to occur at the same **time**
- The relativity of simultaneity states that

  Whether two spatially simultaneous events happen at the same time is not absolute, but depends on the observer's reference frame

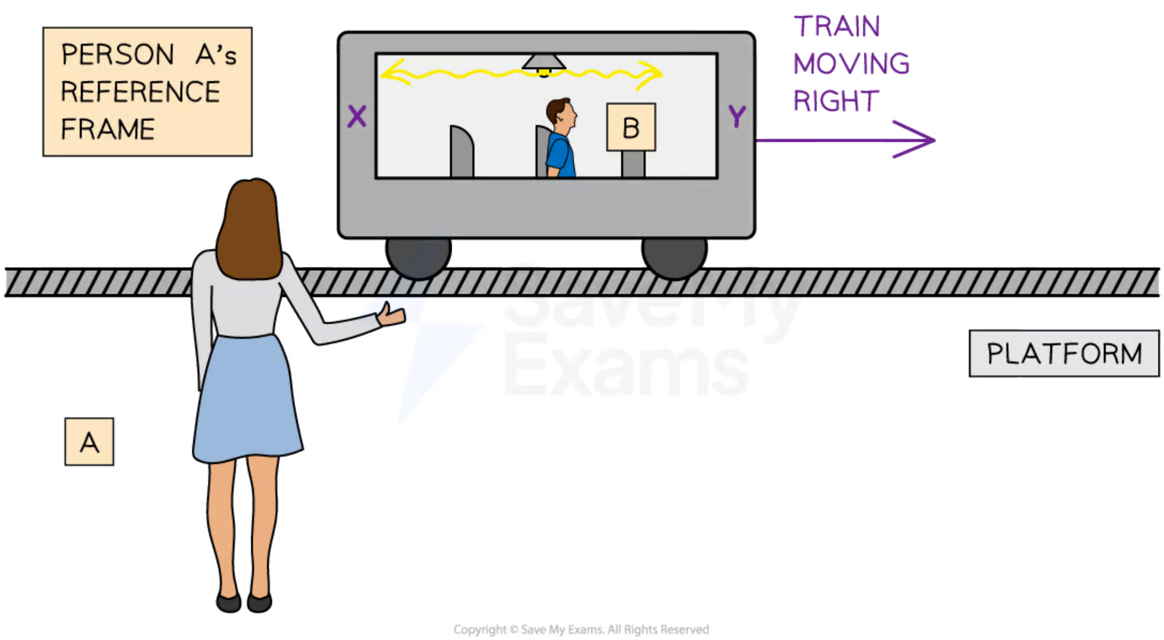
- This means that in one reference frame, two events that occur at **different** points in space seem to happen at the **same** time, whilst in another reference frame **moving** relative to the first the events seem to happen one after another
  - This was not the case in **Galilean** relativity, where simultaneity was **absolute**

- This is best shown in the following example
- Person B is in a train carriage moving to the right at constant velocity
- They switch on a lamp above them and they observe that the light from the lamp reaches the two ends of the carriage, points X and Y, at the same time



Person B sees the light from the lamp reach point X and Y at the same time

- Meanwhile, Person A is stationary on a platform and observes the train travel past
- Person A will still see the light from the lamp move to both ends of the carriage at the same speed (c)
  - This is in line with Einstein's second postulate
- However, Person A will see the light reach point X **before** it reaches point Y
  - This is because the whole carriage is moving to the right (relative to Person A), so the left side of the carriage is moving **towards** the light ray and the right side of the carriage is moving **away** from the light ray
  - This means that Person A will see the light ray reach point Y slightly later



Person A sees the light from the lamp reach point X before point Y

- This image is exaggerated to show the point; the difference between the times will be very very small and is dependent upon the speed of the train carriage
- Simultaneity can be visualised using **space-time** diagrams

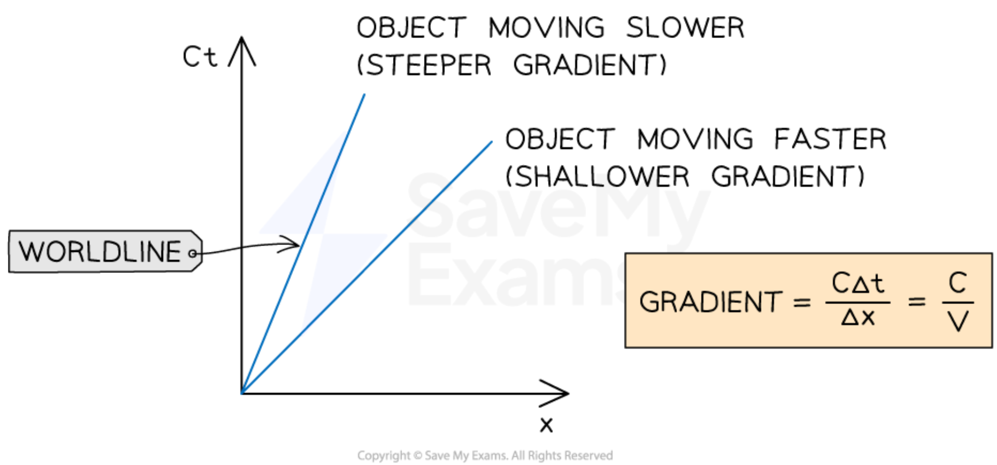
Your notes

Space-Time Diagrams (HL)

Your notes

Space-Time Diagrams

- **Spacetime** (or Minkowski) **diagrams** represent an object's motion in spacetime
- They help to visualise
  - Time dilation
  - Length contraction
  - Simultaneity
- Since 4D (*x*, *y*, *z*, *t*) diagrams cannot be drawn on a 2D page, we collapse 3D space (*x*, *y*, *z*) into 1 spacial dimension and keep time as its own dimension
- This gives a **spacetime diagram**
  - Lines drawn on a spacetime diagram are called **world lines**
- Instead of the usual distance-time graphs, we plot
  - The horiztonal axis as ***x***
  - The vertical axis as **ct**

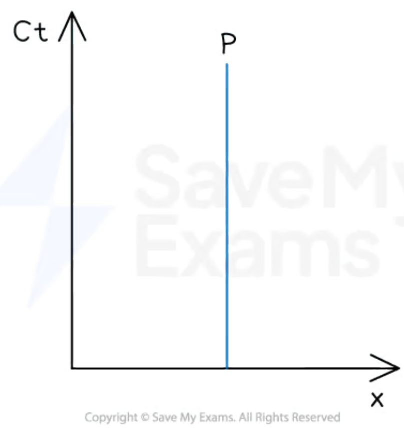


Objects moving slower are represented by a steeper gradient on a spacetime gradient

- Note that both axes have dimensions of **length**
  - This makes it easy to compare values on one axis and another
- This means the worldlines have a gradient of
  - $\frac{c\Delta t}{\Delta x} = \frac{c}{v}$ where the velocity is $v = \frac{\Delta x}{\Delta t}$
- This means:
  - The **steeper** the gradient, the **slower** the object is moving
  - The **shallower** the gradient, the **faster** the object is moving
- *ct* was first seen when the spacetime interval was introduced, and c is chosen deliberately so our diagram is oriented around the speed of light
  - *ct* is a sort of 'distance in time'

