IB · DP · Maths

2 hours 17 questions

Practice Paper 1

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Total Marks **/111**

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1 (a) The line l_1 passes through the points (1, 7) and (5, 5).

Find the equation of l_1. Give your answer in the form of $y = mx + c$.

(2 marks)

(b) A new line, l_2, is perpendicular to l_1 and passes through the point (4, 8).

Find the equation of l_2. Give your answer in the form of $y = mx + c$.

(2 marks)

(c) The point Z is the intersection of l_1 and l_2.

Find the coordinates of Z.

(2 marks)

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2 (a) A function is defined by $f(x) = 4 - \frac{12}{5x+9}$, for $-5 \leq x \leq 5$, $x \neq -\frac{9}{5}$.

Find the range of f.

(3 marks)

(b) Find an expression for the inverse function $f^{-1}(x)$. The domain is not required.

(3 marks)

(c) Write down the range of $f^{-1}(x)$.

(1 mark)

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3 (a) It is claimed that women from Japan are taller on average than women from India. The heights, in cm, of 11 women from each country have been collated in the table below.

Japan	India
173.0	155.2
158.2	157.8
148.5	156.0
150.6	142.7
168.7	149.6
149.8	150.1
158.8	152.6
155.3	148.2
159.2	151.3
158.9	147.6
166.0	168.0

A t-test is to be performed at the 5% significance level.
State the null and alternative hypotheses.

(2 marks)

(b) Find the p-value for this test.

(2 marks)

(c) State whether or not the initial claim is justified. Give a reason for your answer.

(2 marks)

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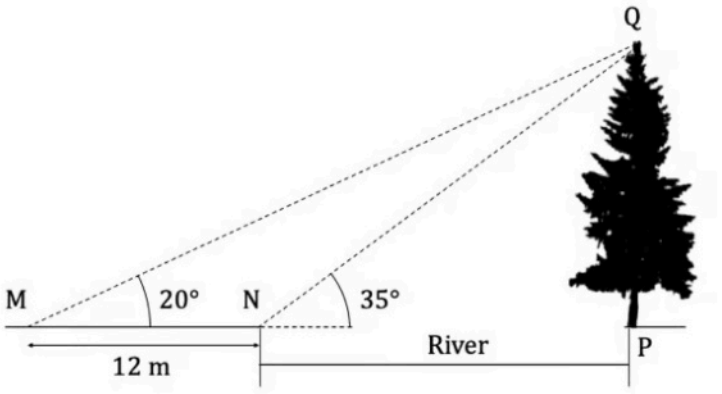
4 (a) Adah would like to estimate the height of a tree located at point P on the edge of a riverbank, with the top of the tree at point Q. However, due to a raging river, she is unable to reach the base of the tree. From point M she measures an angle of elevation of 20° to the top of the tree, and then from point N (which is on the edge of Adah's bank of the river) she measures an angle of elevation of 35° to the top of the tree. Between the points M and N she measures a horizontal distance of 12 m. Points M, N and P all lie on a single horizontal line, and point Q is vertically above point P. The diagram below shows this information.



Calculate the length of NQ.

(3 marks)

(b) Calculate the height of the tree.

(2 marks)

(c) Adah borrows a boat and crosses the river at a rate of 50 metres per 15 minutes.

Assuming that she crosses in a straight line directly from point N to point P, find out how long it takes her to cross the river.

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(3 marks)

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5 (a) A new car costs $20 000 and its value depreciates to $14 792 after 2 years.

Calculate

i) the annual rate of depreciation of the car

ii) the value of the car after 5 years. Give your answer correct to 2 decimal places.

(3 marks)

(b) Find the number of years and months it will take for the car's value to be approximately $4000.

(3 marks)

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6 (a) The function $g(x) = ax^2 + bx + c$ intercepts the y-axis at −16, has an x-intercept when $x = -4$ and can be obtained by an appropriate translation of the graph $y = 2x^2$.

i) Find the values of a, b and c.

ii) Write down $g(x)$.

(4 marks)

(b) Find the other x-intercept of $g(x)$.

(1 mark)

(c) Write down the coordinates of the vertex of $g(x)$.

(2 marks)

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7 (a) Ashley and Emma are attempting to swim a total of 2000 m each by completing laps of a 25 m pool. Ashley swims her first lap in 17 s and takes 0.2 s longer each lap after that. Emma swims her first lap in 16.5 s and takes 1.01 s times longer each lap after that.

i) Find the time Ashley takes to swim her final lap.

ii) Find the time Emma takes to swim her final lap.

(4 marks)

(b) i) State who swims the 2000 m the fastest.

ii) Find the mean lap time for both Ashley and Emma.

