IB · DP · Maths

1 hour 5 questions

Practice Paper 2

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Total Marks **/80**

1 (a) Sharon set up an experiment to investigate the relationship between the mass of a mouse and the time the mouse takes to complete a mini assault course. She conducted the experiment with six mice and recorded her results in the table below.

Mouse mass, x (g)	19.9	18.3	21.1	19.8	17.5	16.3
Time, y (seconds)	18.0	17.7	20.9	18.6	15.0	14.2

i) Calculate Pearson's product-moment correlation coefficient, r.

ii) Describe the relationship between the mass of the mice and the time taken to complete the mini assault course.

(3 marks)

(b) Write down the equation of the regression line of y on x, in the form $y = mx + c$.

(2 marks)

(c) Find the coordinates of the point $\mathrm{M}(\bar{x}, \bar{y})$.

(2 marks)

(d) Show that the point $\mathrm{M}(\bar{x}, \bar{y})$ lies on the line of regression.

(2 marks)

(e) The mass of a seventh mouse is found to be 20.6 g.

i) Using your line of regression, estimate the time that the seventh mouse will take to complete the mini assault course.

ii) Justify whether it is valid to use the line of regression to estimate the result for the seventh mouse.

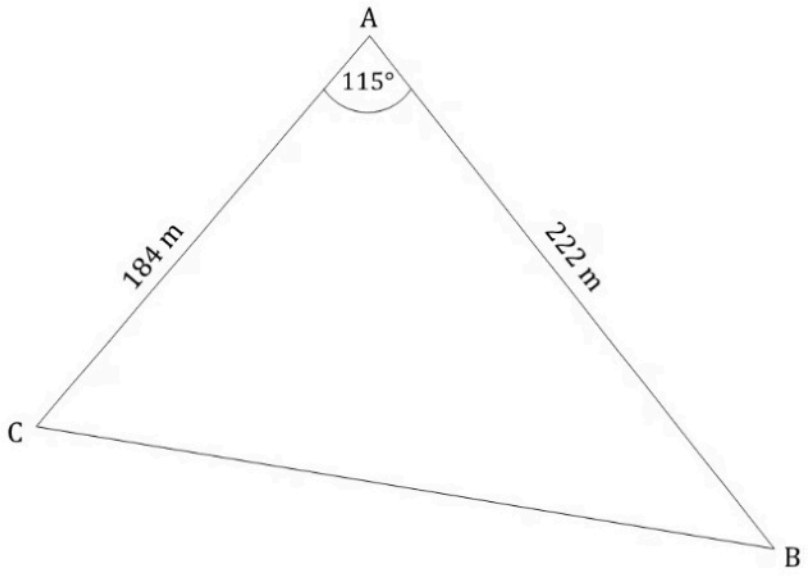
(4 marks)

(f) In the actual experiment, it was found that the seventh mouse took 20.7 seconds to complete the mini assault course.

Calculate the percentage error in the estimated value.

(2 marks)

2 (a) A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point B to point C, and the farmhouse is located at point A. AB and AC form the other two sides of the farm, and the distances from the farmhouse to points B and C are 222 m and 184 m respectively. Angle $C\widehat{A}B$ is 115°, and points A, B and C lie in a horizontal plane.



Calculate the distance along the motorway from B to C.

(2 marks)

(b) The cost of fencing in US dollars (USD) is $89.99 per metre.

Calculate the total cost of fencing the whole perimeter of the farm. Give your answer to 2 decimal places.

(2 marks)

(c) Calculate the area of the farm.

(2 marks)

(d) Find the sizes of angles $A\widehat{B}C$ and $A\widehat{C}B$.

(2 marks)

(e) Calculate the shortest distance from the farmhouse to the motorway.

(3 marks)

(f) A vertical signpost is located at point C, and the top of the signpost is designated as point D. The angle of elevation to the top of the signpost from point B is measured to be 1.4°.

Calculate the distance CD, the vertical height of the signpost.

(2 marks)

(g) Calculate the distance between the top of the signpost, D, and point A.

(2 marks)

(h) The rate of growth of the grass on the farm, G, in inches per month, can be modelled by the function

$$G(T) = -0.015(T - 40)(T - 80)$$

where T is the temperature in degrees Fahrenheit.

Find the maximum rate of grass growth on the farm and the temperature required.

(3 marks)

3 (a) Anna decides she wants to buy a farm and the bank agree to give her a loan provided she makes a 13.9% deposit of $40 000.

Calculate the value of the farm.

(1 mark)

(b) She currently has $15 000 saved up and decides to invest it in some high risk high growth shares forecasted to grow at 65% annually.

Calculate the forecasted number of years it will take for her to be able to afford the deposit.

(2 marks)

(c) 1.5 years later, the shares outperform their forecasted growth rate and Anna is able to afford the deposit on the farm.

Calculate the percentage error between the forecasted annual growth rate and the actual annual growth rate of the shares.

(3 marks)

(d) Anna now takes out the loan from the bank.

Write down the amount of the loan.

(1 mark)

(e) The loan is for 25 years, compounded monthly, with equal monthly payments of $1200.

For this loan, find

i) the amount of interest paid by Anna,

ii) the annual interest rate of the loan.

(5 marks)

(f) After 15 years of paying off this loan, Anna decides to pay the **remainder** in one final payment.