Your notes

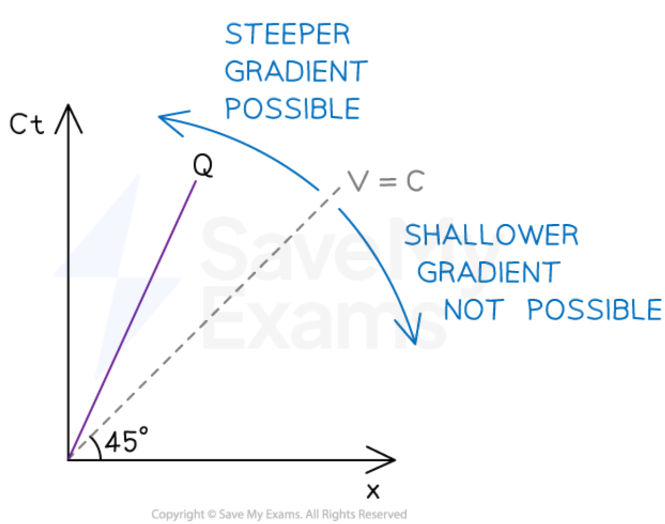
Worldline at Rest



A worldline for an object at rest

- P is an object **at rest**
  - It has an infinite slope meaning v = 0
- It is only moving in time, not in space

Worldline at Constant Velocity



A worldline at constant velocity has to have a gradient steeper than 1

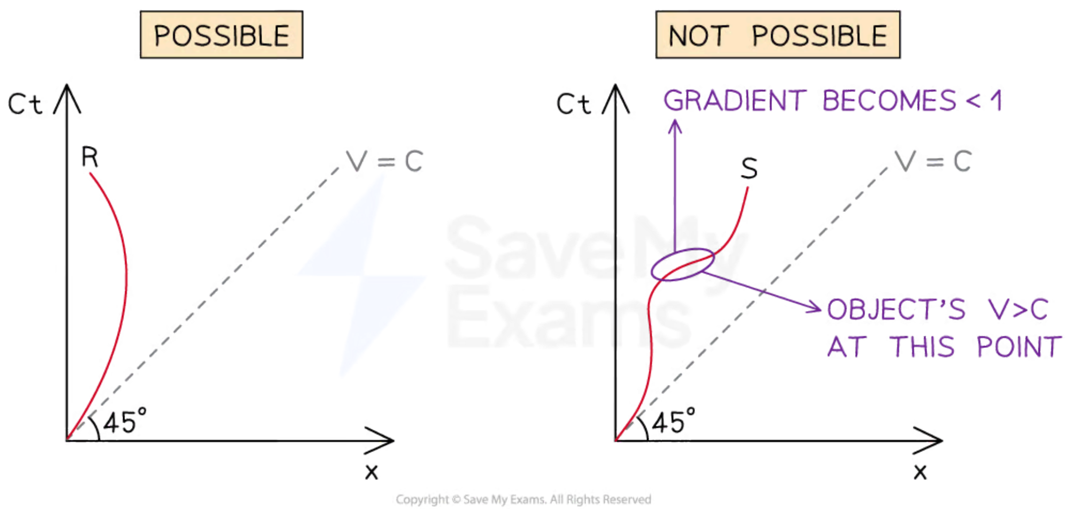
- Q is an object at **constant velocity**
- The object moves some distance over some time
- For an object moving at the speed of light, c (e.g. a massless photon), its worldline has a gradient of

$\frac{c}{v} = \frac{c}{c} = 1$ (a 45° angle)

Your notes

- Since nothing can travel faster than the speed of light (i.e. v cannot be greater than c), then objects can only have a gradient **greater than 1**
  - Therefore, a gradient of **1** is the **lowest** possible gradient
- Q's motion does not need to start at the origin. As long as it has a gradient of less than 1, it can start anywhere on the x-axis

Worldline accelerating



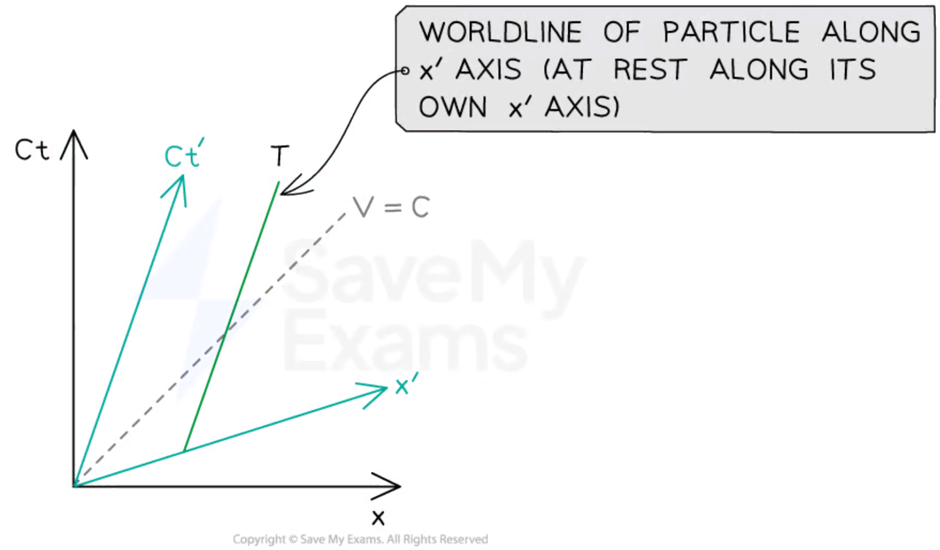
All points on a worldline representing an object accelerating must have a gradient steeper than 1

- R is an object with a **varying velocity** that represents **possible motion**
  - It has a small gradient (larger velocity), which increases (decreasing velocity)
  - Its gradient never gets less than 1
- S is an object with a **varying velocity** that represents **impossible motion**
  - At one point, it has a gradient of less than 1 implying a velocity greater than c
  - Even though it does not physically cross the $v = c$ gradient line, it is still **not** possible because a portion of the line has a gradient of less than one

