IB · DP · Maths

2 hours 18 questions

Practice Paper 1

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Total Marks **/110**

1 (a) Doctor Scotpop is investigating the population of otters in Scotland.

The doctor found, based on historical data, that the population of otters, P, could be modelled by $P = 7500 + A(1.09)^t$ where A is a constant and $t \geq 0$ is the number of years since the start of the year 2000.

At the start of the year 2000, the population of otters in Scotland was 8000. Find the value of the constant A.

(2 marks)

(b) Find the population of otters in Scotland at the start of the year 2020.

(2 marks)

(c) Doctor Scotpop estimates the peak population of otters Scotland can sustain is 15 500. Work out the year in which Doctor Scotpop expects the population of otters to peak.

(2 marks)

2 (a) A store manager wanted to get an idea of how much shoppers were spending. The manager conducted a survey asking shoppers how much they had spent.

The data is shown in the following table.

Amount spent in pounds (£p)	**Number of shoppers**
$£0 \le p < £2$	5
$£2 \le p < £5$	14
$£5 \le p < £10$	20
$£10 \le p < £20$	s
$£20 \le p < £50$	3

Explain why only an ***estimate*** of the mean amount spent by shoppers can be found using the data in the table (even when the value of s is known).

(1 mark)

(b) An estimate of the mean amount spent by the shoppers is $£8.58$.

Find the value of s.

(4 marks)

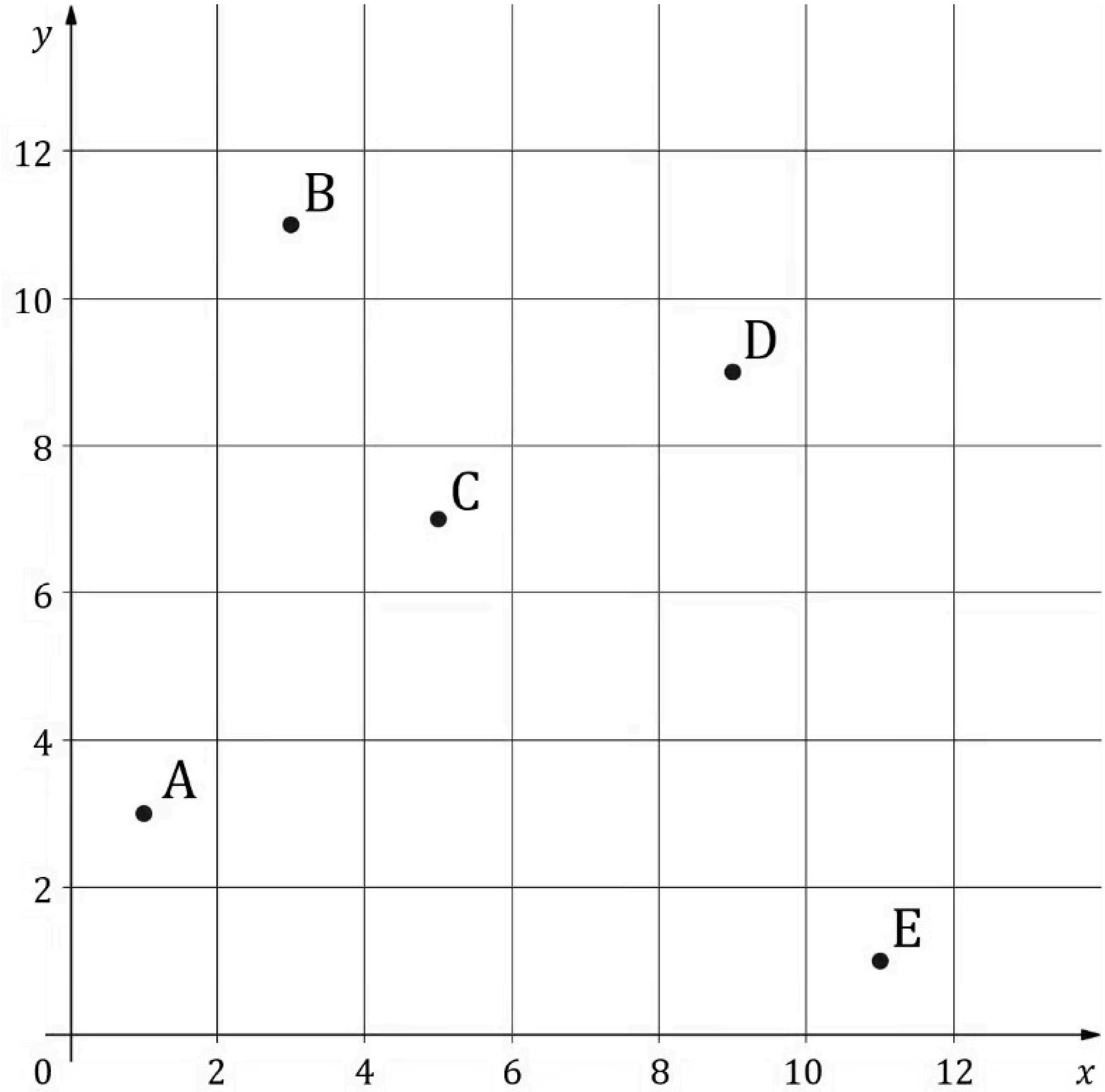
(c) It was not practical to ask every shopper in store on a particular day, so the manager stood at the shop exit for an hour and asked some, but not all, of the shoppers how much they had spent.

