

Practice Paper 2

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Total Marks

/80

- 1 (a)** The following table shows the mean height, y cm, of primary school children who are age x years old.

Age , x years	6.25	7.35	8.5	9.25	10.75
Mean Height , y cm	115	121	129	136	140

The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$.

- i) Find the value of a and the value of b .
- ii) Write down the value of Pearson's product-moment correlation coefficient, r .

(4 marks)

- (b)** Use your regression equation from part (a) (i) to estimate the height of a child aged 9 years old.

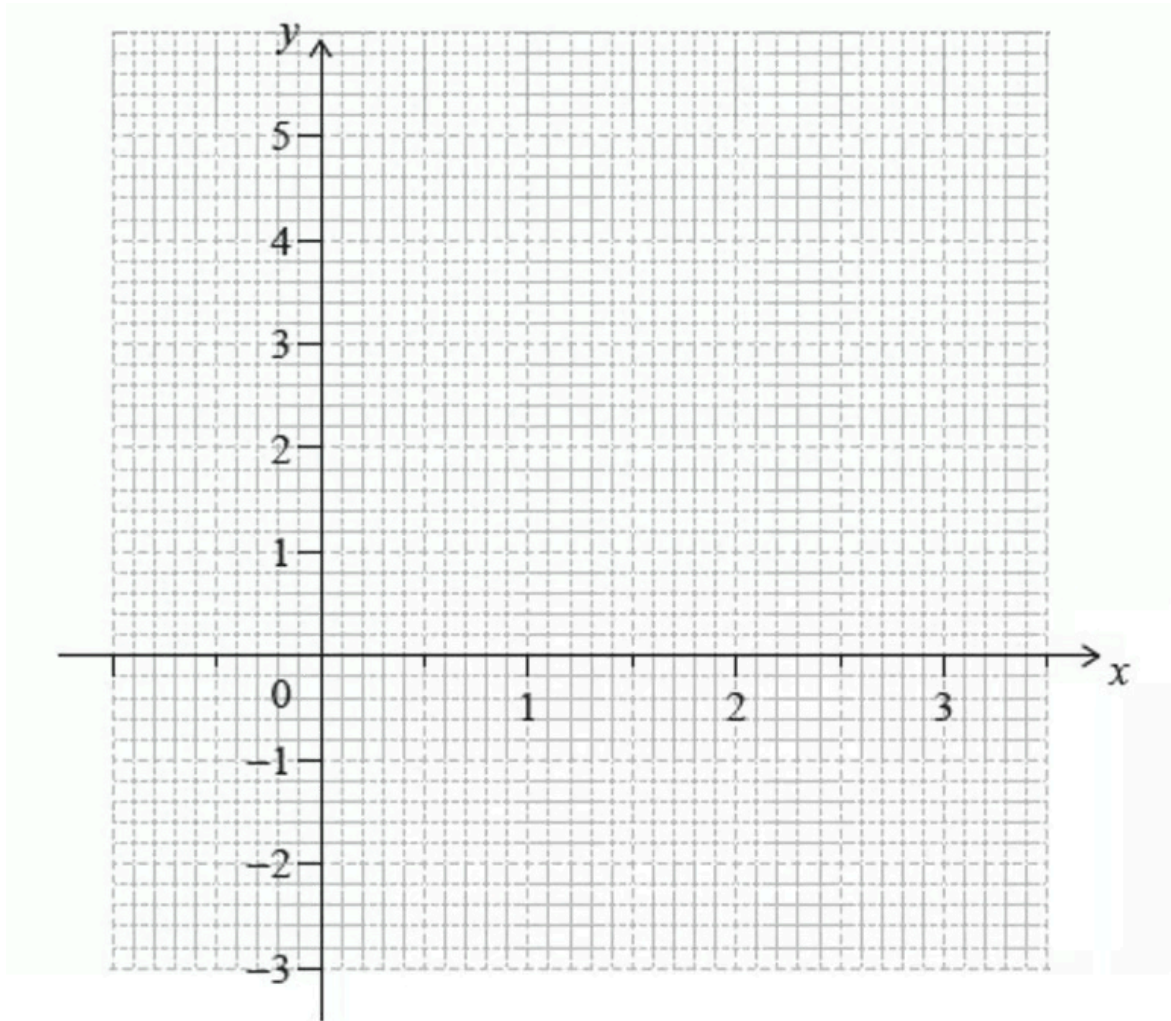
(2 marks)

- (c)** Explain why it is not appropriate to use the regression equation to estimate the age of a child who is 133 cm tall.