Multiple Reference Frames

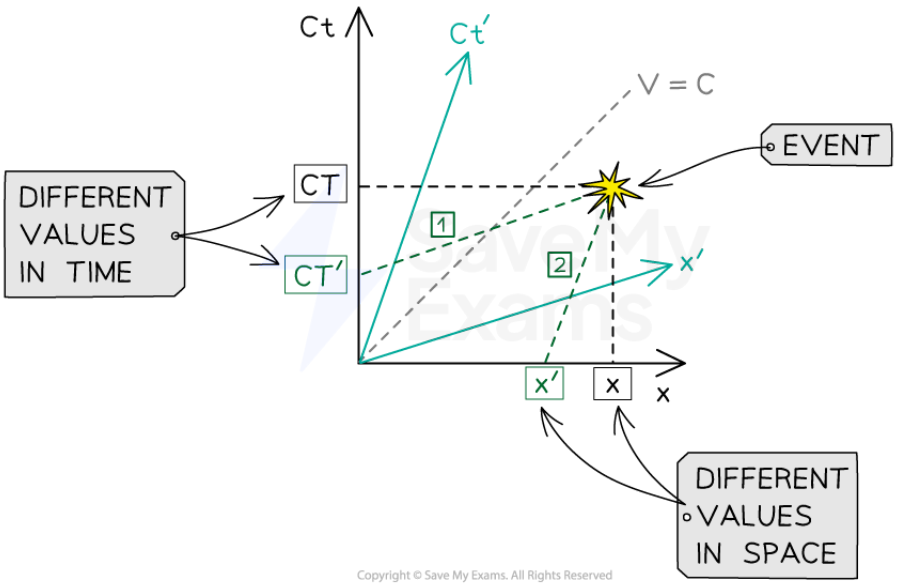
- Every **point** on a spacetime diagram represents an **event**, for example, an object moving
- More than one inertial reference frame can be represented on a spacetime diagram
  - ct and x represent the co-ordinate axes for an observer in frame S
  - ct' and x' represent the co-ordinate axes for an observer in frame S' (moving at speed v with respect to frame S)
- Therefore, we can combine two separate spacetime diagrams for different inertial reference frames moving at constant speed relative to each other
  - The axes for the ct' and x' are at an angle
- The worldline T shows the equivalent worldline of P, but now in the S' reference frame
  - This represents an object at rest in its own co-ordinate system

Your notes



Worldline of a particle at rest in the reference frame S'

- The axes are tilted because the *ct*' reference frame is travelling at speed *v* relative to the *ct* reference frame
  - The *x*' axis must also be tilted in order for the speed of light (the dashed line in the middle) to be the **same** in **both** reference frames
- The **scales** on the time axes ***ct*** and ***ct'*** and on the space axes ***x*** and ***x'*** of two inertial reference frames **moving** relative to one another are **not** the same and are defined by lines of constant spacetime interval
- If an event occurs (such as a flash of light), both reference frames will measure a different time and different position with respect to each other
  - This can be seen on a spacetime diagram

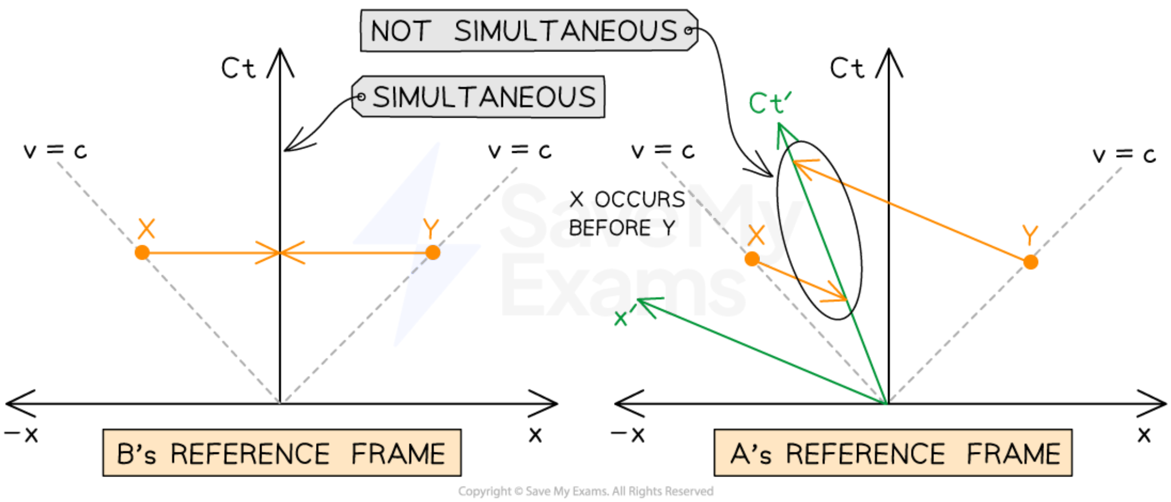


An event is shown in reference frames S and S' with differing values of distance and time

- The event in the S reference frame occurs at (X, cT)
- The event in the S' reference frame occurs at (X', cT')
- The co-ordinates in the S' reference frame are determined by lines 1 and 2
  - Line 1 is a line **parallel** to the *x*' axis
  - Line 2 is a line **parallel** to the *ct'* axis
- The clocks in both frames show zero at the origins where two frames collide i.e. both observers start their clocks at the same time to measure any time intervals

Simultaneity

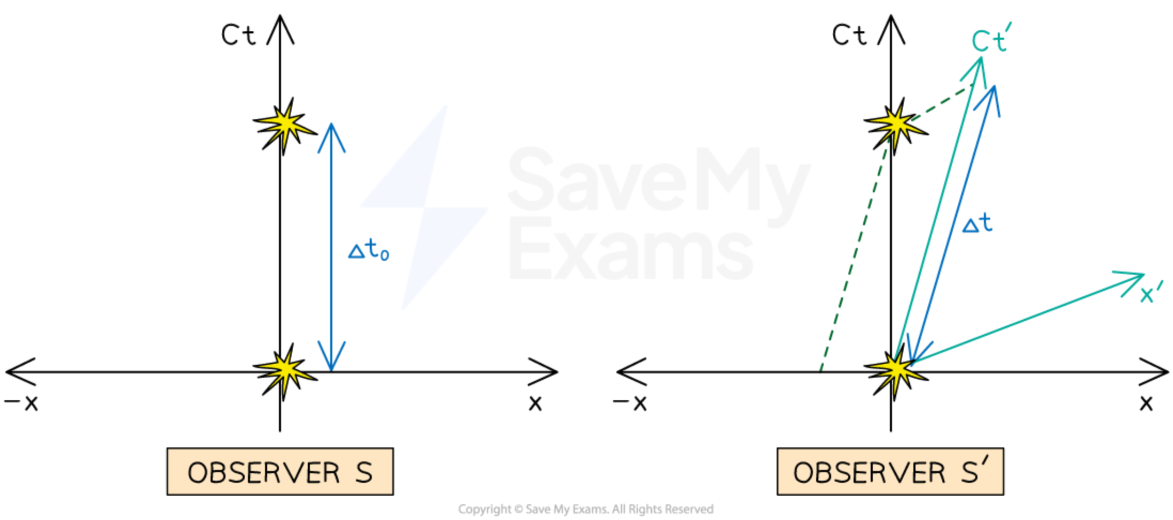
- We can now see that simultaneous events in one frame are not simultaneous in another moving inertial reference frame
- Let's go back to Observers A and B in Simultaneity in Special Relativity
  - We can see that Observer B sees the light reach points X and Y at the same time, whilst Observer A (in the *ct'*-*x'* co-ordinate system) sees the light from the lamp reach point X before point Y on a spacetime diagram



