IB · DP · Maths

2 hours 12 questions

Practice Paper 1

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Total Marks **/110**

1 Prove that the square of an odd number is always odd.

(4 marks)

2 (a) Show that the equation $2\sin^2 x + 3\cos x = 0$ can be written in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers to be found.

(2 marks)

(b) Hence, or otherwise, solve the equation $2\sin^2 x + 3\cos x = 0$ for $-180° \leq x \leq 180°$.

(3 marks)

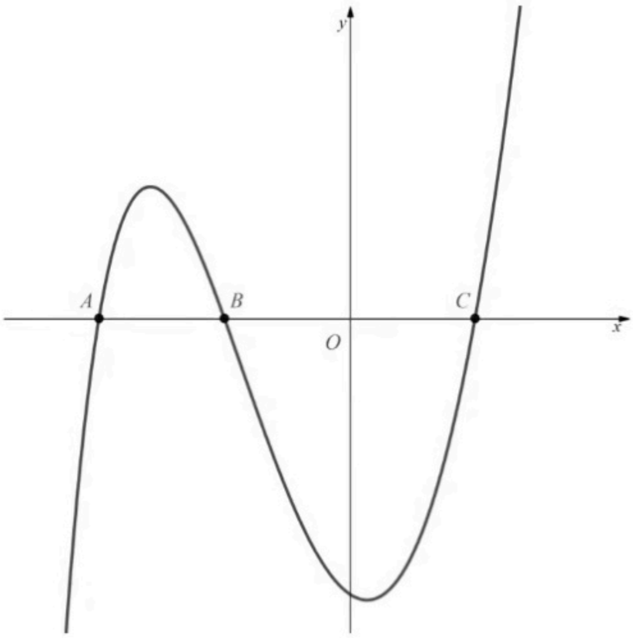
3 In the expansion of $(x + h)^5$, where $h \in \mathbb{R}$, the coefficient of the term in x^3 is 320.

Find the possible values of h.

(5 marks)

4 (a) The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = (x^2 - 1)\,\mathrm{In}(x + 3),\ x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

Find $\mathrm{f}'(x)$.

(4 marks)

(b) Show that the coordinates of point A are (-2, 0).

(2 marks)

(c) Find the equation of the tangent to the curve at point A.

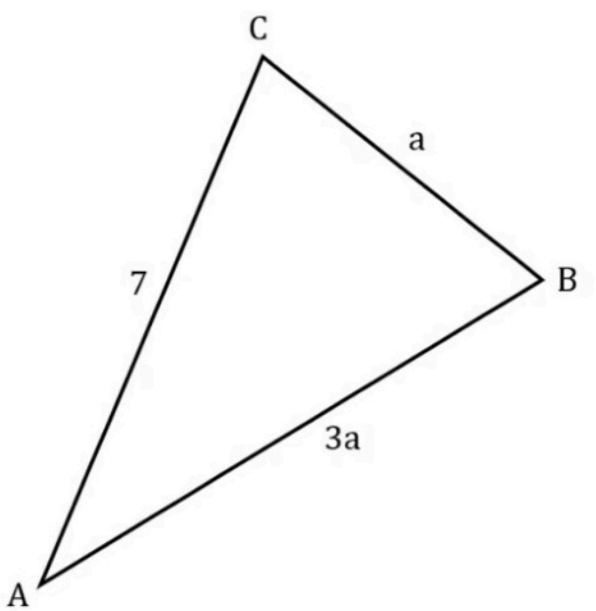
(3 marks)

5 The points **A**, **B**, **C** and **D** form the vertices of a parallelogram with position vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ respectively.

Show that the area of the parallelogram is $|\boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{b} \times \boldsymbol{d} + \boldsymbol{d} \times \boldsymbol{a}|$.

(4 marks)

6 The following triangle shows triangle ABC, with AB = 3a, BC = a and AC = 7.



Given that $\cos \mathrm{A}\widehat{\mathrm{B}}\mathrm{C} = \frac{1}{2}$, find the area of the triangle. Give your answer in the form $\frac{p\sqrt{3}}{r}$ where $p,q \in \mathbb{R}$.

(7 marks)

7 (a) α and β are non-real roots of the equation $x^2 + 3kx + 2k + 1 = 0$, where $k > 0$ is a constant.

Find $\alpha + \beta$ and $\alpha\beta$, in terms of k.

(2 marks)

(b) Given that $\alpha^2 + \beta^2 = 3$, show that $(\alpha + \beta)^2 = 4k + 5$.

(2 marks)

Hence find the value of k .

(c) **(3 marks)**

8 (a) Two lines, l_1 and l_2, are parallel and their vector equations are given below:

$$l_1 : r_1 = \begin{pmatrix} 2 \\ 1 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$$

$$l_2 : r_2 = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ p \\ q \end{pmatrix}$$

(i) State the values of p and q.

