IB · DP · Maths

2 hours · 12 questions

Practice Paper 2

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Total Marks	**/110**

1 (a) Jennifer sells cups of tea at her shop and has noticed that she sells more tea on cooler days.

On five different days, she records the maximum daily temperature, T , measured in degrees Celsius, and the number of cups of teas sold, C. The results are shown in the following table.

Maximum Daily Temperature, T	3	5	8	9	12
Cups of tea sold, C.	37	34	33	26	21

The relationship between T and C can be modelled by the regression line of C on T with equation $C = aT + b$.

i) Find the value of a and the value of b.

ii) Write down the value of Pearson's product-moment correlation coefficient, r.

(4 marks)

(b) Use your regression equation from part (a) (i) to estimate the number of teas that Jennifer will sell on a day when the maximum temperature is 11 °C.

(2 marks)

2 (a) A scientist is studying the movement of snails and has observed that the distribution of their speeds, S, follows a normal distribution with a mean of 48 m/h and a standard deviation of 1.5 m/h.

Sketch a diagram to represent this information.

(2 marks)

(b) Find the probability that a randomly selected snail has a speed of less than 46.5 m/h.

(2 marks)

(c) From a sample of 80 snails, calculate the expected number of snails that would have a speed of less than 46.5 m/h. Give your answer to the nearest integer.

(2 marks)

3 (a) A hamster runs in its exercise wheel, rotating the wheel at a constant speed. The wheel has a diameter of 14 centimetres and the top of the wheel is positioned at a height of k centimetres above the floor of the cage.

A point at the top of the wheel is marked before the hamster starts to run, turning the wheel clockwise. The hamster takes 4 seconds to turn the wheel one complete revolution.

After t seconds, the height of the mark on the wheel above the floor of the cage is given by

$$h(t) = 10 + a\cos\left(\frac{\pi}{2}t\right) \; for \; 0 \le t \le 150$$

After 26 seconds, the mark is 3 cm above the cage floor. Find k.

(2 marks)

(b) Find the value of a.

(3 marks)

4 (a) A particle moves along a straight line with a velocity, $v\ ms^{-1}$, given by $v = 2^t - 2$ where t is measured in seconds such that $0 \le t \le 4$.

Find the acceleration of the particle at time $t = 2$.

(2 marks)

(b) State the time when the particle comes to rest.

(1 mark)

(c) Find the total distance travelled by the particle.

(3 marks)

5 In the expansion of $\left(\frac{1}{2}x + 1\right)^n$, the coefficient of the x^2 term is $8n$, where $n \in \mathbb{Z}^+$.

Find n.

(5 marks)

6 (a) A plane lies parallel to the line with equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 9 \\ 1 \end{pmatrix}$ and contains the points P and X with coordinates $(5, 4, 5)$ and $(-2, 2, 0)$ respectively.

Find the vector $\overrightarrow{\mathrm{PX}}$.

(2 marks)

(b) By appropriate use of the vector product, find the normal to the plane.

(2 marks)

(c) Hence find the Cartesian equation of the plane.

(2 marks)

7 (a) A continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} k \sin 2x, & 0 \le x \le \frac{\pi}{3} \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k.

(2 marks)

(b) Giving your answers to three significant figures, find

(i) the mean of X,

(ii) the mode of X.

(3 marks)

(c) (i) Write down $P\left(X = \frac{\pi}{3}\right)$.

(ii) Show that the median, m, of X lies in the interval $\frac{\pi}{6} < m < \frac{\pi}{3}$.

(2 marks)

8 (a) It is given that that $z_1 = 2e^{i\left(\frac{\pi}{3}\right)}$ and $z_2 = 3\,\text{cis}\left(\frac{n\pi}{12}\right)$, $n \in \mathbb{Z}^+$.

Find the value of $z_1 z_2$ for $n = 3$.