Simultaneity is not possible for two reference frames moving relative to each other

Time Dilation

- Consider two flashes of light at *x* = 0 in the S reference frame that occur one after another
- When these flashes are observed in the S' frame, we can see the time between the flashes is **longer**
  - The time between them has **increased (dilated)**



Spacetime diagrams representing time dilation

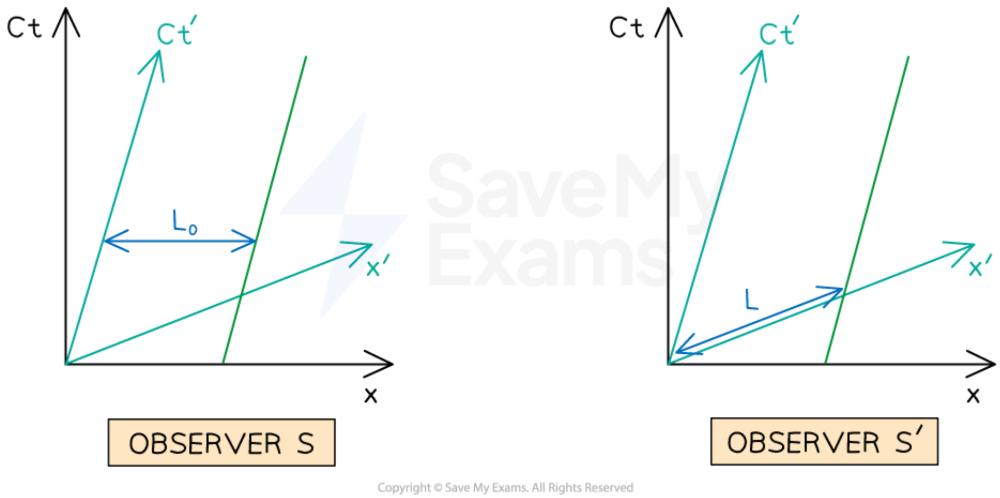
- Another difference is that in the S' reference frame, the first flash now occurs on the -*x* axis
  - This just means it takes place to the **left** of the observer

Length Contraction

- Consider a rod measured in the S reference frame where the rod moving relative to S

Your notes

- The rod has the same speed as it does in the S' reference frame (which is also moving relative to S)



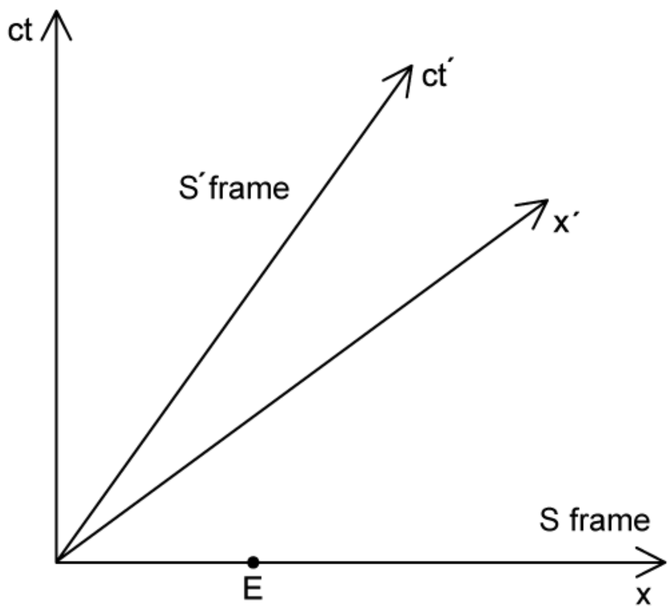
Spacetime diagrams representing length contraction

- The length of the rod is measured by measuring each side at the same time
  - The observer in frame S will measure a length *L*
- When the observer in frame S' measures the rod, they see the rod as stationary (as it is moving at the same speed as the observer)
  - The observer in frame S will measure a length *L*'
- *L* is **shorter** than *L*', which means that the length has been **shortened** (**contracted**) when measured by Observer S, who is **moving** relative to the rod
  - Although it is the rod that is moving, remember, it is at rest **in its own reference frame** and Observer S is moving relative to it
- This occurs from the fact that measurements that are simultaneous in one reference frame are not simultaneous in another

Your notes

Worked example

The spacetime diagram shows the axes of an inertial reference frame S and the axes of a second inertial reference frame S' that moves relative to S with speed 0.6432c. When clocks in both frames show zero the origins of the two frames coincide.



Event E has co-ordinates x = 1.5 m and ct = 0 in frame S.

(a) Label, on the diagram,

(i) the space co-ordinate of event E in the S' frame. Label this event with the letter Q.

(ii) the event that has co-ordinates x' = 1.5 m and ct' = 0. Label this event with the letter R.

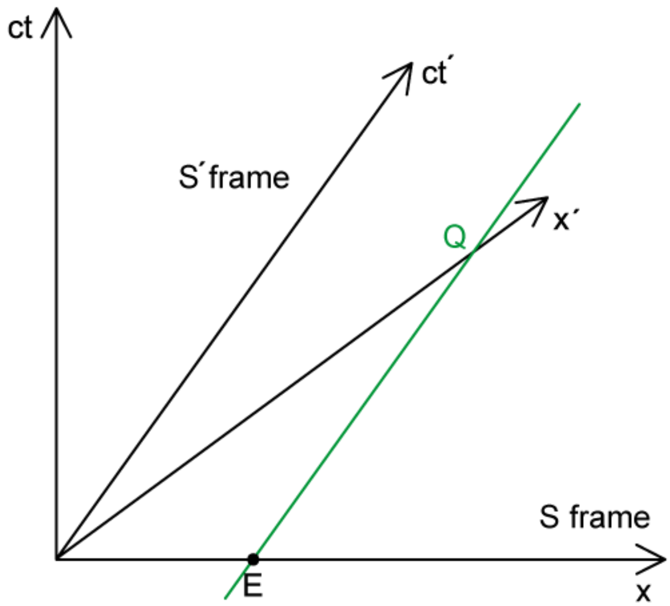
(b) A rod at rest in frame S has a proper length of 1.5 m. At t = 0, the left-hand end of the rod is at x = 0 and the right-hand end is at x = 1.5 m.

Using the spacetime diagram, outline without calculation, why observers in frame S' measure the length of the rod to be less than 1.5 m.