Identify the sampling technique used in the survey.

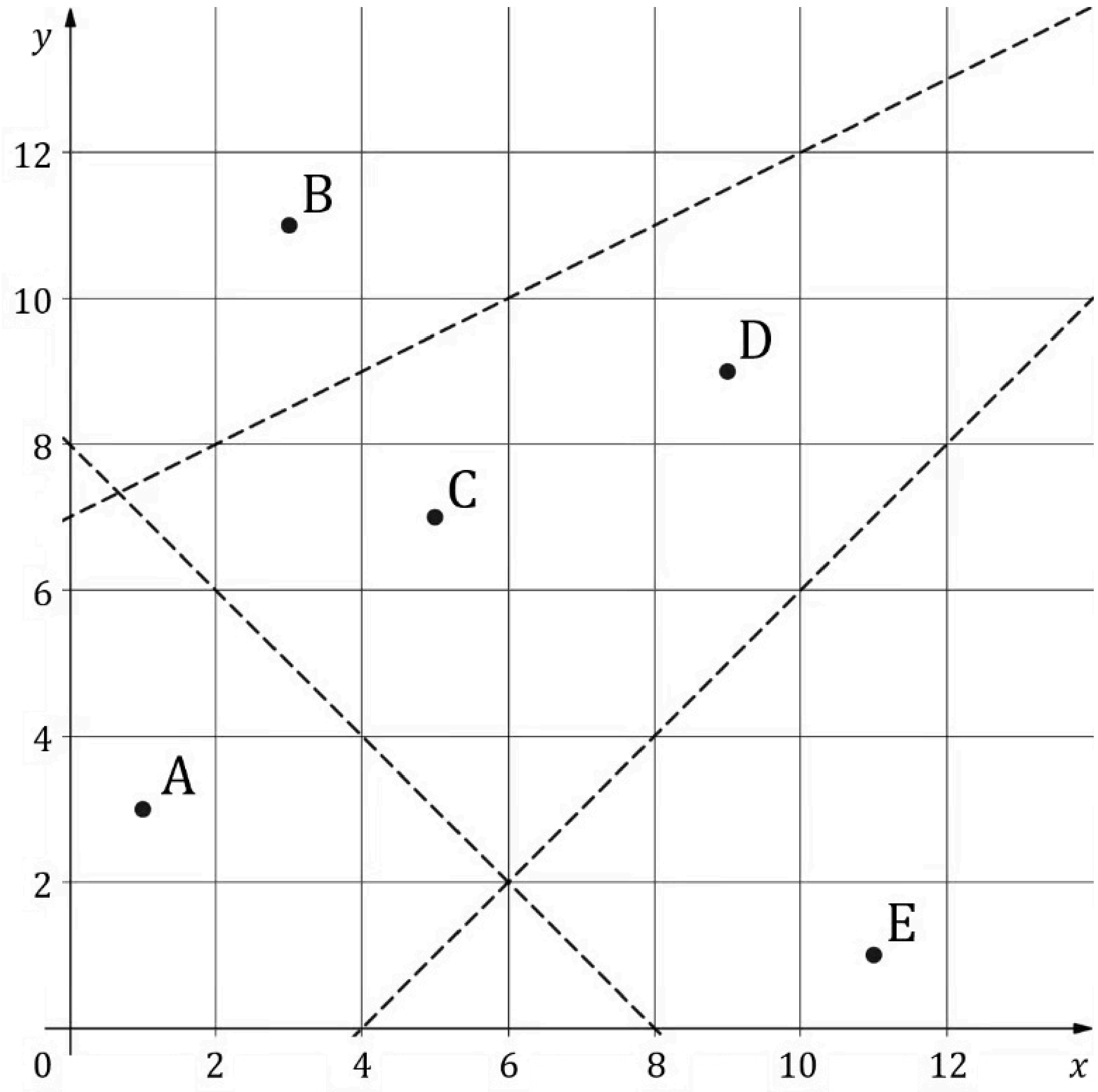
(1 mark)

3 (a) Rangers use aerial imagery to help locate big cats on the savannah. This week the plane is not available so they must use last week's image which shows the last known locations of five male cats at points $\mathrm{A}(1,\ 3)$, $\mathrm{B}(3,\ 11)$, $\mathrm{C}(5,\ 7)$, $\mathrm{D}(9,\ 9)$ and $\mathrm{E}(11,\ 1)$ as illustrated on the following coordinate axes.

Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



Male cats stick to very rigid territories keeping their distance from other males to avoid confrontation. Using the image above, rangers draw three straight lines to form an incomplete Voronoi diagram.



Calculate the gradient of the line segment CD.

(2 marks)

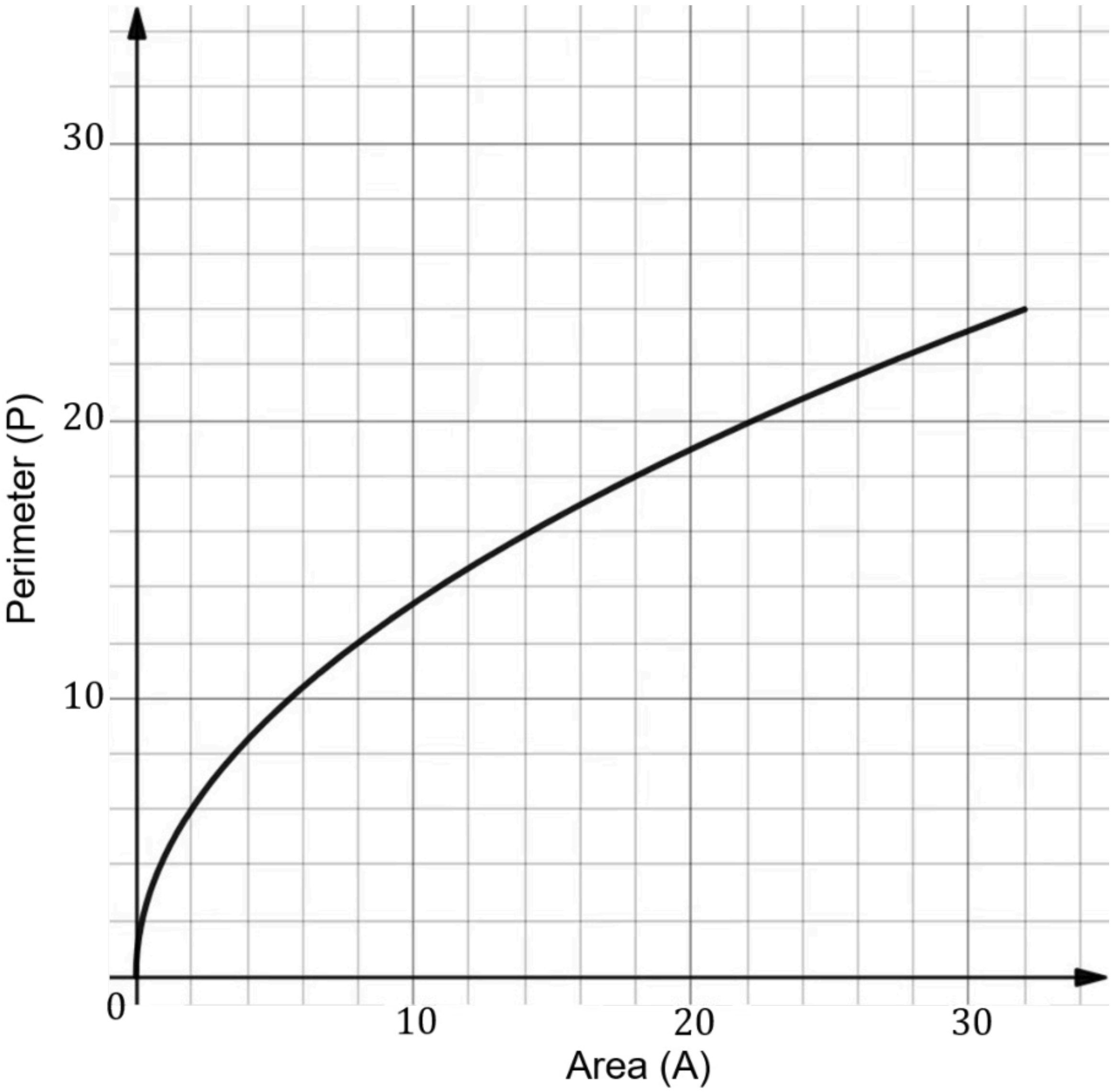
(b) Find the equation of the line which would complete the Voronoi cell containing site *C*. Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

(3 marks)

(c) In the context of the question, explain the significance of the Voronoi cell containing site *C*.

(1 mark)

4 (a) The perimeter of a rectangle P, whose width is double its height, can be represented by the function $P(A) = 6\sqrt{\frac{A}{2}}$, $A \geq 0$, where A is the area of the rectangle. The graph of the function P is shown for $0 \leq A \leq 32$.



Write down the value of $P(32)$.

(1 mark)

(b) On the axes above, draw the graph of the inverse function, P^{-1}.

(3 marks)

(c) In the context of the question, explain the meaning of $P^{-1}(12) = 8$.