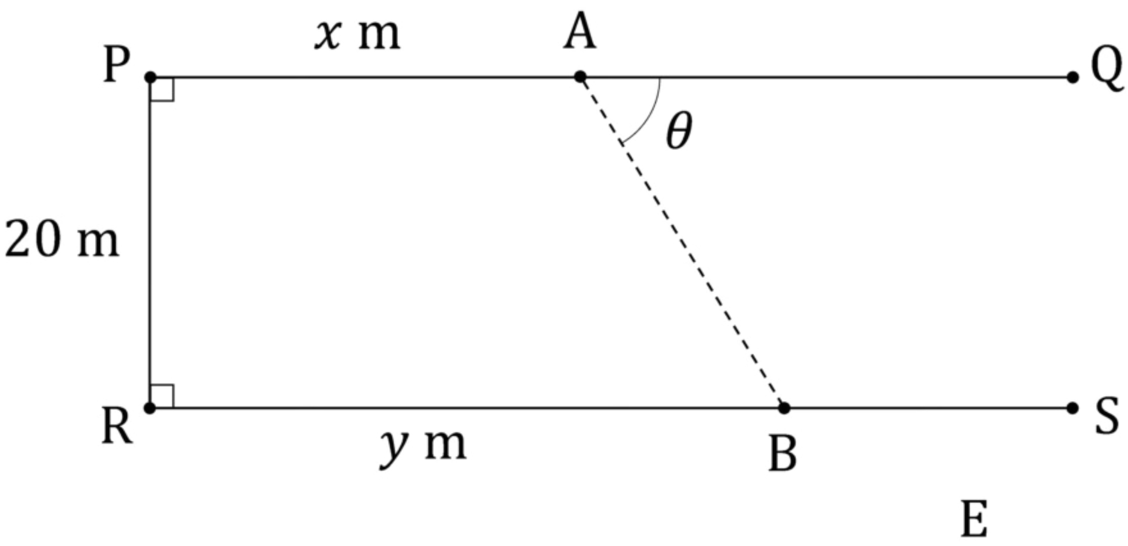
(3 marks)

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$.

(3 marks)

9 (a) In a robotics research facility two robots A and B are moving along parallel tracks [PQ] and [RS]. Robot A begins at point P and moves towards point Q, and robot B begins at point R and moves towards point S. [PR] is perpendicular to both [PQ] and [RS], and the distance from P to R is 20 metres.

Both robots start moving at the same moment, and the distances travelled by robot A and robot B after t seconds are x metres and y metres, respectively. The angle θ is the radian measure of angle $\mathrm{B\hat{A}Q}$, where points A and B indicate the positions of robots A and B respectively at any time t. This information is shown on the following diagram.



Show that $y = x + 20\cot\theta$.

(1 mark)

(b) At time T, the following conditions are true:

Robot B has travelled 4 metres further than robot A.

The speed of Robot A is only two thirds of the speed of robot B.

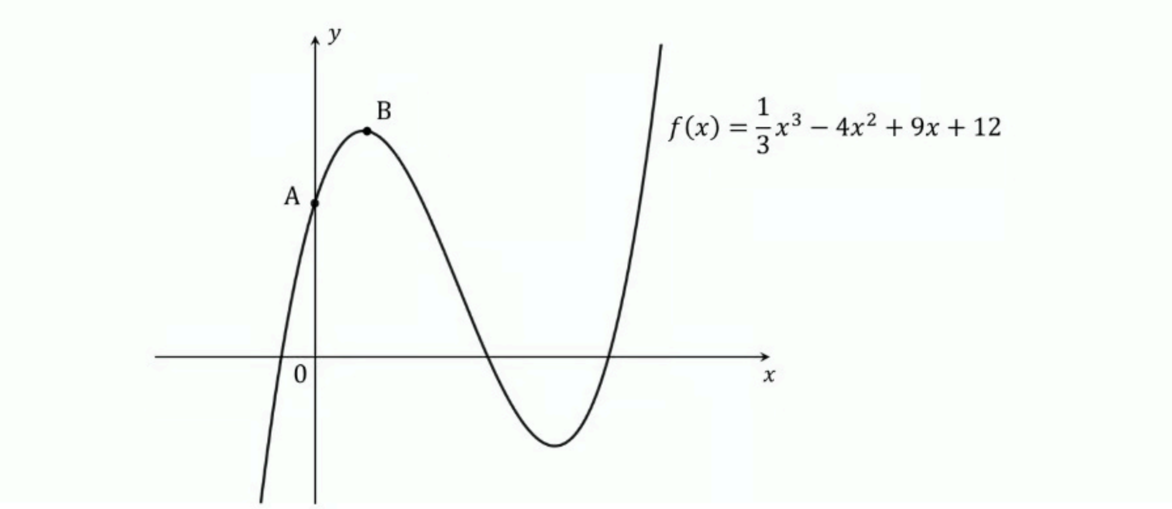
The rate of change of the angle θ is -0.05 radians per second.

