IB · DP · Maths

2 hours 12 questions

Practice Paper 1

Scan here to return to the course
or visit savemyexams.com

Total Marks **/110**

1 (a) (i) Expand $(2k-1)^3$

(ii) Hence, or otherwise, show that $(2k-1)^3-(2k-1)=8k^3-12k^2+4k$

(2 marks)

(b) Thus prove, given $k>1$, $k \in \mathbb{N}$, that the difference between an odd natural number greater than 1 and its cube is always even.

(3 marks)

2 Let A and B be events such that $P(A)=0.3$, $P(B)=0.75$ and $P(A \cup B)=0.9$. Find $P(B|A)$.

(5 marks)

© 2024 Save My Exams, Ltd. Get more and ace your exams at savemyexams.com

3 (a) The functions f and g are defined such that $f(x) = 6x + 7$ and $g(x) = \dfrac{x-5}{3}$.

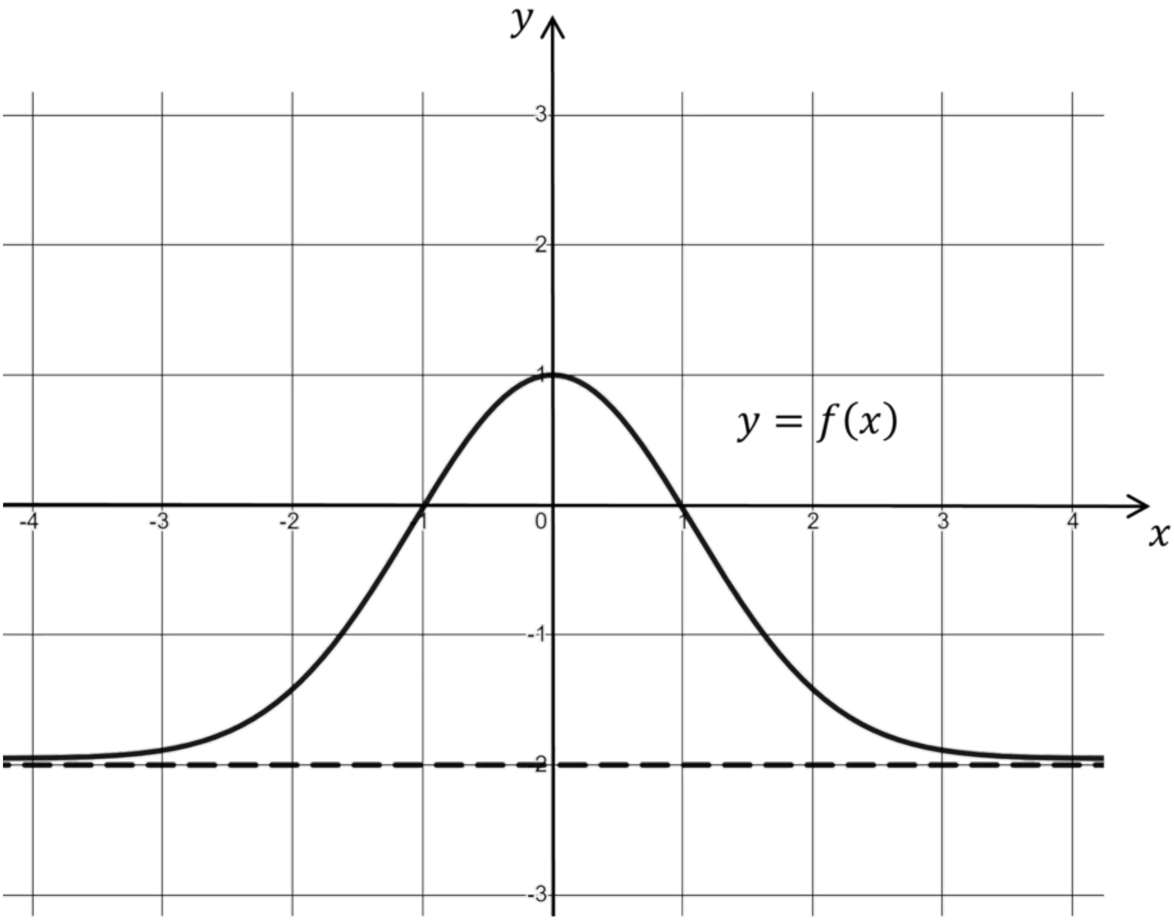
Show that $(f \circ g)(x) = 2x - 3$.

(2 marks)

(b) Given that $(f \circ g)^{-1}(a) = 6$, find the value of a.

