IB · DP · Maths

2 hours

12 questions

Practice Paper 1

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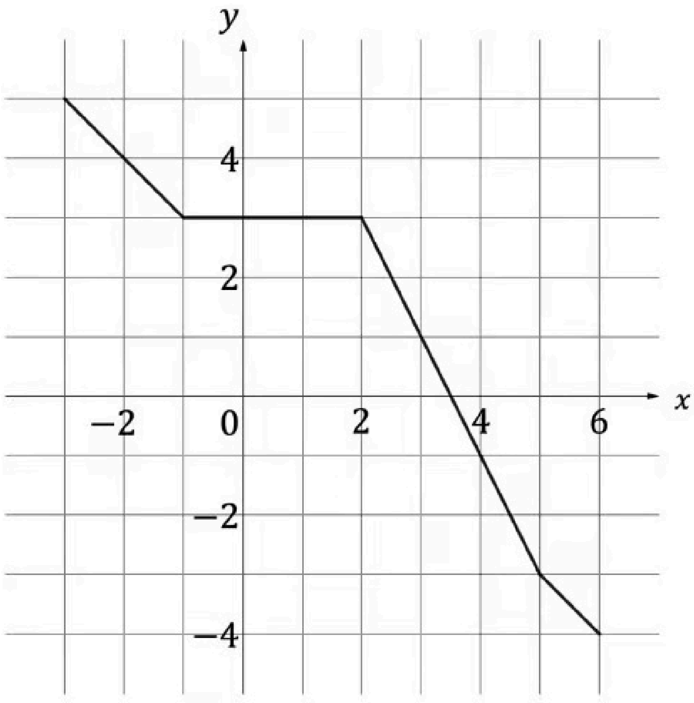
Total Marks /110

1 (a) The following diagram shows the graph of $y = f(x),\ -3 \leq x \leq 6.$



Write down the value of

i) $f(-2)$

ii) $f^{-1}(1).$

(2 marks)

(b) Find the value of $(f \circ f)(0)$.

(1 mark)

(c) Given that $g(x) = f(x+5) - 5$, find the domain and range of g.

(2 marks)

2 (a) Students are arranged for a graduation photograph in rows which follows an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

i) Find the common difference, d, of the arithmetic sequence.

ii) Find the first term of the arithmetic sequence.

(3 marks)

(b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether

(3 marks)

3 (a) The heights, in metres, of a flock of 20 flamingos are recorded and shown below:

0.4	0.9	1.0	1.0	1.2	1.2	1.2	1.2	1.2	1.2
1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.5	1.5	1.6

An outlier is an observation that falls either more than 1.5 x (interquartile range) above the upper quartile or less than 1.5 x (interquartile range) below the lower quartile.

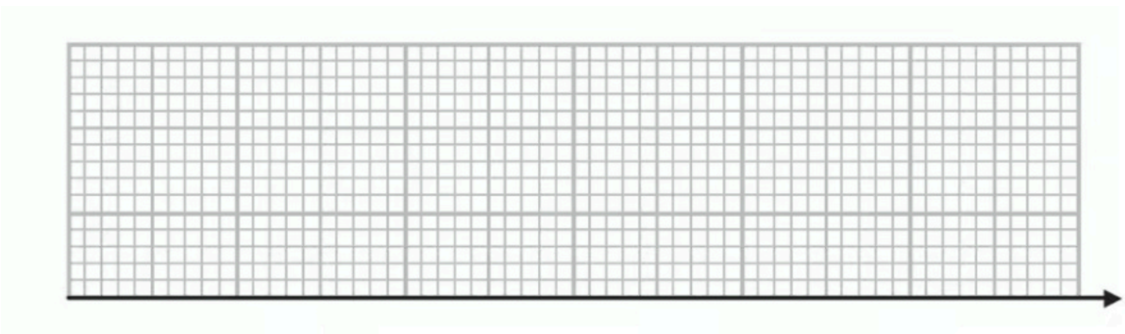
i) Find the values of $Q1$, $Q2$ and $Q3$.

ii) Find the interquartile range.

iii) Identify any outliers.

(4 marks)

(b) Using your answers to part (a), draw a box plot for the data.



(3 marks)

4 Let $f(x) = \dfrac{g(x)}{h(x)}$, where $g(2) = 4$, $h(2) = -1$, $g'(2) = 0$ and $h'(2) = 2$.

Find the equation of the tangent of f at $x = 2$.

(6 marks)

5 (a) Prove that $\sqrt{3}\sin 2\theta + \cos 2\theta - 1 = 2\sin\theta(\sqrt{3}\cos\theta - \sin\theta)$

(3 marks)

(b) Hence solve $\sqrt{3}\sin 2\theta + \cos 2\theta + 3\cos\theta - \sqrt{3}\sin\theta = 1$, *where* $0 \leq \theta \leq 360°$

(5 marks)

6 It is given that $\sec\theta = \frac{7}{3}$, where $\pi < \theta < 2\pi$.

Find the exact value of $\operatorname{cosec}\theta$.

