IB · DP · Maths

2 hours   12 questions

Practice Paper 2

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Total Marks /110

1 (a) On 1st January 2021, Nerys invests $\$P$ in an account that pays a nominal annual interest rate of 4.2%, compounded **monthly**.

The amount of money in Nerys' account **at the end of each year** follows a geometric sequence with common ratio, r.

Find the value of r, giving your answer to four decimal places.

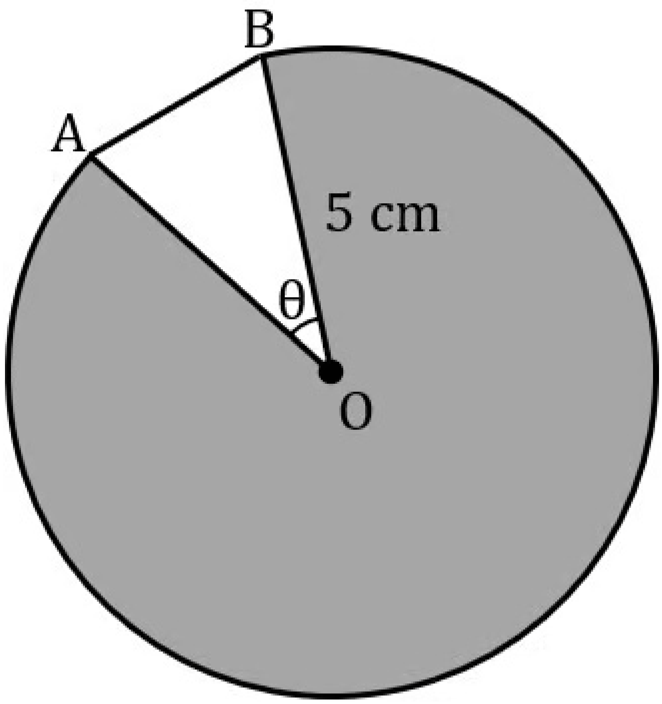
(3 marks)

(b) Nerys makes no further deposits to or withdrawals from the account.

Find the year in which the amount of money in Nerys' account will become double the amount she invested.

(3 marks)

2 (a) A circular pond with radius 0.8 m contains 16 lily pads. The diagram below shows the shape of each lily pad as part of a circle with centre O and radius 5 cm, $A\widehat{O}B = \theta$.



The lily pads cover 5% of the pond's surface.

Find the surface area of each lily pad.

(2 marks)

(b) Find the value of θ, giving your answer in radians.

(2 marks)

(c) Find the area of the triangle AOB.

(2 marks)

3 (a) A car safety expert is investigating a possible link between the tread depth of a car's tyres and the car's stopping distance.

Using the same car on the same track under the same weather conditions the expert records the average tread depth, $(x \text{ mm})$, from the car's four tyres and the stopping distance,$(y \text{ m})$, when the car's brakes are applied at a particular speed.

Tread depth (x)	6.8	1.4	4.1	0.9	5.7	1.9	3.5	2.6	2.9
Stopping distance (y)	29	45	33.5	49.5	31	42	34	36.5	36

(i) Calculate the Pearson product moment correlation coefficient for these data.

(ii) State the type of linear correlation that is shown between tread depth and stopping distance.

(2 marks)

(b) Let L be the regression line of y on x.

(i) Find the equation of L in the form $y = a + bx$.

(ii) Give an interpretation of the values of a and b in the context of the investigation.

(3 marks)

(c) The researcher remembers that he had also done a test of the car when its tyres had an average tread depth of 4.8 mm, but that he had forgotten to record the stopping distance for that tread depth. Use an appropriate regression equation to estimate the value of the missing stopping distance.

(2 marks)

4 (a) A six-sided biased die is weighted in such a way that the probability of obtaining a “one” is $\frac{3}{7}$.

The die is tossed 10 times. Find the probability of obtaining

at most four “ones”.

(3 marks)

(b) the fourth “one” on the tenth toss

(3 marks)

5 The complex numbers w and z satisfy the equations

$$\frac{w}{z} = -\mathrm{i}$$

$$w - 4z^* = -3 + 18\mathrm{i}.$$

Find w and z in the form $a + b\mathrm{i}$ where $a,\ b \in \mathbb{R}$.

(7 marks)

6 **(a)** The velocity, $v \text{ m s}^{-1}$, of a particle, at time t seconds, is given by $v(t) = 10e^{0.5t}\sin 2t$, $0 \le t \le \pi$.

Find the maximum speed of the particle and at what time this occurs.

(3 marks)

(b) Find the initial acceleration of the particle.