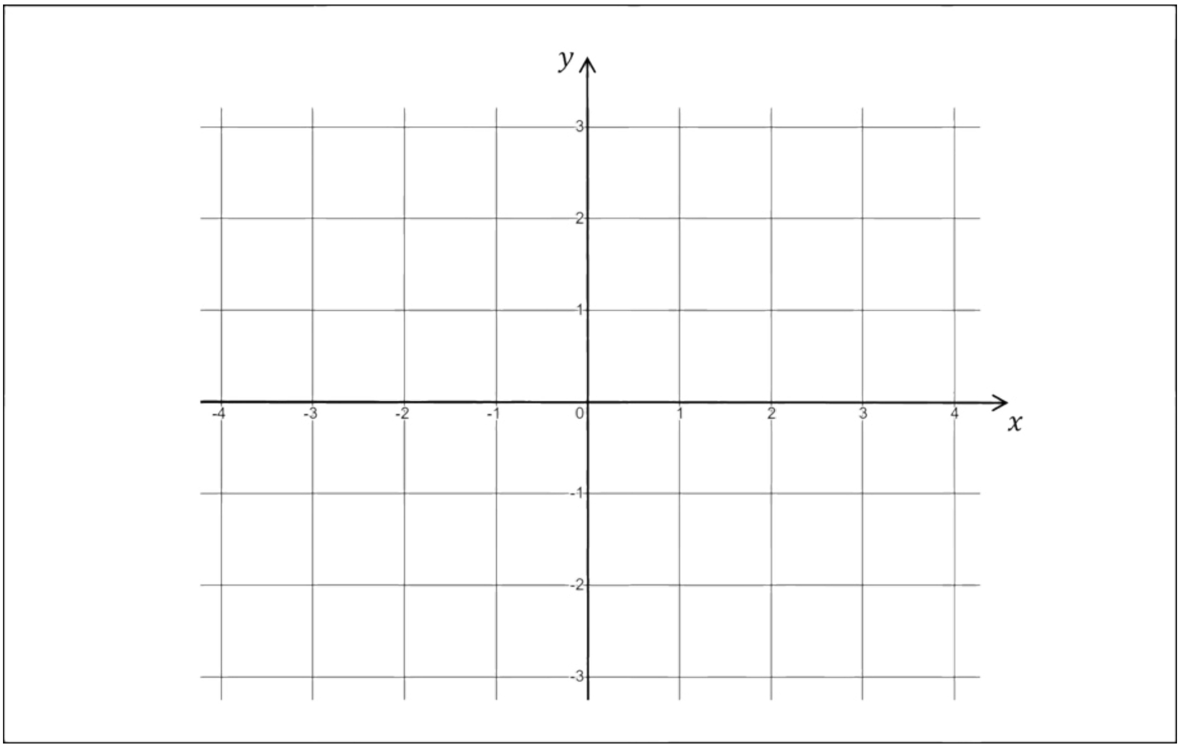
(3 marks)

4 The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -2$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 1$.



On the following set of axes, sketch the graph of $y = [f(x)]^2 - 2$, clearly showing any asymptotes with their equations along with the coordinates of any local maxima or minima.

© 2024 Save My Exams, Ltd.
Get more and ace your exams at savemyexams.com



(5 marks)

5 Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\cos\left(3x^3 + \frac{\pi}{2}\right)$ and that the graph of y passes through the point $(0, -1)$, find an expression for y in terms of x.

(5 marks)

6 The plane Π has the Cartesian equation $5x - y - z = 15$.

The line L has the vector equation $r = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ -2 \\ 2 \end{pmatrix}$, $\lambda, k \in \mathbb{R}$. The acute angle between the line L and the plane Π is $60°$.

Find the possible values of k.

(7 marks)