Answer:

(a)

(i) Draw a line parallel to the *ct'* axis



(ii)

Step 1: List the known quantities

- Speed of the spacecraft, *v* = 0.6432c
- Position of event in frame S, *x* = 1.5 m

Step 1: Calculate the *x*' co-ordinate of point Q

- To convert between a position (or time) from one co-ordinate system and another, we can use **Lorentz** transformations

$$x' = \gamma(x - vt)$$

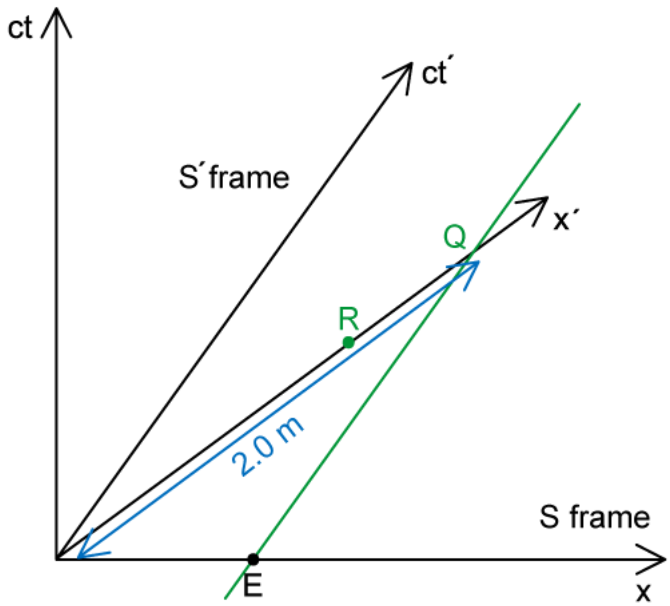
$$x' = \frac{1}{\sqrt{1 - \frac{(0.6432c)^2}{c^2}}}(1.5 - 0) = 1.959 = 2.0 \text{ m}$$

- Since there are no other objects involved, speed *v* = 0

Step 2: Label this point on the axes as R

- The co-ordinates are x' = 1.5 m and ct' = 0
- Point R (at 1.5 m) is roughly $\frac{2}{3}$ of the distance of Q (at 2.0 m)

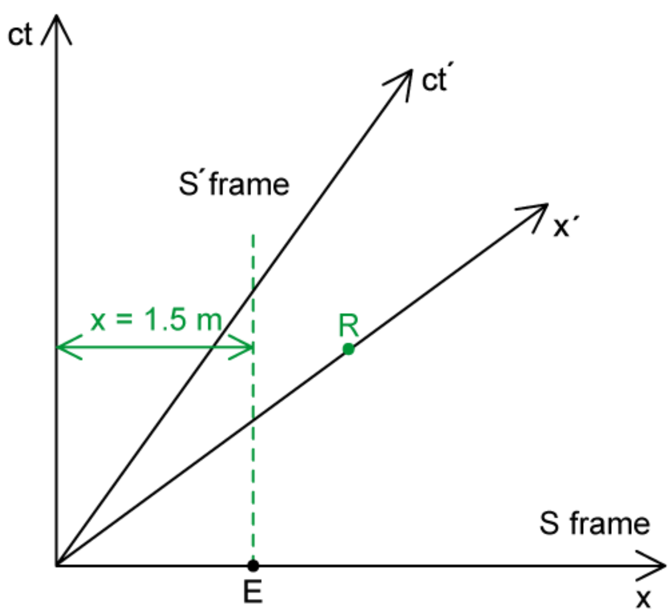
Your notes



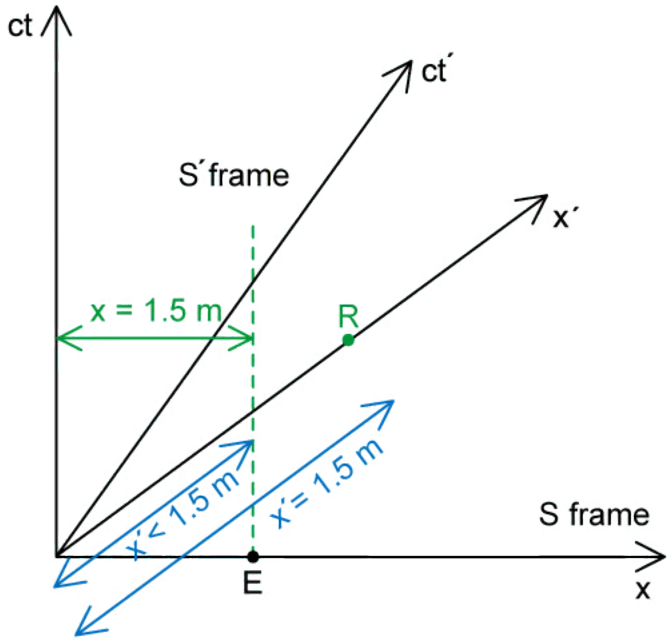
(b)

Step 1: Outline why observers in frame S' measure the length of the rod to be less than 1.5 m

- The ends of the rod must be recorded at the same time in frame S'
- This is shown on the spacetime diagram:



- The right-hand side of the rod intersects the *x*' axis at a co-ordinate that is less than 1.5 m



Examiner Tip

This all might sound counter-intuitive because we're used to thinking of position versus time with distance-time graphs, rather than time versus position. Remember, now the gradient is $\frac{1}{velocity}$ instead of equating to the velocity.

The important thing about worldlines is not their value but their **gradient**. Where they start doesn't matter, whether at the origin or along the *x* axis, their gradients cannot be less than 1.

Make sure you never write c', as there is no such thing. c is the same in all reference frames.

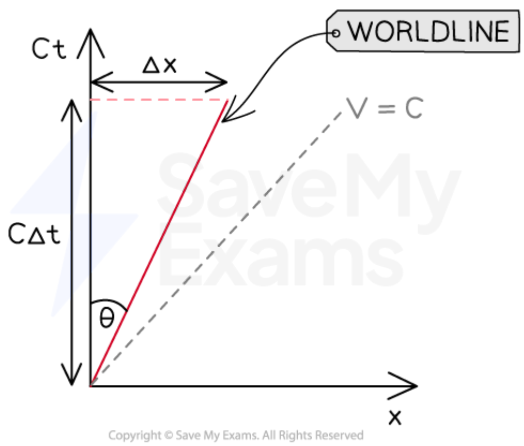
Notice that reading from the *ct*' and *x*' co-ordinate axis is actually no different reading from *ct* and *x*, it's just that they're slanted so it looks a bit different, but the principles are still the same.

Exam questions will generally have the units of *ct* and *x* in **light years (ly)**, so make sure you're comfortable with this definition.