(ii) Show that l_1 and l_2 are not collinear.

(4 marks)

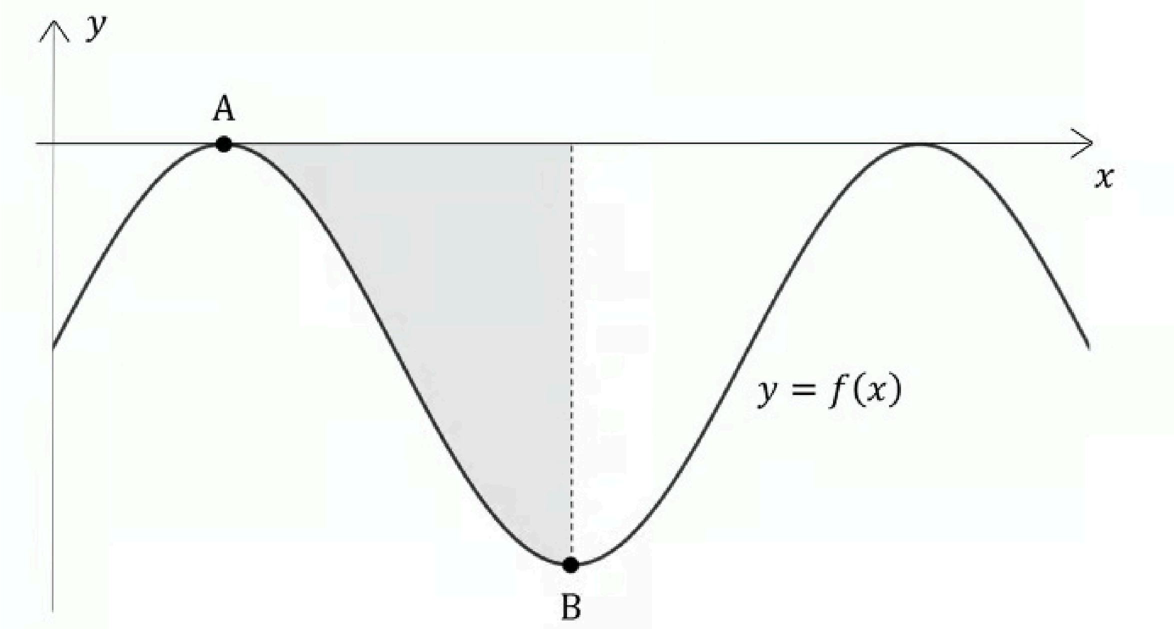
Find the minimum distance between l_1 and l_2.

(b) **(5 marks)**

9 Use the substitution $u = \cos x$ to find $\int \frac{\sin x \cos x}{\cos^2 x + 3\cos x - 4} \, dx$.

(7 marks)

10 (a) Consider the function f defined by $f(x) = 3\sin x - 3$, for $0 \le x \le 3\pi$
The following diagram shows the graph of $y = f(x)$



The graph of f touches the x-axis at point A as shown. Point B is a local minimum of f.

The shaded region is the area between the graph of $y = f(x)$ and the x-axis, between the
points A and B.

Find the x-coordinates of A and B.

(4 marks)

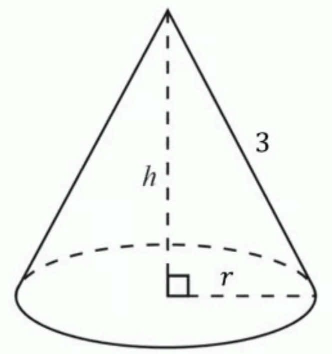
(b) Show that the area of the shaded region is 3π units^2.

(5 marks)

(c) The right cone in the diagram below has a curved surface area of twice the shaded area in

the previous part of the question.

The cone has a slant height of 3, base radius r, and height h.



Find the value of r.

(2 marks)

(d) Hence find the volume of the cone.

(4 marks)

11 (a) A particle is moving in a vertical line and its acceleration, in ms^{-2} , at time *t* seconds, $t \geq 0$ is given by $a = -\dfrac{1-v}{2}$, where v is the velocity in meters per second and $v < 1$.

The particle starts at a fixed origin O with initial velocity $v_o \text{ ms}^{-1}$.

By solving a suitable differential equation, show that the particle's velocity at time *t* is given by $v(t) = 1 - e^{-\frac{t}{2}}(1 - v_o)$.

(6 marks)

(b) The particle moves down in the negative direction, until its displacement relative to the origin reaches a minimum. Then the particle changes direction and starts moving up, in a positive direction.

(i) If the initial velocity of the particle is -3 ms^{-1}, find the time at which the minimum displacement of the particle from the origin occurs, giving your answer in exact form.

(ii) If *T* is the time in seconds when the displacement reaches its smallest value, show that $T = 2\ln(1 - v_o)$.