© 2024 Save My Exams, Ltd. | Get more and ace your exams at savemyexams.com

7 (a) Show that $\log_4(\cos 2x + 13) = \log_2\sqrt{\cos 2x + 13}$

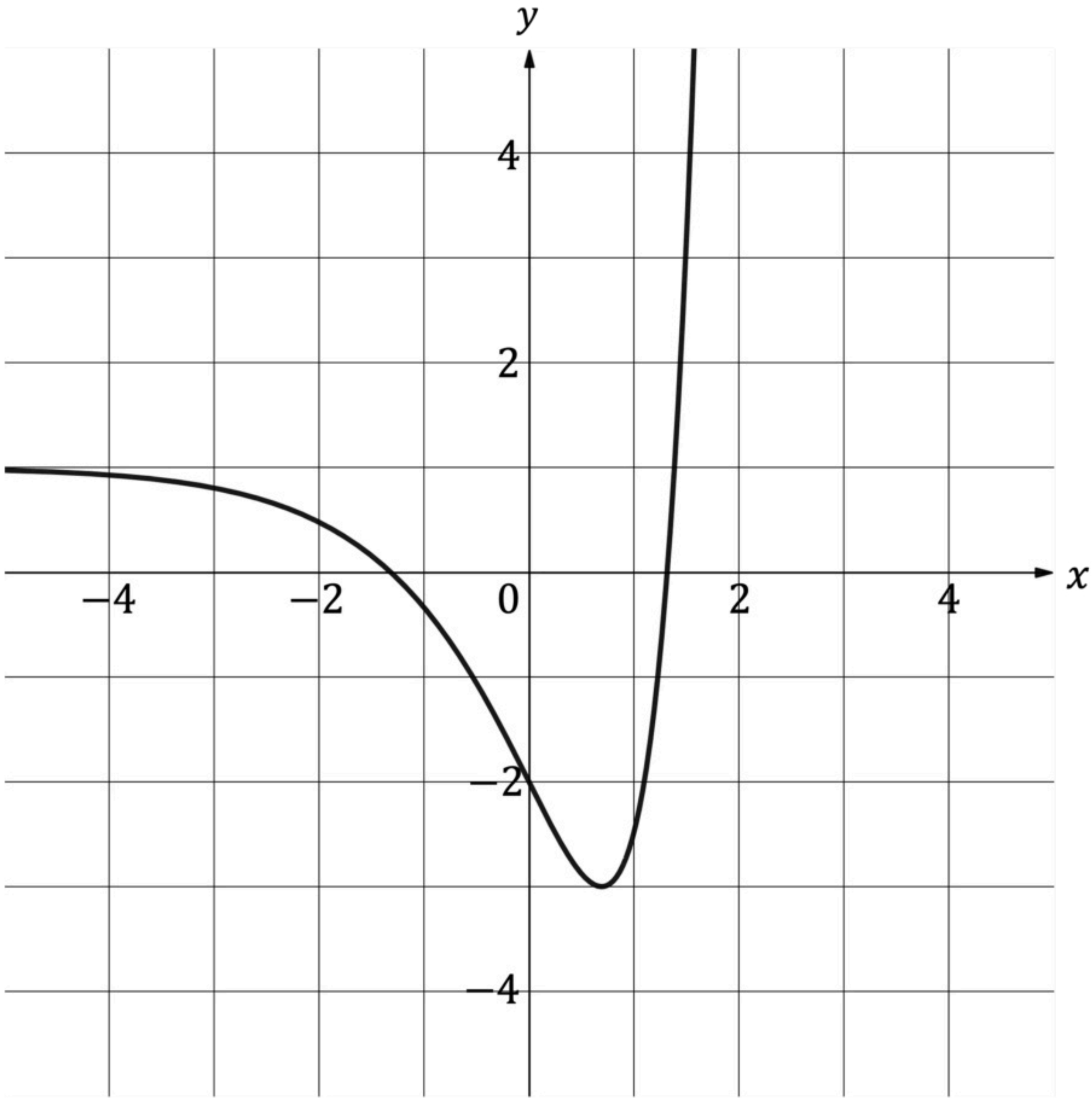
(3 marks)

(b) Hence or otherwise $\log_2\left(3\sqrt{2}\cos x\right) = \log_4(\cos 2x + 13)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(5 marks)

© 2024 Save My Exams, Ltd.
Get more and ace your exams at savemyexams.com

8 (a) The function f is defined by $f(x) = e^{2x} - 4e^x + 1,\ x \in \mathbb{R},\ x \leq a$. The graph of $y = f(x)$ is shown in the following diagram.



Find the largest value of a such that f has an inverse function.

(3 marks)

(b) For this value of a, find an expression for $f^{-1}(x)$, stating its domain.

(5 marks)

9 A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} \frac{\pi x}{81}\sin\left(\frac{\pi x}{9}\right), & 0 \le x \le 9 \\ 0, & \text{otherwise} \end{cases}$$

Find $P(3 \le X \le 6)$.

(7 marks)

© 2024 Save My Exams, Ltd.
Get more and ace your exams at savemyexams.com

10 **(a)** Express $-2-2\sqrt{3}\mathrm{i}$ in the form $re^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$.

(5 marks)

(b) Let the roots of the equation $z^3=-2-2\sqrt{3}\mathrm{i}$ be u, v and w.

Find u, v and w expressing your answers in the form $re^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$.

(5 marks)

(c) On an Argand diagram u, v and w are represented by the points U, V and W respectively.

Find the area of triangle UVW.

© 2024 Save My Exams, Ltd.
Get more and ace your exams at savemyexams.com

(4 marks)

(d) By considering the sum of the roots u, v and w, show that

$$\cos\left(\frac{2\pi}{9}\right)+\cos\left(\frac{4\pi}{9}\right)+\cos\left(\frac{8\pi}{9}\right)=0$$

(4 marks)

11 (a) The function f is defined by $f(x) = 2e^{\sin 2x}$.

Find the first two derivatives of $f(x)$ and hence find the Maclaurin series for $f(x)$ up to and including the x^2 term.