Find the amount of Anna's final payment.

(3 marks)

(g) Find how much money Anna saved by making one final payment after 20 years.

(3 marks)

4 (a) It is believed that the time in minutes, T, that a customer spends on hold during a call when calling a customer service line can be modelled by a normal distribution, with $T \sim N(17, 2.8^2)$.

Using the model find, correct to four decimal places, the probability that during a call a customer chosen at random spends

i) less than 15 minutes on hold

ii) more than 23 minutes on hold.

(3 marks)

(b) 500 customer service calls are monitored and the length of time that a customer is put on hold for during each call is measured.

By again using the model find, correct to one decimal place, the expected number of the 500 calls in which a customer is put on hold for between

i) 15 and 20 minutes

ii) 20 and 23 minutes.

(3 marks)

(c) For the 500 monitored calls, the measured lengths of time that a customer was put on hold for during each call are summarised in the following table.

Length of time on hold, T	Number of calls
Less than 15 minutes	98
Between 15 and 20 minutes	309
Between 20 and 23 minutes	79
More than 23 minutes	14

It is decided to perform a χ^2 goodness of fit test at the 10% level of significance to decide whether the length of time that a customer is put on hold for during a call can indeed be modelled by a normal distribution, with $T \sim N(17, 2.8^2)$.

State the null and alternative hypotheses.

(2 marks)

(d) Find the p-value for the test.

(3 marks)

(e) State the conclusion of the test. Give a reason for your answer.

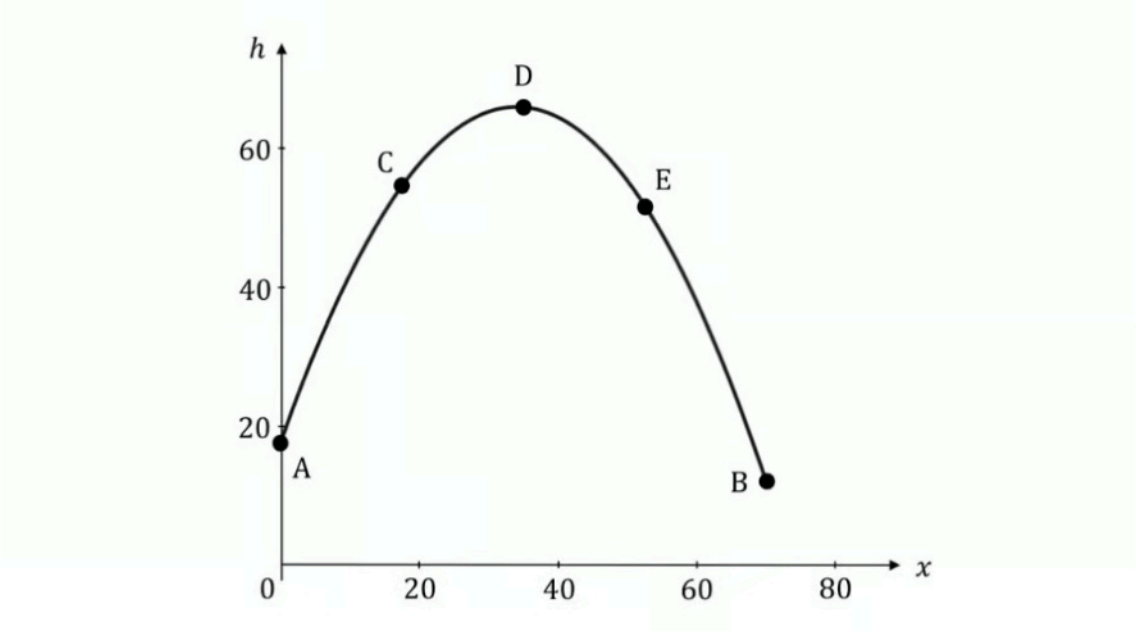
(2 marks)

5 (a) The cross-sectional profile of a hill is modelled by the function

$$h(x) = \frac{1}{6}\left(17x + 107 - \frac{x^2}{4}\right), \qquad 0 \leq x \leq 70,$$

where h is the altitude above mean sea level, in metres, and x is the horizontal distance, in metres, from a fixed point O.

The cross-sectional profile of the hill can be seen in the diagram below.



Point A has coordinates (0, 17.8) correct to 3 significant figures, and point B has exact coordinates (70, 12).

Calculate the altitude at $x = 3$.

(2 marks)

(b) A point P is at an altitude of 40 m.

Find the possible values of its horizontal distance from O.

(3 marks)

(c) Find $h'(x)$.

(2 marks)

(d) Hence calculate the maximum altitude of the hill.

(2 marks)

(e) When $x = 17.5$, the altitude of the hill is 54.7 m, when $x = 35$, the altitude of the hill is 66.0 m, and when $x = 52.5$ the altitude of the hill is 51.7 m. These points are shown on the diagram as C, D and E respectively, and the altitudes in each case are given correct to 3 significant figures.

Use the trapezoidal rule with four intervals to estimate the cross-sectional area of the hill

(3 marks)

(f) i) Write down the integral which can be used to find the cross-sectional area of the hill.

ii) Hence find the cross-sectional area of the hill to the nearest square metre.

(4 marks)