(1 mark)

2 (a) Let $f(x) = 4x - 3^{0.25}x^2$ for $0 \leq x \leq 3$.

Sketch the graph of $f(x)$ on the grid below.



(3 marks)

(b) Find the value of x for which $f'(x) = 0$.

(2 marks)

3 (a) An arithmetic sequence with a common difference -3.5 has first term 77 .

Given that the r th term of the sequence is zero, find the value of r .

(2 marks)

(b) Find the maximum value of the sum of the first n terms of the sequence.

(3 marks)

4 A and B are independent events, such that $P(A) = 0.25$ and $P(B) = 0.52$. C is another event, such that B and C are mutually exclusive and $P(A \cap C) = 0.09$.

Given that $P(A \cup B \cup C) = 0.95$, find

- i) $P(A \cap B)$
- ii) $P(C)$
- iii) $P(A' \cap B')$
- iv) $P(A|C')$

(9 marks)

5 (a) Let $f(x) = \frac{5-x^2}{3}$ and $g(x) = 4 - \frac{3}{x}$ where each function has the largest possible valid domain.

Write down the range of f .

(1 mark)

(b) Write down the domain and range of g .

(2 marks)

(c) Find

i) $(f \circ g)(x)$

ii) $(g \circ f)(x)$

(3 marks)

6 (a) The number of bacteria, n , in a dish, after t minutes is given by $n = 5231e^{0.12t}$.

Find the initial amount of bacteria.

(2 marks)

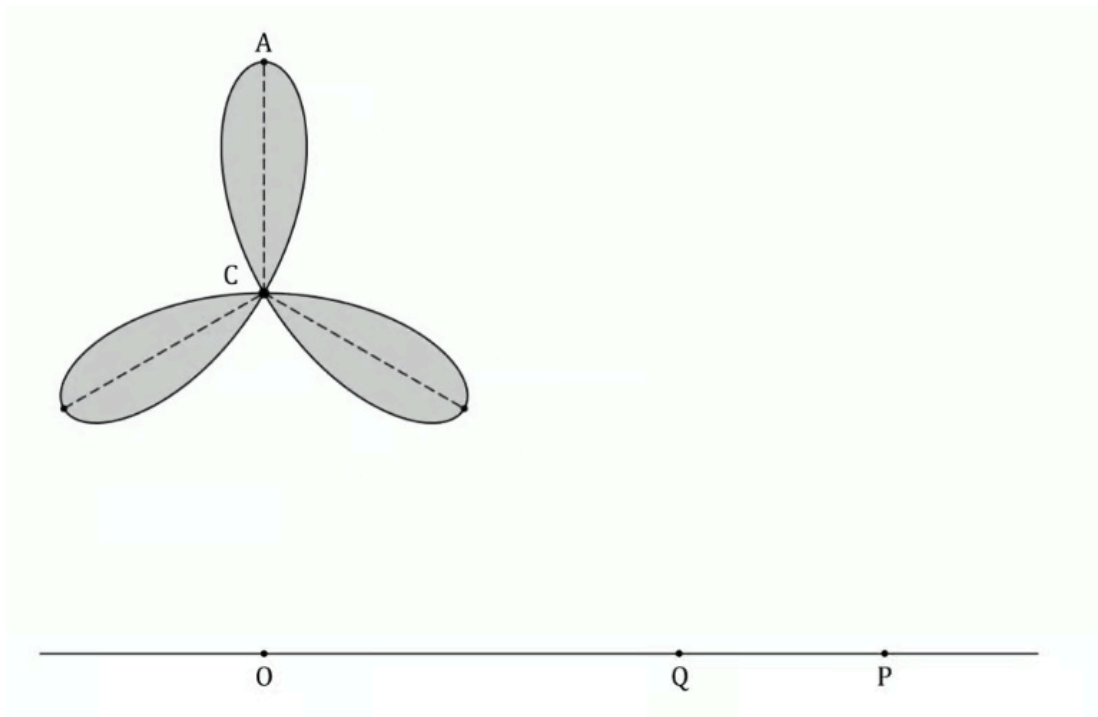
(b) Find the amount of bacteria after 12 minutes. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.