Your notes

Velocity on a Space-Time Diagram

- The worldline for a moving particle on a spacetime diagram using the *x*-*ct* axis is a diagonal line



- The velocity of the particle can be calculated by the angle of the moving particle's worldline with the *ct* axis
- When we are using the *x*-*ct* axis, we can see that:

$$\tan\theta = \frac{opposite}{adjacent} = \frac{\Delta x}{c\Delta t}$$

- From mechanics, we know that velocity is the rate of change of displacement:

$$\frac{\Delta x}{\Delta t} = v$$

- Therefore:

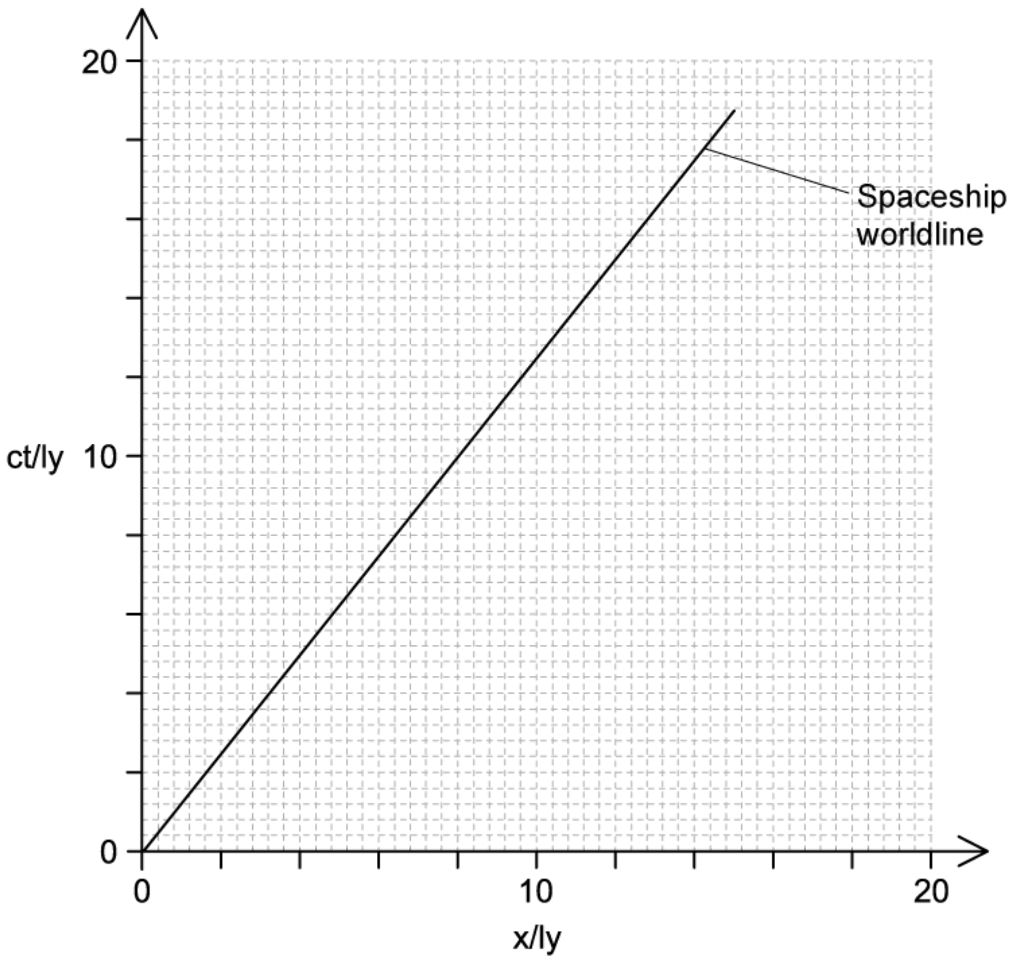
$$\tan\theta = \frac{v}{c}$$

- Where:
  - θ = angle between the world line and the *ct* axis (°)
  - v = velocity of the object (m s^{-1})
  - c = speed of light

Your notes

Worked example

A spaceship travels away from Earth in the direction of a nearby planet. A spacetime diagram for the Earth's reference frame shows the worldline of the spaceship. Assume the clock on the Earth, the clock on the planet, and the clock on the spaceship were all synchronized when ct = 0.



Show, using the spacetime diagram, that the speed of the spaceship relative to the Earth is 0.80c.

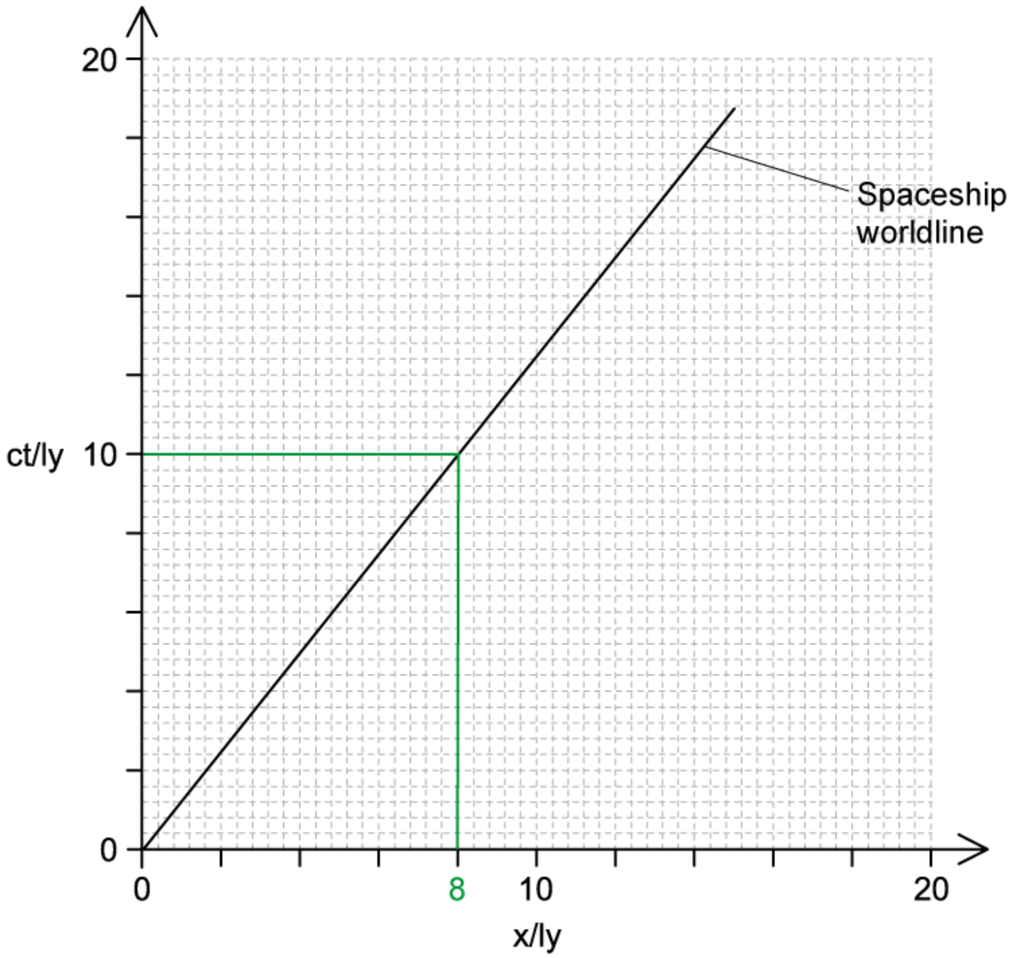
Answer:

Method 1: Using the gradient

Step 1: Choose a co-ordinate pair

- Choose any corresponding value of *ct* and *x* e.g. *ct* = 10, *x* = 8

Your notes



Step 2: Calculate the gradient

- The gradient of a spacetime graph is $\frac{c}{v}$

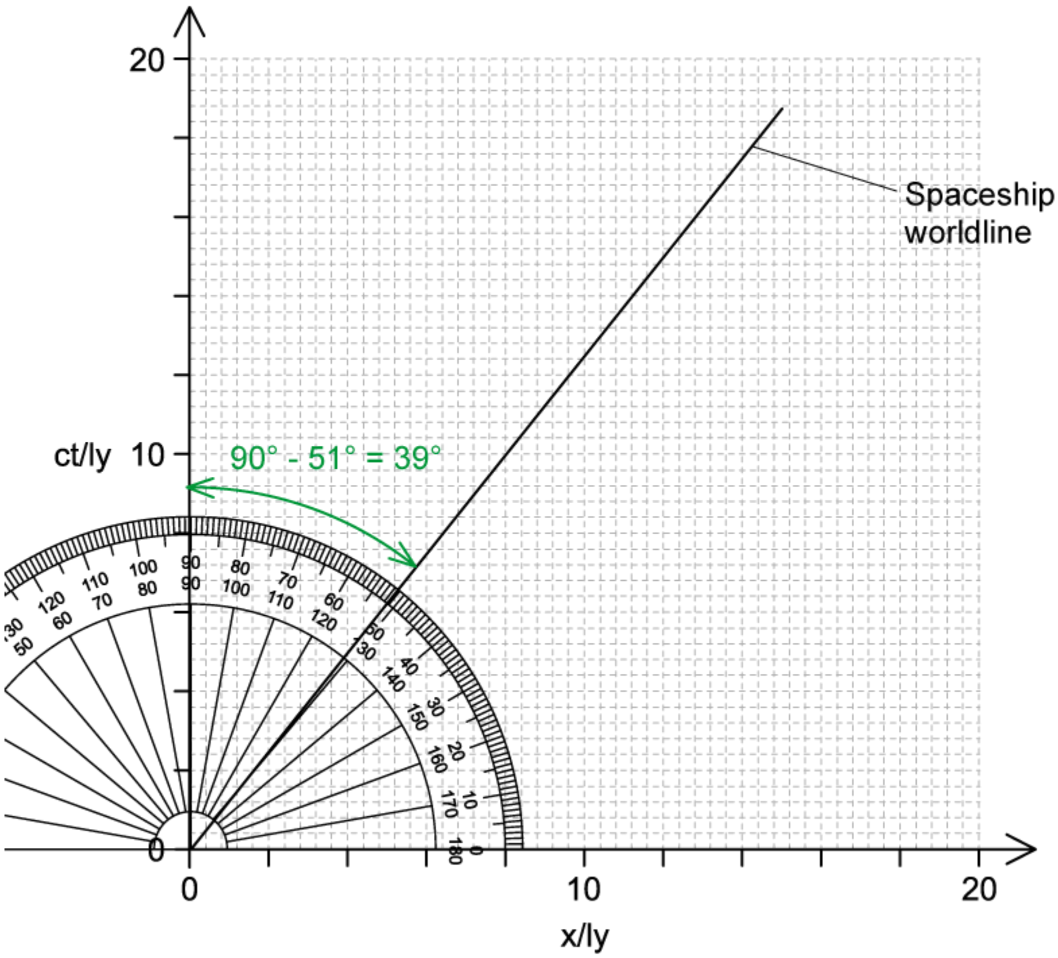
$$gradient = \frac{c}{v} = \frac{10}{8}$$

Step 3: Calculate the velocity, *v*

$$v = \frac{8}{10}c = 0.8\ c$$

Method 2: Measure the angle

Step 1: Measure angle *θ* using a protractor



- $\theta = 39°$

Step 2: Substitute into the velocity equation

$$\tan\theta = \tan(39) = 0.8 = \frac{v}{c}$$

$$v = 0.8\,c$$

Examiner Tip

Any discussion of world lines of moving particles will only be limited to **constant** velocity in your exam.

Make sure you have your **protractor** with you in your exam. An exam question could ask you to measure the angle θ yourself from the diagram on your exam paper.

Your notes

Muon Lifetime Experiment (HL)

Muon Lifetime Experiment

- Muon decay experiments provide **experimental evidence** for time dilation and length contraction
- **Muons** are unstable, subatomic particles that are around 200 times heavier than an electron and are produced in the upper atmosphere as a result of pion decays produced by cosmic rays
- Muons travel at 0.98c and have a half-life of 1.6 μs (or mean lifetime of 2.2 μs)
  - The distance they travel in one half-life is around 470 m
- A considerable number of muons can be detected on the Earth's surface, which is about 10 km from the distance they are created
  - Therefore, according to Newtonian Physics, very few muons are expected to reach the surface as this is about 21 half-lives!
- The detection of the muons is a product of time dilation (or length contraction, depending on the viewpoint of the observer)

Muon Decay From Time Dilation

- According to the reference frame of an observer on Earth, time is dilated so the muon's half-life is **longer**
- We can see this from the time dilation equation

$$\Delta t = \gamma \Delta t_0$$

- Where:
  - The gamma factor, $\gamma = \frac{1}{\sqrt{1 - (0.98)^2}} = 5$
  - Δt = the half-life measured by an observer on Earth
  - Δt_0 = the proper time for the half-life measured in the muon's inertial frame
- Therefore, in the reference frame of an observer on Earth, the muons have a lifetime of

$$\Delta t = 5 \times 1.6 = 8 \text{ μs}$$

- The time to travel 10 km at 0.98c is 33 μs or 4.1 half-lives, so a significant number of muons remain undecayed at the surface

Muon Decay From Length Contraction

- According to the muon's reference frame, length is contracted so the distance the need to travel is **shorter**
- We can see this from the length contraction equation