(2 marks)

(c) Show that the distance travelled by the particle is 48.0 m to the nearest 0.1 m.

(2 marks)

7 (a) In a particular game a team begins each round with a full squad of 6 players. During each round it is possible that one or more of the players will be eliminated, with the number of players remaining at the end of a round following the probability distribution in the table below.

X	1	2	3	4	5	6
$P(X=x)$	0.25	0.42	0.15	0.12	0.05	0.01

The team receives a basic 5 points at the end of each round, plus an additional 2 points for each player still remaining at the end. Let S represent the total number of points scored per round.

Find $E(S)$.

(4 marks)

(b) Given that $\text{Var}(X) = 0.64$, find $\text{Var}(S)$.

(2 marks)

8 Consider the graphs of $y = \dfrac{3x^2}{x-2}$ and $y = m(x+2)$, $m \in \mathbb{R}$.

Find the set of values for m such that the two graphs have at least one point of intersection.

(5 marks)

9 Two players, A and B, are on the level putting green of a golf course with the hole located at point $O(0,0)$. The players hit their balls along the ground at the same time, such that the position vectors of their balls relative to the hole t seconds after being hit are given respectively by

$$r_A = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$r_B = \begin{pmatrix} 3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 9 \\ -2 \end{pmatrix}$$

where distances are measured in cm.

Find the minimum distance between the two golf balls.

(5 marks)

10 (a) In a chemistry lab a tank originally contains a pure solvent. Solvent containing a chemical, X, is allowed to flow into the tank. The solution is kept uniform by a stirring mechanism, and excess solution leaves the tank through an outlet at its base. Let x grams represent the amount of chemical X in the tank at time t minutes after the solvent with the chemical began flowing into the tank. The rate of change of the amount of chemical X in the tank, $\frac{\mathrm{d}x}{\mathrm{d}t}$, is described by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}y} = 25\mathrm{e}^{-\frac{t}{2}} - \frac{x}{t+4}.$$

Show that $t+4$ is an integrating factor for this differential equation.

(2 marks)

(b) Hence, by solving the differential equation, show that $x(t) = \dfrac{300 - 50\mathrm{e}^{-\frac{t}{2}}(t+6)}{t+4}$.

(8 marks)

(c) Sketch the graph of x versus t for $0 \le t \le 60$ and hence find the maximum amount of chemical X in the tank and the value of t at which this occurs.

(5 marks)

(d) Find the value of t at which the amount of chemical X in the tank is decreasing most rapidly.

(2 marks)

(e) The rate of change of the amount of chemical X leaving the tank is equal to $\frac{x}{t+4}$.

Find the amount of chemical X that left the tank during the first 60 minutes.

(4 marks)

11 (a) Show that $3\cot 2\theta = \dfrac{3(1-\tan^2\theta)}{2\tan\theta}$

(2 marks)

(b) Verify that $x = -\tan\theta$ and $x = -6\cot 2\theta$ satisfy the equation $x^2 + (3\cot\theta - 2\tan\theta)x + (3 - 3\tan^2\theta) = 0$.

(4 marks)

(c) Hence show that the value of $\tan\frac{\pi}{8}$ must satisfy the equation

$$\tan^3\frac{\pi}{8} - 4\tan^2\frac{\pi}{8} - 13\tan\frac{\pi}{8} + 6 = 0$$

.

(5 marks)

(d) Use the identity from part (a) to show that the exact value of $\tan\frac{\pi}{8} = \sqrt{2} - 1$, and confirm that this satisfies the equation from part (c).

(4 marks)

(e) Using the results from parts (a) and (d), find the exact value of $\frac{3-3\tan^2\theta}{\tan\theta}$ when $\theta=\frac{\pi}{16}$. Give your answer in the form $a+b\sqrt{2}$ where $a,b\in\mathbb{Z}$.

(4 marks)

12 (a) For cans of a particular brand of soft drink labelled as containing 330 ml, the actual volume, V ml, of soft drink in a can is normally distributed with mean 330 and variance σ^2.

The probability that V is greater than 336 is 0.1288.

Find $P(330 < V < 336)$.

(2 marks)

(b) (i) Find σ, the standard deviation of V.

(ii) Hence, find the probability that a can of soft drink selected at random will contain less than 320 ml of soft drink.

(5 marks)

(c) Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans that contain less than 320 ml of soft drink.

Find $E(L)$.

(3 marks)

(d) Find the probability that exactly two of the cans contain less than 320 ml of soft drink.

(2 marks)

(e) A can selected at random contains more than 320 ml of soft drink.

Find the probability that it contains between 330 ml and 335 ml of soft drink.

(3 marks)