(1 mark)

5 (a) A particle, A, moves such that its velocity ($v\ \mathrm{ms}^{-1}$) at time t seconds is given by $v = 3\cos t,\ t \geq 0$.

The kinetic energy (E) of particle is measured in joules (J) and is given by $E = 2v^2$.

Write down an expression for E as a function of time.

(1 mark)

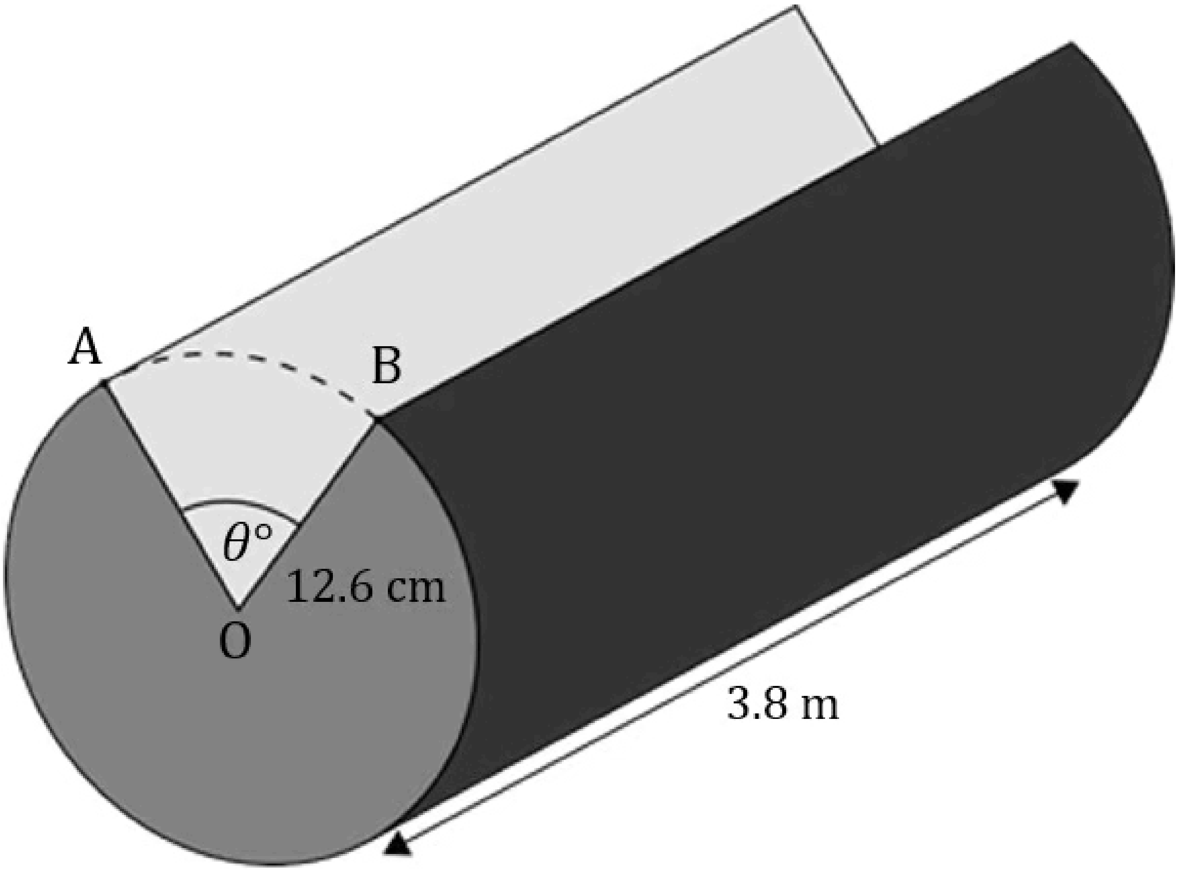
(b) Hence find $\dfrac{\mathrm{d}E}{\mathrm{d}t}$.

(2 marks)

(c) Hence or otherwise find the first time at which the kinetic energy is changing at a rate of 10 Js^{-1}.

(2 marks)

6 (a) Alex has been commissioned to create an art sculpture using 50 cylindrical bars of metal with a length-ways wedge cut out. Each piece will have length 3.8 m and radius 12.6 cm as illustrated in the following diagram, where O indicates the centre of the circular cross-section.



The whole sculpture will use 7.15 m^3 of metal.