(4 marks)

8 Consider the lines l_1 and l_2 defined by the equations:

$$l_1: \begin{cases} x = 2 + 6\lambda \\ y = 2 + q\lambda \\ z = -8 - 5\lambda \end{cases}$$

$$l_2: r = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix} + \lambda \begin{pmatrix} -24 \\ 12 \\ 20 \end{pmatrix}$$

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Given that l_1 and l_2 are identical, find the value of p and q.

(6 marks)

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9 (a) A carpet salesman in interested how his sales are distributed and records his sales results over a period of six months. The data is shown in the table.

Month	January	February	March	April	May	June
Number of sales	16	12	14	20	15	19

A chi-squared goodness of fit test is to be performed on the data at the 5% significance level to find out whether the data fits a uniform distribution.

The critical value for the test is 11.070 and the hypotheses are

H_0: The data satisfies the model.
H_1: The data does not satisfy the model.

Find an estimate of how many carpets the salesman expects to sell each month.

(1 mark)

(b) Write down the number of degrees of freedom for this test.

(1 mark)

(c) State the conclusion of the test. Give a reason for your answer.

(4 marks)

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10 (a) Let D be a normally distributed random variable that represents the distance travelled in metres by a slug in one day. The distance covered by a random sample of 21 slugs on a randomly selected day can be summarized as follows

$$\Sigma d = 341, \Sigma d^2 = 5881.$$

Find an unbiased estimate of the mean, μ, of D.

(1 mark)

(b) Use the formula $s_{n-1}^2 = \dfrac{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}{n-1}$ to find an unbiased estimate of the variance of D.

(1 mark)

(c) Find a 95% confidence interval for μ.

(2 marks)

(d) Justin believes that the average slug travels 15 m per day.

State whether or not Justin's statement is valid. Give a reason for your answer.

(2 marks)

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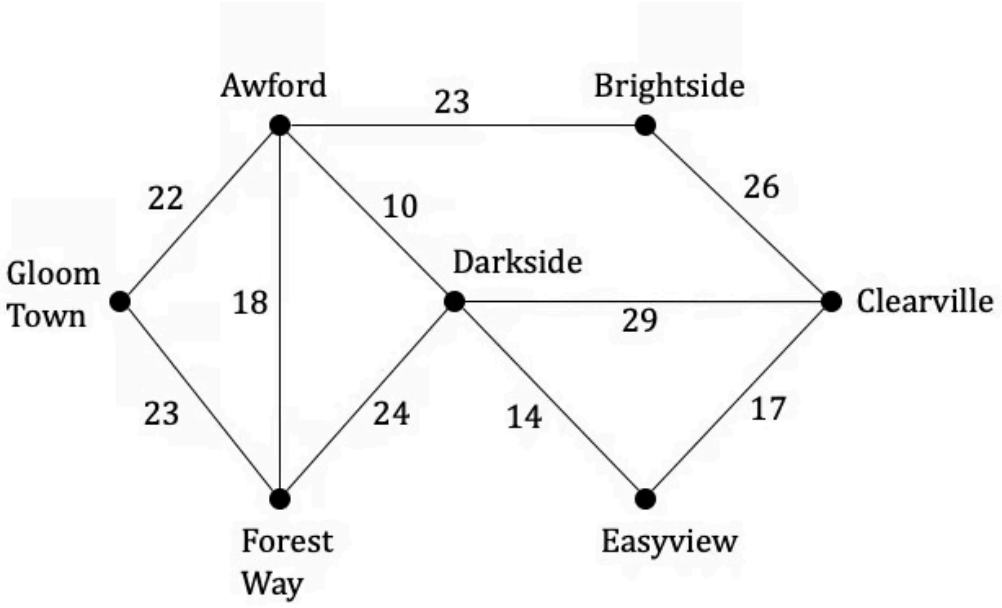
11 (a) The graph G below shows 7 towns and the train tracks that the connect them, with the vertices representing the towns and the weighting of each edge indicating the time taken in minutes to walk along the section of track.



State the degree of each vertex.

(2 marks)

(b) Explain why G does not contain an Eulerian circuit.

(1 mark)

(c) The railway company in charge of maintaining the track wishes to inspect all sections of the track for defects after a storm event.

Find the minimum time it would take for a railway worker who was walking to inspect all of the track.

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(4 marks)

12 (a) It is given that that $z_1 = 2e^{i\left(\frac{\pi}{3}\right)}$ and $z_2 = 3cis\left(\frac{n\pi}{12}\right)$, $n \in \mathbb{Z}^+$.

Find the value of $z_1 z_2$ for $n = 3$.

(3 marks)

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$.