(8 marks)

(b) Show that the coefficient of x^3 in the Maclaurin series for $f(x)$ is zero.

(4 marks)

(c) Using the Maclaurin series for $\arctan x$ and $e^x - 1$, find the Maclaurin series for $\arctan(e^{2x} - 1)$ up to and including the x^3 term.

© 2024 Save My Exams, Ltd. Get more and ace your exams at savemyexams.com

(6 marks)

(d) Hence, or otherwise, find $\lim_{x \to 0} \dfrac{f(x) - 2}{\arctan(e^{2x} - 1)}$

(3 marks)

© 2024 Save My Exams, Ltd. Get more and ace your exams at savemyexams.com

12 (a) Let $f(x) = \dfrac{\ln px}{qx}$ where $x > 0,\ \ p,\ q \in \mathbb{R}^+$.

Show that $f'(x) = \dfrac{1 - \ln px}{qx^2}$.

(3 marks)

(b) The graph of f has exactly one maximum point A.

Find the x-coordinate of A.

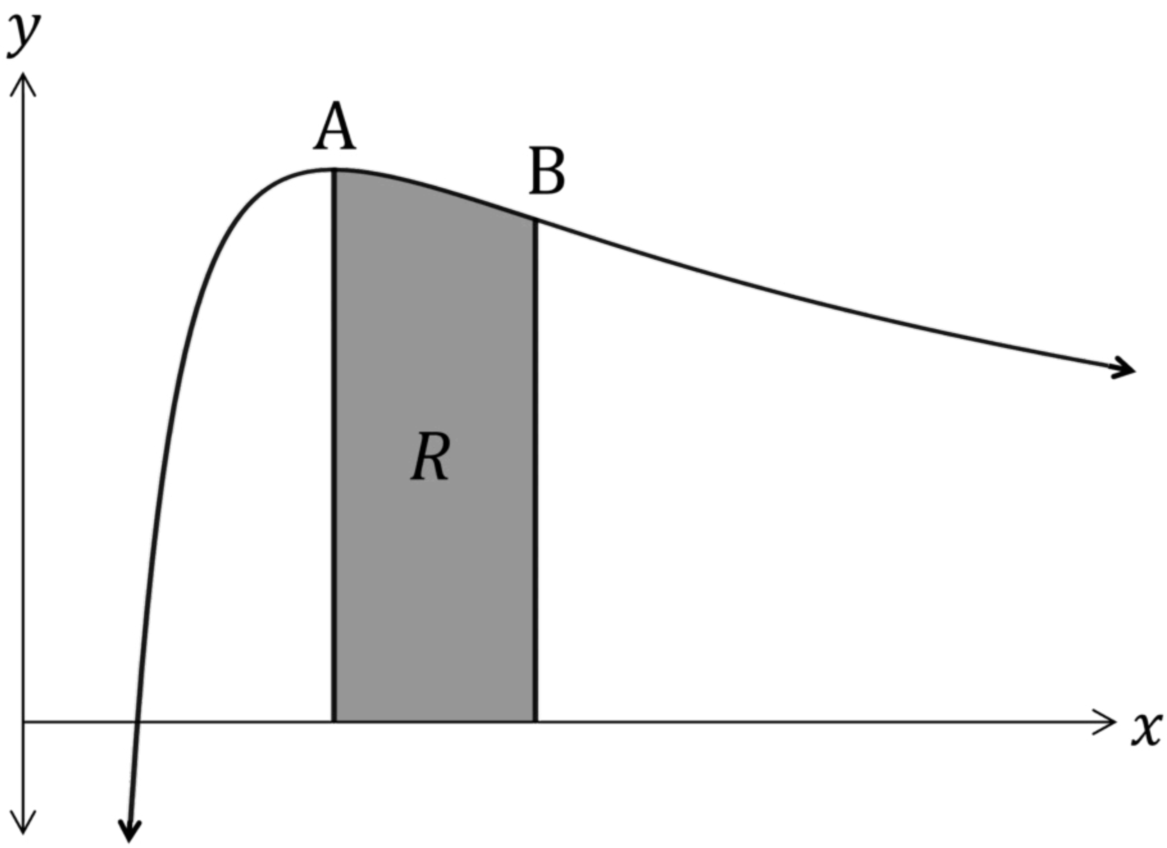
(3 marks)

(c) The second derivative of f is given by $f''(x) = \dfrac{2\ln px - 3}{qx^3}$. The graph of f has exactly one point of inflexion B.

Show that the x-coordinate of B is $\dfrac{e^{\frac{3}{2}}}{p}$.

(3 marks)

(d) The region R is enclosed by the graph of f, the x-axis, and the vertical lines through the maximum point A and the point of inflexion B.



Calculate the area of R in terms of q and show that the value of the area is independent of p.

(7 marks)

© 2024 Save My Exams, Ltd.