DP IB Maths: AI SL

Your notes

2.1 Linear Functions & Graphs

Contents

Your notes

2.1.1 Equations of a Straight Line

Equations of a Straight Line

How do I find the gradient of a straight line?

- Find two points that the line passes through with coordinates (x_1, y_1) and (x_2, y_2)
- The gradient between these two points is calculated by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

  - This is given in the **formula booklet**
- The gradient of a straight line measures its **slope**
  - A line with gradient 1 will go up 1 unit for every unit it goes to the right
  - A line with gradient -2 will go down two units for every unit it goes to the right

What are the equations of a straight line?

- $y = mx + c$
  - This is the **gradient-intercept form**
  - It clearly shows the gradient m and the y-intercept (0, c)
- $y - y_1 = m(x - x_1)$
  - This is the **point-gradient form**
  - It clearly shows the gradient m and a point on the line (x_1, y_1)
- $ax + by + d = 0$
  - This is the **general form**
  - You can quickly get the x-intercept $\left(-\frac{d}{a}, 0\right)$ and y-intercept $\left(0, -\frac{d}{b}\right)$

How do I find an equation of a straight line?

- You will need the gradient
  - If you are given two points then first find the gradient
- It is easiest to start with the **point-gradient form**
  - then rearrange into whatever form is required
    - multiplying both sides by any denominators will get rid of fractions
- You can check your answer by using your GDC
  - Graph your answer and check it goes through the point(s)
  - If you have two points then you can enter these in the **statistics mode** and find the regression line $y = ax + b$

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Examiner Tip

- A sketch of the graph of the straight line(s) can be helpful, even if not demanded by the question
  - Use your GDC to plot them
- Ensure you state equations of straight lines in the format required
  - Usually $y = mx + c$ or $ax + by + d = 0$
  - Check whether coefficients need to be integers (they usually are for $ax + by + d = 0$)

Your notes

Worked example

The line l passes through the points $(-2,\ 5)$ and $(6,\ -7)$.

Find the equation of l, giving your answer in the form $ax + by + d = 0$ where $a,\ b$ and d are integers to be found.

Find the gradient between (-2,5) and (6,-7)

Formula booklet

$m = \frac{-7 - 5}{6 - -2} = -\frac{3}{2}$

Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$

Use the point-gradient formula

Formula booklet

Equations of a straight line	$y - y_1 = m(x - x_1)$

$(x_1, y_1) = (-2, 5) \quad m = -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x - -2)$ Simplify

$y - 5 = -\frac{3}{2}(x + 2)$

Multiply by denominator

$2(y - 5) = -3(x + 2)$

Expand

$2y - 10 = -3x - 6$

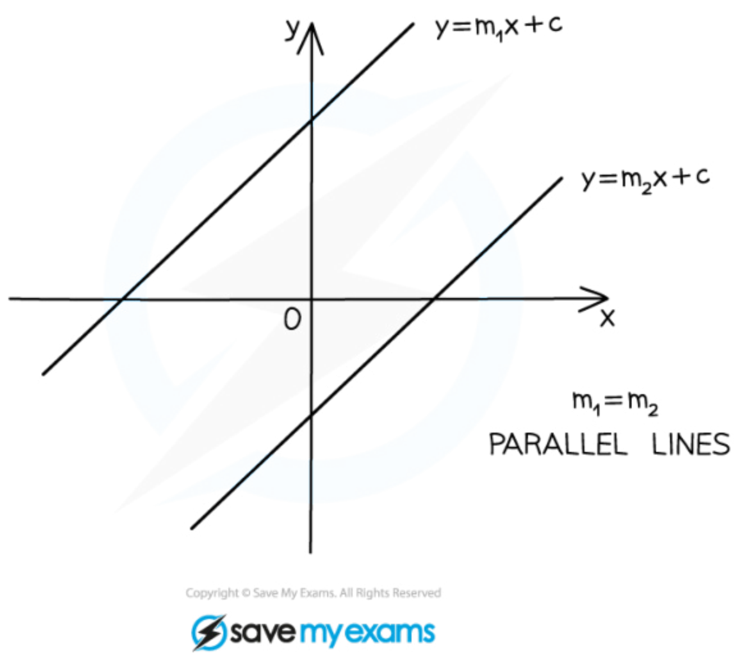
Rearrange

$3x + 2y - 4 = 0$

Parallel Lines

How are the equations of parallel lines connected?

- **Parallel lines** are always equidistant meaning they never intersect
- Parallel lines have the same gradient
  - If the gradient of line l_1 is m_1 and gradient of line l_2 is m_2 then...
    - $m_1 = m_2 \Rightarrow l_1$ & l_2 are parallel
    - l_1 & l_2 are parallel $\Rightarrow m_1 = m_2$
- To determine if two lines are parallel:
  - Rearrange into the gradient-intercept form $y = mx + c$
  - Compare the coefficients of x
  - If they are equal then the lines are parallel



Your notes

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Worked example

The line l passes through the point $(4, -1)$ and is parallel to the line with equation $2x - 5y = 3$.