$$L = \frac{L_0}{\gamma}$$

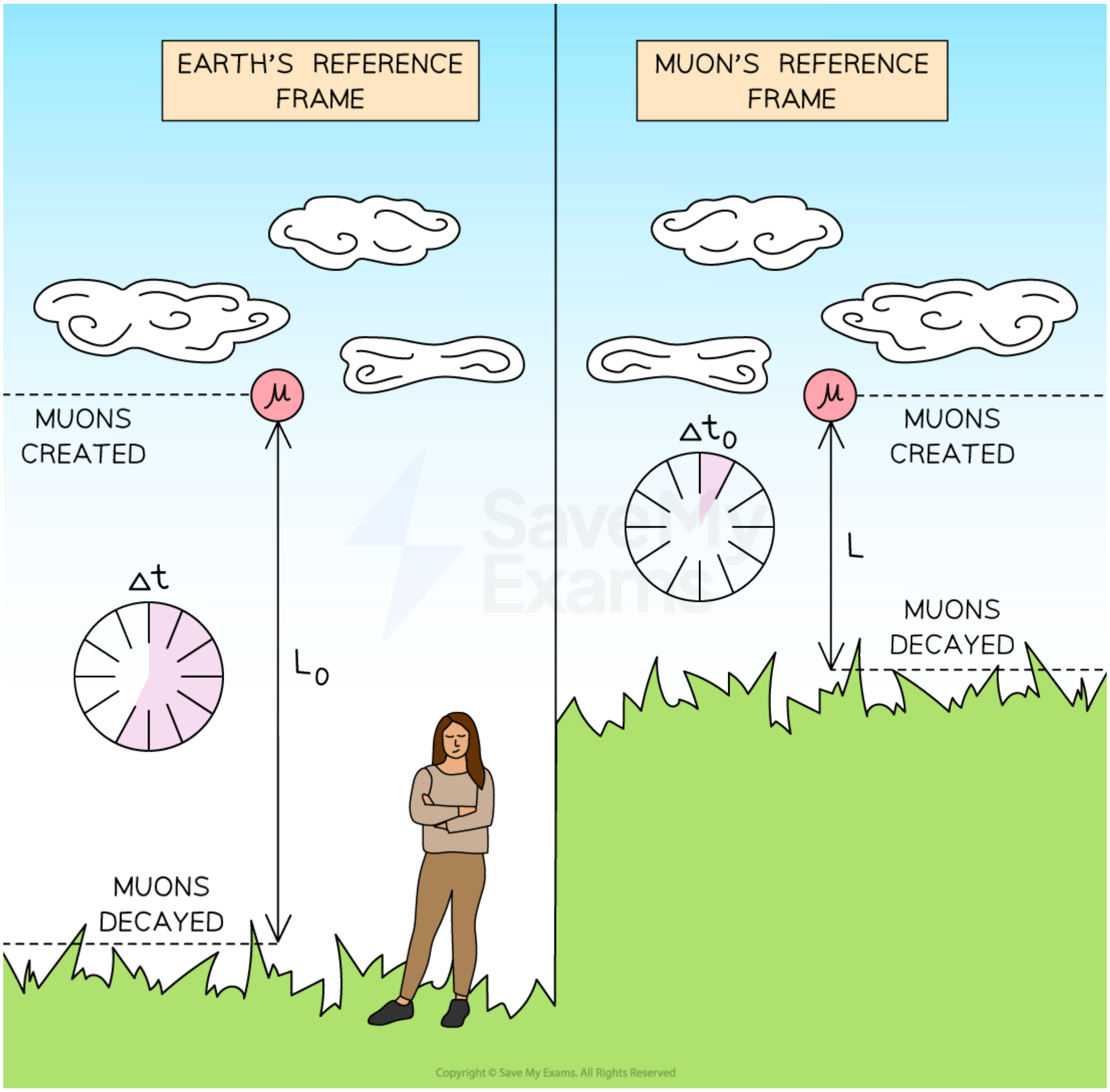
- Where:
  - L_0 = the proper length for the distance measured in the muon's inertial frame
  - L = the distance measured by an observer on Earth
- Therefore, in the reference frame of the muons, they only have to travel a distance:

$$L = \frac{10\,000}{5} = 2000 \text{ m}$$

- To travel this distance takes a time of $\frac{2000}{0.98c}$ = 6.8 μs which is about 4.3 half-lives again, so a significant number of muons remain undecayed at the surface

Your notes

Your notes



Muon decay from the Earth's and a muon's reference frame

Your notes

Worked example

Muons are created at a height of 4250 m above the Earth's surface. The muons move vertically downward at a speed of 0.980c relative to the Earth's surface. The gamma factor for this speed is 5.00. The half-life of a muon in its rest frame is 1.6 μs.

(a) Estimate the fraction of the original number of muons that will reach the Earth's surface before decaying, from the Earth's frame of reference, according to:

(i) Newtonian mechanics

(ii) Special relativity

(b) Demonstrate how an observer moving with the same velocity as the muons, accounts for the answer to (a)(ii).

Answer:

(a) (i)

Step 1: List the known quantities

- Height of muon creation above Earth's surface, h = 4250 m
- Speed of muons, v = 0.980c
- Lifetime of muon, t = 1.6 μs = 1.6×10^{-6} s

Step 2: Calculate the time to travel for the muon

$$time = \frac{distance}{speed} = \frac{h}{v}$$

$$t = \frac{4250}{0.98 \times (3.0 \times 10^8)} = 1.45 \times 10^{-5} \text{ s}$$

Step 3: Calculate the number of half-lives

$$\frac{1.45 \times 10^{-5}}{1.6 \times 10^{-6}} = 9 \text{ half-lives}$$

Step 4: Calculate the fraction of the original muons that arrive

$$\frac{1}{2^9} \times 100\ \% = 0.2\ \%$$

(a) (ii)

Step 1: List the known quantities

- Time for the muon to travel, Δt_0 = 1.45 × 10^{-5} s

Step 2: Calculate the time travelled in the muons rest frame

$$\Delta t = \gamma \Delta t_0$$

$$\Delta t = 5 \times (1.6 \times 10^{-6}) = 8 \times 10^{-6}$$

Step 3: Calculate the number of half-lives

$$\frac{1.45 \times 10^{-5}}{8 \times 10^{-6}} = 1.8 \text{ half-lives}$$

Step 4: Calculate the fraction of the original muons that arrive

$$\frac{1}{2^{1.8}} \times 100\ \% = 29\ \%$$

(b)

Step 1: Analyse the situation

- An observer moving with the same velocity as the muons will measure the **distance** to the surface to be **shorter** by a factor of $\left(\frac{9}{1.8}\right)$ = 5 **OR** length contraction occurs

Step 2: Calculate the distance travelled in the muon's rest frame

$$L = \frac{L_0}{\gamma}$$

$$L = \frac{4250}{5} = 850 \text{ m}$$

Step 3: Calculate the time to travel

$$\text{time taken} = \frac{distance}{speed} = \frac{850}{0.98 \times (3 \times 10^8)} = 2.9 \times 10^{-6}$$

Step 4: Calculate the number of half-lives

Your notes

$$\frac{2.9 \times 10^{-6}}{1.6 \times 10^{-6}} = 1.8$$ half-lives (same as (b))

Your notes

Examiner Tip

Remember that it is the observer on **Earth** that viewed the muons' lifetime or half-life as **longer** (time dilation), whilst it is the **muons'** reference frame that views the distance needed to travel as **shorter** (length contraction).

Always do a sense check with your answer, you must always end up with a longer time or shorter distance for the muons to be observed on the Earth's surface.

Any exam questions on this topic will only use the following equations:

- Time dilation
- Length contraction
- $speed = \frac{distance}{time}$

Calculating half-lives through $\frac{1}{2^{number\ of\ half-lives}}$ is a common way to calculate the number of muons remaining:

- After 1 half-life, $\frac{1}{2}$ the original muons remain
- After 2 half-lives, $\frac{1}{4}$ or $\frac{1}{2^2}$ of the original muons remain
- After 3 half-lives, $\frac{1}{8}$ or $\frac{1}{2^3}$ of the original muons remain, and so on