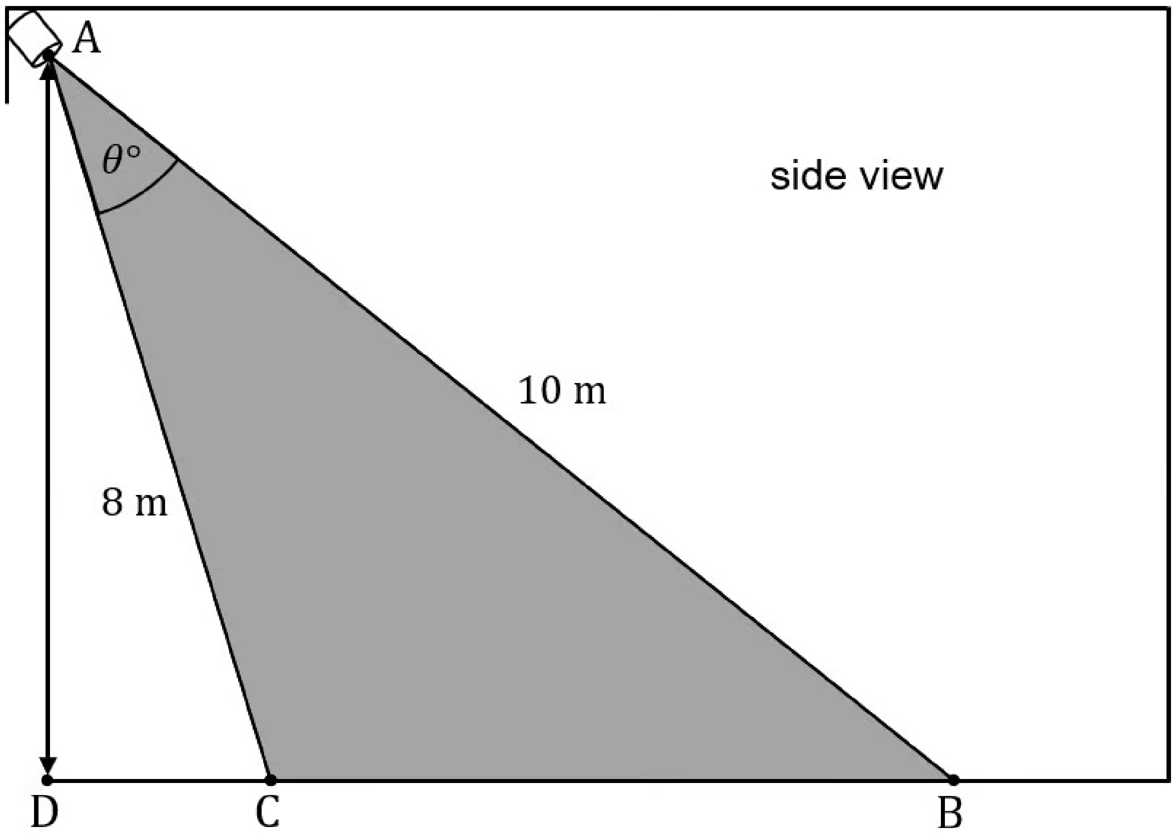
Find the angle $\theta°$ defining the size of the wedge that each bar must have cut out of it.

(4 marks)

(b) Convert your answer in part (a) into radians.

(1 mark)

7 (a) A new spotlight is being installed on a theatre lighting rig. The lighting crew must adjust the angle of the beam and mark out where actors can stand to ensure they are properly lit. The spotlight is located at point A directly above point D at the front of the stage. The area covered by the light is shown by the shaded region enclosed by triangle ABC in the following diagram and can be adjusted by changing the angle $\text{C}\widehat{\text{A}}\text{B}$.



The lighting crew have adjusted the light so that the distance from A to B is 10 m, the distance from A to C is 8 m , and the length of the stage floor covered between points B and C is 5.2 m.

Find the angle the lighting crew have adjusted $\text{C}\widehat{\text{A}}\text{B}$ to.

(2 marks)

(b) Point C is 1.2 m from the front of the stage at point D. To ensure actors know where to stop when walking towards point B from the direction of point C, the lighting crew mark a point on the floor at which any actor under 1.9 m tall can stand and remain fully lit.

Find the furthest distance **from the front of the stage** that the lighting crew should place their mark.

(6 marks)

8 (a) Leonardo has constructed a biased spinner with six sectors labelled $0,\ 1,\ 1,\ 2,\ 3$ and 5.

The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

Find the exact value of p.

(1 mark)

(b) Leonardo is playing a game with his biased spinner. The score for the game is the number which the spinner lands on after being spun.

Leonardo plays the game once.

Calculate the expected score

(2 marks)

(c) Leonardo plays the game twice and adds the two scores together.

Find the probability Leonardo has a **total** score of 2.

(3 marks)

9 (a) A plane is flying with velocity $v = \begin{pmatrix} -3 \\ -12 \\ 4 \end{pmatrix}$ into a wind with velocity $w = \begin{pmatrix} 4 \\ k \\ 2 \end{pmatrix}$, $k \in \mathbb{R}$.

Given that v is perpendicular to w, find the value of k.

(2 marks)

(b) The lift force on the plane acts perpendicular to both the plane and the wind and is given by the vector equation $\boldsymbol{L} = av \times \boldsymbol{w}$, $a \in \mathbb{R}^+$.