Find the equation of l, giving your answer in the form $y = mx + c$.

Rearrange into $y = mx + c$ to find the gradient

$5y = 2x - 3 \Rightarrow y = \frac{2}{5}x - \frac{3}{5} \quad \therefore \text{gradient} = \frac{2}{5}$

Parallel lines $\Rightarrow m_1 = m_2$

$m = \frac{2}{5}$

Use the point-gradient formula

Formula booklet

Equations of a straight line	$y - y_1 = m(x - x_1)$

$(x_1, y_1) = (4, -1) \quad m = \frac{2}{5}$

$y + 1 = \frac{2}{5}(x - 4)$

$y + 1 = \frac{2}{5}x - \frac{8}{5}$

$y = \frac{2}{5}x - \frac{13}{5}$

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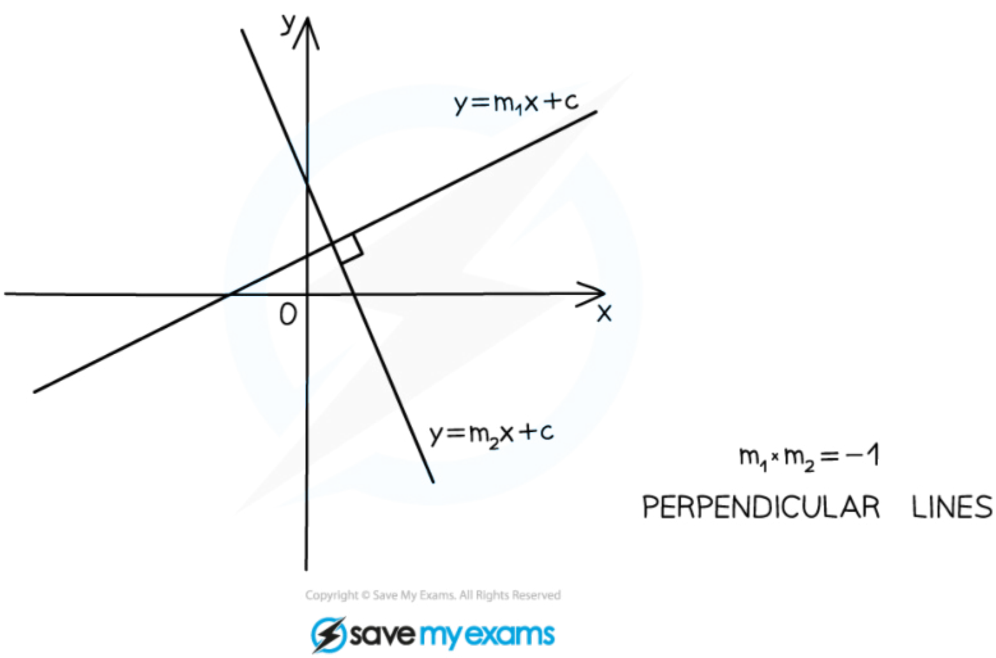
Perpendicular Lines

How are the equations of perpendicular lines connected?

- **Perpendicular lines** intersect at right angles
- The gradients of two perpendicular lines are negative reciprocals
  - If the gradient of line l_1 is m_1 and gradient of line l_2 is m_2 then...
    - $m_1 \times m_2 = -1 \Rightarrow l_1 \ \& \ l_2$ are perpendicular
    - $l_1 \ \& \ l_2$ are perpendicular $\Rightarrow m_1 \times m_2 = -1$
- To determine if two lines are perpendicular:
  - Rearrange into the gradient-intercept form $y = mx + c$
  - Compare the coefficients of x
  - If their product is -1 then they are perpendicular
- Be careful with horizontal and vertical lines
  - $x = p$ and $y = q$ are perpendicular where *p* and *q* are constants



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Worked example

The line l_1 is given by the equation $3x - 5y = 7$.

The line l_2 is given by the equation $y = \frac{1}{4} - \frac{5}{3}x$.

Determine whether l_1 and l_2 are perpendicular. Give a reason for your answer.

Rearrange l_1 into $y = mx + c$ form

$5y = 3x - 7 \Rightarrow y = \frac{3}{5}x - \frac{7}{5}$

Identity gradients

$m_1 = \frac{3}{5}$ $m_2 = -\frac{5}{3}$

$m_1 \times m_2 = -1 \Rightarrow$ Perpendicular lines

$\frac{3}{5} \times -\frac{5}{3} = -1$

l_1 and l_2 are perpendicular as $m_1 \times m_2 = -1$