IB · DP · Maths

2 hours   7 questions

Practice Paper 2

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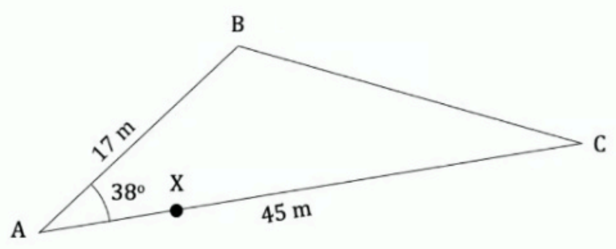
Total Marks	/110

1 (a) The diagram below shows a triangular field on a farm. $AB = 17$ m, $AC = 45$ m and angle $B\widehat{A}C = 38°$.

X is a point on AC, such that $AX : XC$ is $1 : 4$.



The field is going to be used for livestock, so a fence is to be installed around its perimeter.

Calculate the total length of fencing required.

(4 marks)

(b) The owner of the field had estimated the length of fence required to be 98 m.

Calculate the percentage error in her estimation.

(2 marks)

(c) The field is to be divided into two parts by installing a new fence connecting B to X.

Calculate the area of BXC.

(4 marks)

(d) The farmer decides that field ABX is too small and wishes instead to divide the original field by adjusting the position of X such that angle $\text{A}\widehat{\text{B}}\text{X} = 32°$

Determine how much less fencing is required for BX given the new position of X.

(6 marks)

2 (a) The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

Calculate

i) the mean number of baskets scored by a player

ii) the standard deviation.

(2 marks)

(b) Find the median number of baskets scored.

(1 mark)

(c) Find the interquartile range.

(2 marks)

(d) Determine if a player who scored 8 baskets would be considered an outlier.

(2 marks)

(e) Two players are randomly chosen.

Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

(4 marks)

(f) The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.

i) Calculate the probability that a player trains less than 6 hours a week.

ii) Calculate the probability that a player trains less than 4 hours a week.

iii) Calculate the expected number of players that train between 4 and 6 hours a week.

(3 marks)

3 (a) Chun-hee is creating some packaging in the shape of a square based pyramid where the base has length x cm and the perpendicular height of the pyramid is h cm. Chun-hee wants to keep the distance from the apex of the pyramid to the midpoint of the base edge fixed at 7 cm.

Write down an equation for the volume, V, of the packaging in terms of x and h.

(1 mark)

(b) Show that V can be expressed by $\frac{196}{3}h - \frac{4}{3}h^3$.

(3 marks)

(c) Find $\frac{dV}{dh}$.

(2 marks)

(d) Find the value of h for which the volume of the pyramid is maximised.

(2 marks)

(e) Find the value of x when the volume of the pyramid is maximised.

(2 marks)

(f) Chun-hee decides to make the packaging using the dimensions required to maximise the volume. The material for the packaging costs 4 KRW / cm^2.

Calculate the number of units that Chun-hee can make given that she has 90, 000 KRW.

(4 marks)

(g) Chun-hee takes out a 3 year loan for 90,000 KRW at a nominal annual interest rate of 2.3% compounded monthly. Repayments are made at the end of each month.

Find the value of the repayments that Chun-hee must make to pay off the loan.

(3 marks)

4 (a) In the town of Manh, all the residents belong to either one or the other of the town's two fitness clubs – Giang's House of Fitness (G) or Thu's Wonder Gym (T). Each year 30% of the members of G switch to T and 25% of the members of T switch to G. Any other losses or gains of members by the two fitness clubs may be ignored.

Write down a transition matrix $\boldsymbol{T}$ representing the movement of members between the two clubs in a particular year.

(2 marks)

(b) Find the eigenvalues and corresponding eigenvectors of $\boldsymbol{T}$.

(4 marks)

(c) Hence write down matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ such that $\boldsymbol{T} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1}$.

(2 marks)

(d) Initially there are 2500 members of G and 800 members of T.

Using the matrix power formula, show that the numbers of members of G and T after n years will be $(1500 + 1000\,(0.45^n))$ and $(1800 - 1000(0.45^n))$, respectively.

(6 marks)

(e) Hence write down the number of customers that each of the fitness clubs can expect to have in the long term.

(2 marks)

5 (a) In a game, enemies appear independently and randomly at an average rate of 2.5 enemies every minute.

Find the probability that exactly 3 enemies will appear during one particular minute.

(1 mark)

(b) Find the probability that exactly 10 enemies will appear in a five-minute period.

(2 marks)

(c) Find the probability that at least 3 enemies will appear in a 90-second period.

(2 marks)

(d) The probability that at least one enemy appears in k seconds is 0.999. Find the value of k correct to 3 significant figures.

(2 marks)

(e) A 10-minute interval is divided into ten 1-minute periods (first minute, second minute, third minute, etc.). Find the probability that there will be exactly two of those 1-minute periods in which no enemies appear.

(4 marks)

(f) On the next level of the game, there is a boss enemy and a number of additional henchmen to fight against.

The number of times that the boss enemy appears in a one-minute period can be modelled by a Poisson distribution with a mean of 1.1.

The number of times that an individual henchman appears in a one-minute period can be modelled by a Poisson distribution with a mean of 0.6.

It may be assumed that the boss enemy and the henchmen each appear randomly and independently of one another.

Each time that the boss enemy or any particular henchman appears, it is counted as one 'enemy appearance'.

Determine the least number of henchmen required in order that the probability of 40 or more 'enemy appearances' occurring in a 3-minute period is greater than 0.38. You may assume that neither the boss enemy nor any of the henchmen are able to be totally eliminated from the game during this 3-minute period.

(4 marks)

6 (a) James throws a throws ball to his friend Mia. The height, h, in metres, of the ball above the ground is modelled by the function

$$h(t) = -1.05t^2 + 3.84t + 1.97, \qquad t \geq 0$$

where t is the time, in seconds, from the moment that James releases the ball.

Write down the height of the ball when James releases it.

(1 mark)

(b) After 4 seconds the ball is at a height of metres above the ground.

Find the value of q.

(2 marks)

Find $h'(t)$

(c) **(2 marks)**

(d) Find the maximum height reached by the ball and write down the corresponding time t.

(3 marks)

(e) James then drives a remote-controlled car in a straight horizontal line from a starting position right in front of his feet. The velocity of the remote-controlled car in ms^{-1} is given by the equation

$$v(t) = \frac{5}{4}t^3 - \frac{19}{2}t^2 + 18t - 2$$

Find an expression for the horizontal displacement of the remote-controlled car from its starting position at time t seconds.

(4 marks)

(f) Find the total horizontal distance that the remote-controlled car has travelled in the first 5 seconds.

(3 marks)

7 (a) Consider the following system of differential equations:

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = -3x - 4y$$

Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 2 \\ -3 & -4 \end{pmatrix}$.

(6 marks)

(b) Hence write down the general solution of the system.

(2 marks)

(c) When $t = 0$, $x = 2$ and $y = 4$.

Use the given initial condition to determine the exact solution of the system.

(3 marks)

(d) (i) Find the value of $\frac{dy}{dx}$ when $t = 0$.

(ii) Find the values of x, y and $\frac{dy}{dx}$ when $t = \ln\frac{9}{7}$.

(3 marks)

(e) Hence sketch the solution trajectory of the system for $t \geq 0$.

(3 marks)