(3 marks)

(c) Find the value of t when $n = 2.7 \times 10^4$.

(2 marks)

- 7 (a) Badon Iron Works is building a new ship called the Gargantuan, which will be a full- sized replica of the original RMS Titanic. Eleanor is an engineer at the company, and is involved with construction and testing of the ship's screws (commonly known as 'propellers'). The diagram below depicts one of the ship's screws mounted in the testing facility.



Point C is the centre of the screw, which is fixed in place so that the screw is able to rotate about it. Point A is the marked tip of one of the three identical blades of the screw. Point O is the point on the horizontal floor of the testing facility that lies directly below point C . Points O , A , C , P and Q lie at all times in the same plane.

The height, h m, of point A above the testing facility floor once the screw begins to rotate may be modelled by the function

$$h(t) = 5.59 + 3.6 \cos(k\pi t)$$

where t is the time in seconds since the screw began rotating, and k is a constant.

Use the above information to determine:

- The distance of point A from point C .
- The height of point C above point O .

(2 marks)

- (b)** Given that the tips of the three blades of the screw are located at equal distances from each other around the circumference of a circle with centre C , determine the exact distance of point A from the tip of one of the other blades of the screw.

(3 marks)

- (c)** When it is rotating, the screw makes 75 complete revolutions every minute.

Given that the argument of the cosine in the equation for $h(t)$ is measured in radians, use this information to determine the value of the constant k .

(3 marks)

- (d)** Paul, a mathematician, has been hired as a consultant on the Gargantuan project.

Because of his height of 1.96 m, Eleanor is concerned about whether he will be able to walk safely beneath the screw while it is rotating.

Determine whether Eleanor is right to be concerned, giving a mathematical reason for your answer.

(2 marks)

- (e) The screw has been locked in place so that point **A** is at its highest possible position above the floor. Paul is standing at point **P**, which is at a distance of 9.69 m from point **O**. He walks towards point **O** until he arrives at point **Q**, which is located such that

$$\tan \widehat{OQC} = \frac{3}{2} \tan \widehat{OPC}$$

Determine the distance of point **Q** from point **P**.

(3 marks)

- 8 (a)** The Strike A Light! matchstick company produces matchsticks with a length, X mm, that is normally distributed with mean 45 and variance σ^2 .

The probability that X is greater than 45.37 is 0.1714.

Find $P(44.63 < X < 45.37)$.

(2 marks)

- (b)** i) Find σ , the standard deviation of X .
- ii) Hence, find the probability that a randomly selected matchstick has a length less than 44.5 mm.

(5 marks)

- (c)** Andrew has a box of Strike A Light! matches with fifteen matchsticks remaining in it. Those matchsticks may be assumed to be a random sample. Let Y represent the number of matchsticks in Andrew's box with lengths less than 44.5 mm.

Find $E(Y)$.

(3 marks)

- (d) Find the probability that exactly one of the matchsticks in Andrew's box has a length less than 44.5 mm.

(2 marks)

- (e) A Strike A Light! matchstick is selected at random and is found to have a length greater than 44.5 mm.

Find the probability that the length of the matchstick is between 44.63 mm and 45.37 mm.

(3 marks)

9 (a) A village committee decide to hold a fundraising lottery. They sell tickets for \$3 each.

Each ticket wins \$ W , and the head of the committee, Ms Led, proposes a probability distribution for W as below. The Star Prize is \$500.

w	0	3	15	30	500
$P(W = w)$	d	0.25	0.1	0.01	0.001

Calculate d , the probability of not winning any money having bought a raffle ticket.

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(2 marks)

(b) In the first week they sell 1000 tickets. Miss Givins is worried that the above distribution may not be profitable for the village committee.

Calculate the expected profit or loss for the village committee if they use the above probability distribution.

(4 marks)

(c) Ms Led changes the lottery rules so that the probability of winning the Star Prize is 10 times less likely. The probability of winning the \$3, \$15 and \$30 prizes remain the same.

After the first week the Star Prize is not won, so to encourage more players, Ms Led announces that the Star Prize will triple each week until it is won. The probabilities remain the same from week to week.

If the Star Prize continues not to be won, write an expression in terms of n for the value of the Star Prize in the n th week of the lottery.

(2 marks)

- (d)** If the Star Prize continues not to be won, in which week does the lottery become a fair game?

(5 marks)