(6 marks)

7 α and β are non-real solutions of the equation $2x^2-(2k-3)x+2k=0$.

Given that $\alpha^2+\beta^2=\dfrac{9}{4}$ and $k\neq 0$, find the value of k.

(6 marks)

8 (a) Consider the following limit:

$$\lim_{x \to 0} \frac{-1 + \cos 2x}{x^2}$$

Explain why it is appropriate to use l'Hôpital's rule to attempt to evaluate this limit.

(2 marks)

(b) Show that employing l'Hôpital's rule once leads to an indeterminate form when you attempt to evaluate the limit.

(2 marks)

(c) By employing l'Hôpital's rule a second time, show that the limit exists and find its value.

(2 marks)

9 (a) Frank plays a game involving a biased six-sided die.
The faces of the die are numbered 1 to 6.
The score of the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score, x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{2}p$	$\frac{1}{8}$	$\frac{3}{2}p$	$\frac{1}{12}$	$3p$

Calculate the exact value of p.

(2 marks)

(b) Frank plays the game once.

Calculate the expected score.

(3 marks)

(c) Frank plays the game twice and adds the scores together.

Find the probability Frank has a total score of 4, giving your answer as a fraction.

(3 marks)

10 (a) Frank has a biased six-sided die.
The faces of the die are numbered 1 to 6.
Frank's score, X, is the number which lands face up after his die is rolled.
The following table shows the probability distribution for X.

Score, x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{10}$	$\frac{1}{2}p$	$\frac{1}{5}$	$\frac{3}{2}p$	$\frac{1}{5}$	$3p$

Calculate the exact value of p.

(2 marks)

(b) Frank plays the game once.

Calculate the expected score.

(3 marks)

(c) Frank plays the game twice and adds the scores together.

Find the probability Frank has a total score of 4, giving your answer as a fraction.

(3 marks)

(d) Jenny has a different biased six-sided die.
On Jenny's die, the faces are numbered as multiples of 3.

Jenny's score, Y, is the number which lands face up after her die is rolled.
The following table shows the probability distribution for Y.

Score, y	3	6	9	12	15	18
$P(Y = y)$	a	a	b	b	b	b

It is given that the range of possible values for a is $0 < a < \frac{1}{2}$.

i) Find the range of possible values for b.
ii) Hence, find the range of possible values for $E(Y)$.

(4 marks)

(e) Frank and Jenny each roll their die once. The probability that Frank's score is at least as high as Jenny's is $\frac{23}{80}$.

Find the value of $E(Y)$.

(6 marks)

11 (a) The points A(2, 3, 0), B(-2, 4, 1), C(1, -1, 3) and D(5, -2, 2) lie on the plane Π_1 and form a parallelogram, where AB and CD are one pair of parallel edges and BC and AD are the other pair of parallel edges. Each unit on the coordinate grid is equivalent to 1 cm in length.

Find the vector product of $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$.

(3 marks)

(b) Hence, or otherwise, find the Cartesian equation of the plane Π_1.

(2 marks)

(c) A second plane Π_2 contains the point with position vector $\begin{pmatrix} 5 \\ -3 \\ 5 \end{pmatrix}$ and also the line *L*,

which has vector equation $r = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$.

Show that Π_1 and Π_2 are parallel.

(4 marks)

(d) A parallelepiped is a 3D object made up of six faces that are parallelograms lying in pairs of parallel planes. EFGH is a parallelogram on Π_2 that is congruent to ABCD, and points A, B, C and D on Π_1 are joined to points E, F, G and H respectively on Π_2 to form a parallelepiped.

Given that the coordinates of E are (3, 6, 0), find the coordinates of point H.

(3 marks)

(e) The volume of a parallelepiped can be found using the formula $|(\boldsymbol{a} \times \boldsymbol{b}).\boldsymbol{c}|$ where $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are vectors corresponding to three edges meeting at a single vertex of the parallelepiped.

Show that the volume of the parallelepiped ABCDEFGH is 40 cm^3.

(5 marks)

12 (a) A mathematical function f is defined by $f(x) = xe^{2x}$.

Show that $f''(x) = (4x + 4)e^{2x}$.

(3 marks)

(b) Prove by mathematical induction that if $f(x) = xe^{2x}$, then $f^{(n)}(x) = (2^n x + n2^{n-1})e^{2x}$.

(7 marks)

(c) Let $g(x) = \ln(1 + mx),\ m \in Z^+$.

Consider the function h defined by $h(x) = f(x) \times g(x)$.

Given that the term in x^4 of the Maclaurin series for $h(x)$ has coefficient 6, find the value of m.

(7 marks)