Given that $|\boldsymbol{L}| = 88$, find the value of a.

(4 marks)

10 (a) Scientists are growing colonies of two types of bacteria, X and Y, in a large culture dish in their lab. The rates of change of the areas covered by the two types of bacteria, where x is the area covered by X and y is the area covered by Y, are given by the following equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 19x - 9y$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 9x - 11y$$

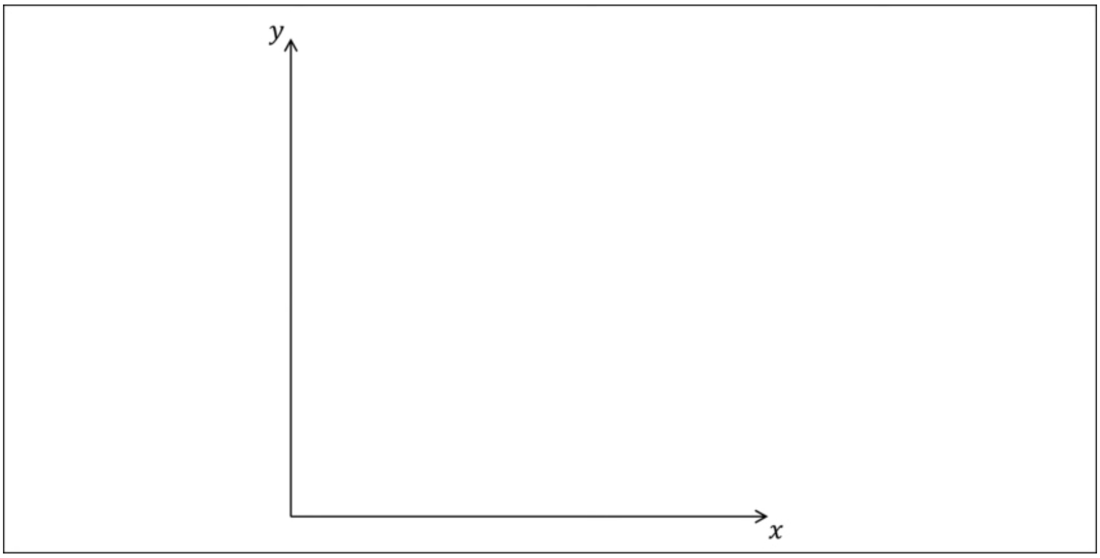
The matrix $\begin{pmatrix} 19 & -9 \\ 9 & -11 \end{pmatrix}$ has eigenvalues of 16 and -8 with corresponding eigenvectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

Initially $x = 5 \text{ cm}^2$ and $y = 7 \text{ cm}^2$.

Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $t = 0$.

(2 marks)

(b) On the following axes, sketch a possible trajectory for the growth of the two colonies of bacteria, making clear any asymptotic behaviour.



(4 marks)

11 The length of time that a phone call to a customer complaint line lasts can be modelled by a normal distribution with a mean of 2.4 minutes and a variance of 0.63 minutes^2.

Jennifer completes 5 customer phone calls, with one immediately following the other.

Find the probability that the total time taken to complete these phone calls will be more than 13 minutes.

Clearly state any assumptions you have made.

(6 marks)

12 (a) The graph of $y = -x^3$ is transformed onto the graph of $y = 45 - 0.015x^3$ by a translation of a units vertically and a stretch parallel to the x-axis of scale factor b.