(4 marks)

(c) (i) Find a general expression for the displacement, in terms of t and v_o.

(ii) Combine this general expression with the result from part (b)(ii) to find an expression for the minimum displacement of the particle in terms of v_o.

(5 marks)

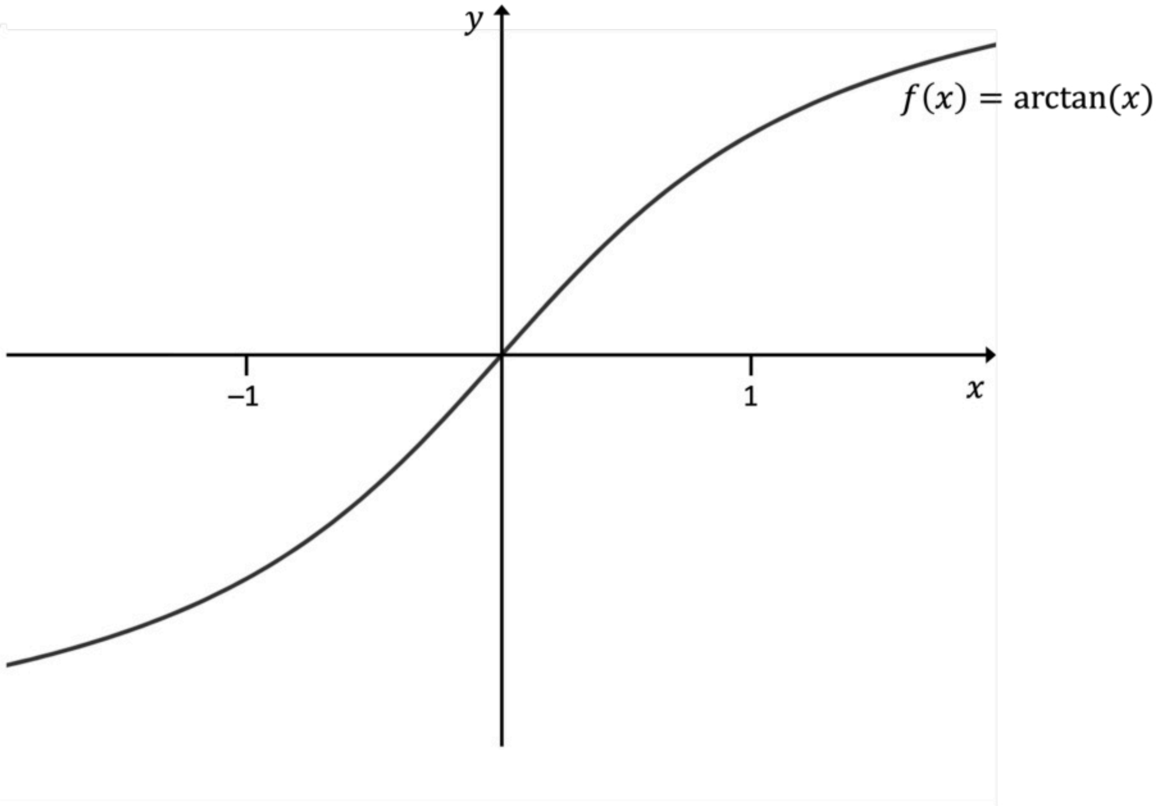
(d) Let $v(T-k)$ represent the particle's velocity *k* seconds before the minimum displacement and $v(T+k)$ the particle's velocity *k* seconds after the minimum displacement.

(i) Show that $v(T-k) = 1 - e^{\frac{k}{2}}$.

(ii) Given that $v(T+k) = 1 - e^{-\frac{k}{2}}$, show that $v(T-k) + v(T+k) \geq 0$.

(5 marks)

12 (a) The diagram below shows the graph of $f(x) = \arctan(x)$, $x \in \mathbb{R}$. The graph has rotational symmetry of order 2 about the origin.



A different function, g, is described by $g(x) = -\arctan(x-1)$, $x \in \mathbb{R}$.

(i) Describe the sequence of transformations that transforms $f(x)$ to $g(x)$.

(ii) Sketch the graph of $g(x)$ on the axes above.

(iii) Using your answers to parts (i) and (ii) to help you, describe the relationship between $\int_0^1 \arctan(x)\,dx$ and $\int_0^1 -\arctan(x-1)\,dx$. .

(5 marks)

(b) (i) Prove that $\arctan p - \arctan q = \arctan\left(\frac{p-q}{1+pq}\right)$.

(ii) Show that $\arctan\left(\frac{1}{x^2-x+1}\right)$ can be written as $\arctan(x) - \arctan(x-1)$.

(6 marks)

(c) Using the results from parts (a) and (b), evaluate $\int_0^1 \arctan\left(\frac{1}{x^2-x+1}\right)\mathrm{d}x$, leaving your answer in exact form.

(7 marks)