(3 marks)

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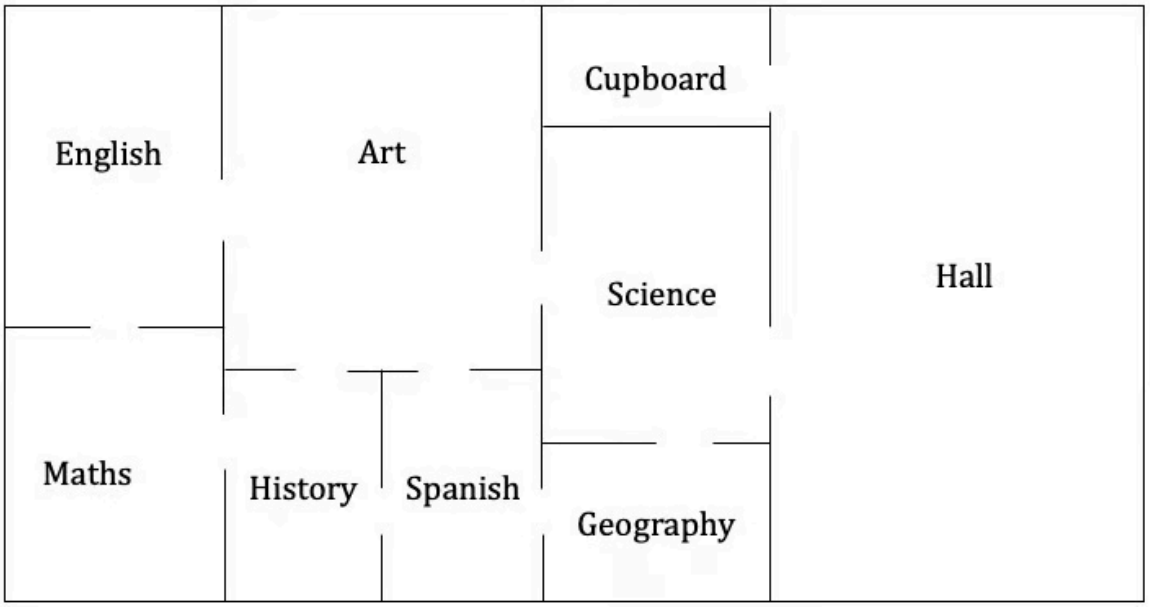
13 (a) The diagram below shows the plan of a school building.



Construct a graph to represent this information, using vertices to represent rooms and edges to represent the connecting doors.

(2 marks)

(b) For a prank, a student releases a monkey into the English classroom at 8:30 pm on Thursday. The monkey continues to wander at random through the school building all night until the cleaner arrives at 6 am on Friday morning.

By inspecting the steady state probabilities, write down the room in which the cleaner is most likely to discover the monkey.

(2 marks)

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(c) Stating clearly any assumptions you have made, find an approximation for the total length of time the monkey is likely to have spent in the Maths classroom before the cleaner arrives.

(2 marks)

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14 (a) Points in a plane are subjected to a transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x' \\ y' \end{pmatrix}$, where T is defined by:

$$T: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} p \\ 6 \end{pmatrix}$$

Given that a point $A(4, q)$ is mapped to $A'(7-\sqrt{3}, 2\sqrt{3}+7)$, find p and q, where $p, q \in \mathbb{R}$.

(2 marks)

(b) Given that T comprises two individual transformations describe in full the composite transformation T.

(4 marks)

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15 (a) The number of spam emails that Arturo receives per day is modelled by a Poisson distribution with a mean of 25 spam emails per day.

After changing the settings on his spam filter, Arturo decides to test whether the new settings have reduced the number of spam emails he receives. To do this he records the number of spam emails he receives over a period of one week. He decides to use a 5% level of significance for his test.

State the null and alternative hypotheses for the test.

(1 mark)

(b) (i) Find the critical value and the critical region for Arturo's test.

(ii) Hence find the probability that Arturo will make a Type I error in determining the conclusion of his test.

(5 marks)

(c) During the 1-week period, Arturo receives 149 spam emails.

State Arturo's conclusion to his test, being sure to justify your answer.

(2 marks)

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16 (a) A particle P moves in a straight line, such that its displacement x at time t is defined by the differential equation $\dot{x} = x(e^{\cos t - 1})\sin t,\ t \geq 0$. At time $t = 0,\ x = e$.

By using Euler's method with a step length of 0.2, find an approximate value for x when $t = 0.6$.

(3 marks)

(b) Solve the differential equation to find the actual value of x when $t = 0.6$.

(4 marks)

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17 (a) A manufacturing process takes place inside a sealed chamber and produces a pollutant that decays over time. After the process is completed, at time t = 0 seconds, the amount of pollutant, P ppm (parts per million) in the chamber is modelled by

$$P = P_B + P_A e^{kt}$$

Write down an expression for

(i) the background level of the pollutant in the chamber,

(ii) the amount of pollutant in the chamber at the moment the process completes,

(iii) the half-life of the pollutant.

(3 marks)

(b) The half-life of the pollutant is $t_{0.5} = e^4$.

Find the value of k.

(2 marks)

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