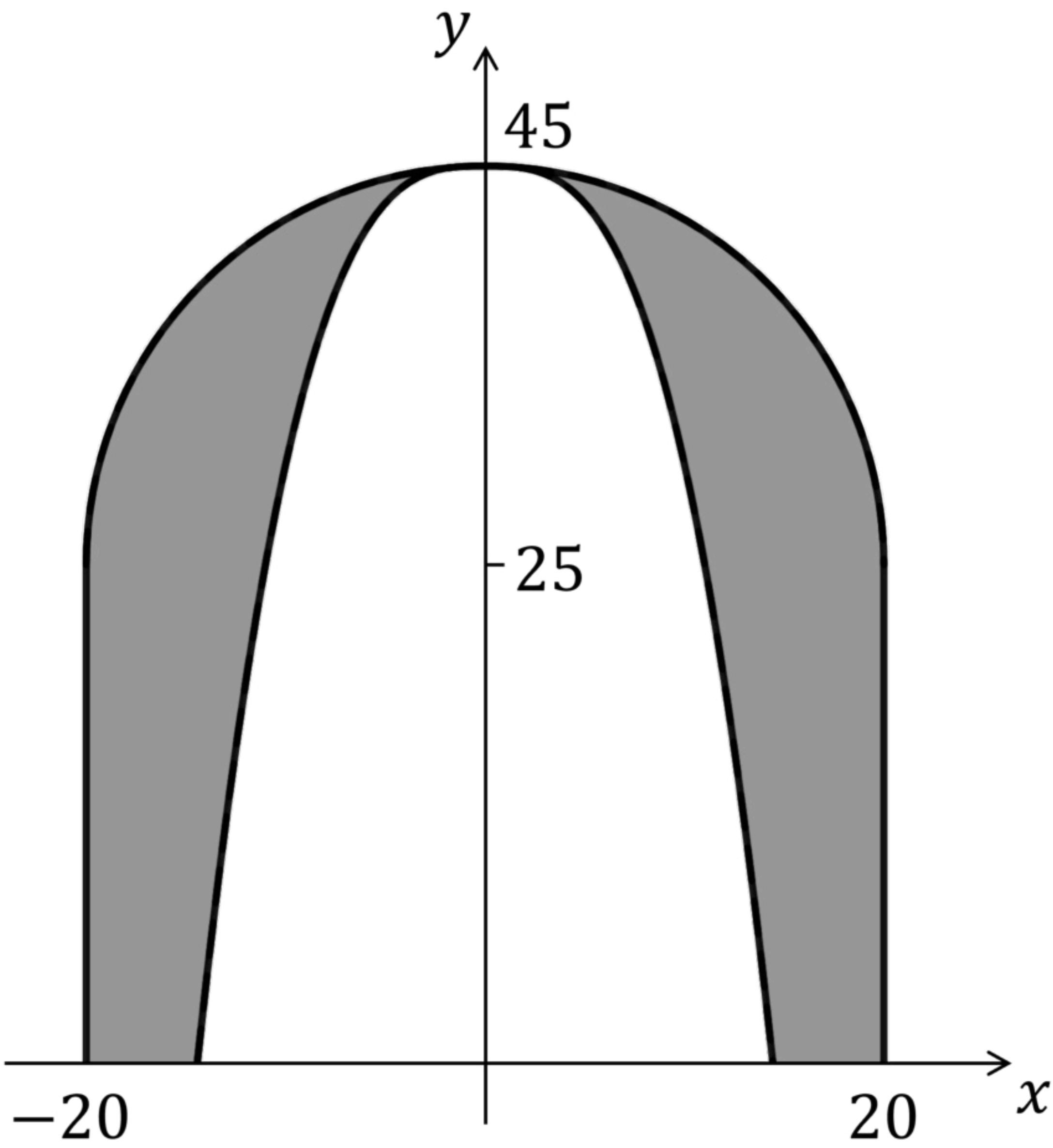
(i) Write down the value of a.

(ii) Find the value of b.

(3 marks)

Rocco is building a hollow mountain fortress for his miniatures wargaming club. The outer surface of the mountain is in the shape of a hemisphere of diameter 40 cm, supported by vertical walls of height 25 cm. The inner surface of the mountain can be modelled by rotating the curve $y = 45 - 0.015x^3$ through $360°$ about the y-axis between $y = 0$ and $y = 45$, as indicated in the diagram.

(b)



Find the volume of the space between the inner and outer surfaces of the mountain.

(5 marks)

13 (a) Marica is the owner of a prize truffle-hunting pig. The number of truffles that her pig can discover in one hour in a particular area of woodland can be modelled by a Poisson distribution.

Marica claims that the average number of truffles her pig can discover in an hour is twelve.

Ricardo, the owner of a rival pig, believes that the average for Marica's pig is less than twelve. He decides to set up a test. Marica's pig will be given one hour to hunt for truffles, and if the pig discovers less than eight truffles then Ricardo will reject Marica's claim.

State suitable null and alternative hypotheses for Ricardo's test.

(1 mark)

(b) Find the probability of a Type I error.

(2 marks)

(c) The average number of truffles found by Marica's pig in an hour is actually 11.3.

Find the probability of a Type II error.

(3 marks)

14 (a) A company manufactures fitness trackers with built-in rechargeable batteries. For quality assurance purposes a sample of 25 fully-charged trackers is tested, to determine the battery life of a fully-charged fitness tracker. The mean battery life of the sample is found to be 70.6 hours, with a standard deviation of 2.3 hours.

Find s_{n-1} for this sample.

(2 marks)

(b) Find a 95% confidence interval for the population mean.

(2 marks)

(c) The company claims that a fully-charged fitness tracker can last for 3 complete days before needing to be recharged.

Comment on this claim with reference to your answer in part (b).

(1 mark)

15 A biologist believes that variation of the percentage of silica (s) in a sediment sample affects the population per cm^3 of a particular type of bacteria (B). He collects samples from three different areas and the results are shown in the following table.

Area	Silica, s(%)	Bacteria, B (no. per cm^3)
1	25	32
2	34	186
3	12	3

The biologist believes that the relationship between the concentration of silica and the population of the bacteria can be modelled by the equation

$$B(s) = ase^{bs} \qquad a, b \in \mathbb{R}$$

Two competing models based on the biologist's equation are suggested, with differing parameters a and b as shown below:

Model : $a = 0.006,\ b = 0.2$

Model : $a = 0.09,\ b = 0.12$

The biologist will choose the model that has the smallest value for the sum of square residuals.