Find the speed of robot A at time T.

(6 marks)

10 (a) The diagram below shows a part of the graph of the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 9x + 12$$



Point A is the point of intersection between the graph and the y-axis. Write down the coordinates of point A.

(1 mark)

(b) Find $f'(x)$.

(2 marks)

(c) Using the graph, explain why the equation $f'(x) = 0$ must have exactly two distinct real solutions.

(3 marks)

(d) Point B is the point on the graph with x-coordinate $\frac{8-\sqrt{26}}{2}$.

Find the gradient of the tangent line to the graph at point B.

(2 marks)

(e) Points C and D are the points on the graph at which the tangent lines are perpendicular to the tangent line at point B.

By first determining the gradient of the tangents at points C and D, find the x-coordinates of points C and D.

(4 marks)

(f) Given that point C lies between points A and B on the graph, find the equation of the tangent line to the graph at point C.

(4 marks)

11 (a) The function f is defined by $f(x) = \dfrac{4x+3}{9x^2-4}$, for $x \in \mathbb{R}$, $x \neq p$, $x \neq q$.

Given that $p < q$, find the value of p and the value of q.

(2 marks)

(b) Find an expression for $f'(x)$.

(3 marks)

(c) The graph of $y = f(x)$ has exactly one point of inflection.

Find the x-coordinate of the point of inflection.

(2 marks)

(d) Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$, showing the values of any axes intercepts, the coordinates of any local maxima and local minima, and giving the equations of any asymptotes.

(5 marks)

(e) The function g is defined by $g(x) = \dfrac{9x^2 - 4}{4x + 3}$, for $x \in \mathbb{R}$, $x \neq -\dfrac{3}{4}$.

Find the equations of all the asymptotes on the graph of $y = g(x)$.

(4 marks)

(f) By considering the graph of $y = f(x) - g(x)$, or otherwise, solve $f(x) < g(x)$ for $x \in \mathbb{R}$.

(4 marks)

12 (a) The derivative of the function f is given by $f'(x) = \dfrac{1}{x(k-x)}$, $x \in \mathbb{R}$, $x \neq 0$, $x \neq k$, where $k > 0$ is a real constant.

By finding appropriate constants a and b in terms of k, show that the expression for $f'(x)$ can be written in the form $\dfrac{a}{x} + \dfrac{b}{k-x}$, where $a, b \in \mathbb{R}$.

(3 marks)

(b) Hence find an expression for $f(x)$.

(3 marks)

(c) Consider a population of lizards, P, which has an initial size of 800. The rate of change of the population can be modelled by the differential equation $\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{P(k-P)}{25k}$, where t is the time measured in years, $t \geq 0$, and k is the maximum sustainable population.

By solving the differential equation, show that

$$P = \frac{800k}{(k-800)e^{-\frac{t}{25}} + 800}$$

(8 marks)

(d) At $t = 12$ the lizard population has reduced in size to three fourths of its original value.

Find the value of k, giving your answer correct to four significant figures.

(3 marks)

(e) Find the value of t when the population is decreasing at a rate of 16 lizards per year.

(3 marks)