Determine which model the biologist will choose.

(7 marks)

16 The rate, *A*, of a chemical reaction at a fixed temperature is related to the concentration of two compounds, *B* and *C*, by the equation

$$A = k\,B^x\,C^y, \text{ where } x, y, k \in \mathbb{R}.$$

A scientist measures the three variables three times during the reaction and obtains the following values.

Experiment	$A\left(\text{mol l}^{-1} s^{-1}\right)$	$B\left(\text{mol l}^{-1}\right)$	$C\left(\text{mol l}^{-1}\right)$
1	7.21	1.8	3.1
2	3.25	1.3	2.7
3	0.827	0.4	1.7

Find x, y and k.

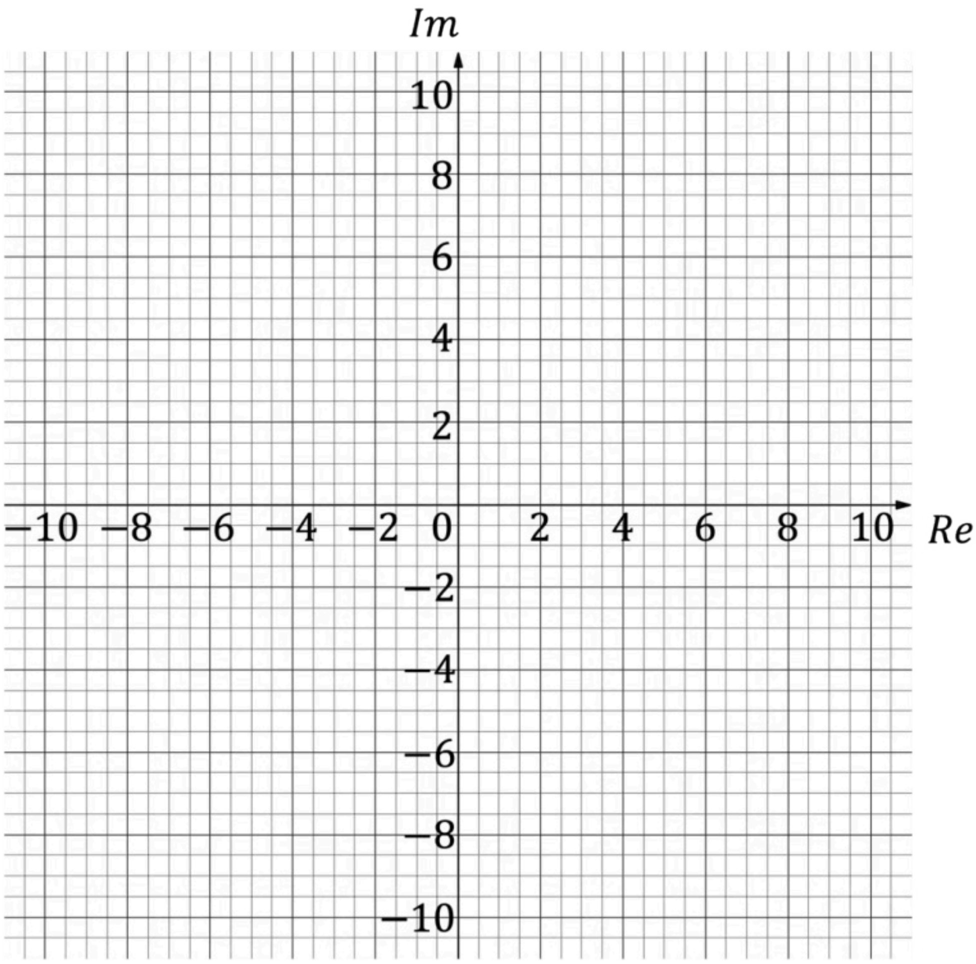
(6 marks)

17 (a) Let $w = ae^{\frac{\pi}{3}i}$, where $a \in \mathbb{R}^+$.

For $a = \sqrt{3}$,

(i) find the values of w^2, w^3 and w^4;

(ii) draw w^2, w^3 and w^4 on the following Argand diagram.



(5 marks)

(b) Let $z = \frac{w}{1+i}$.

Find the value of for which successive powers of z lie on a circle.

(2 marks)

18 (a) Leila has a collection of 26 crystals. There are 8 white crystals, 5 red crystals and the rest are blue.

Each day Leila chooses a crystal at random to put on her desk.

Find the probability that she will pick a white crystal 10 times in January.

(2 marks)

(b) Leila combines her crystal collection with her sister's so together they have a total of 54 crystals. Leila calculates that the probability of picking a blue crystal 5 times in the first week is 0.201063 correct to 6 decimal places.

Find the number of blue crystals in the combined collection.

(4 marks)