IB · DP · Maths

2 hours 17 questions

Practice Paper 1

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Total Marks /110

1 (a) Lucy loves the cinema and goes on average four times a week. The number of times she goes to the cinema in a week can be modelled as a Poisson distribution with a mean of four times.

Find the probability that Lucy goes to the cinema exactly five times in a week.

(2 marks)

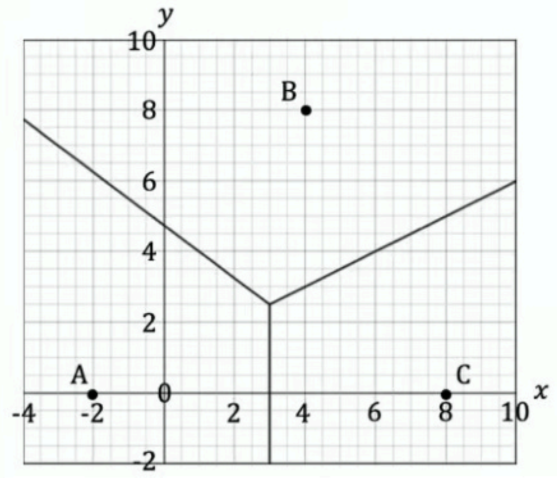
(b) Find the probability that Lucy goes to the cinema no more than four times in a fortnight.

(2 marks)

2 (a) A town is built in a rectangular area bounded by the lines $y = -2, y = 10$, $x = -4$ and $x = 10$ as shown on the Voronoi diagram below.

Each horizontal and vertical unit on the grid below represents 1 km.

Points $\text{A}(-2, 0)$, $\text{B}(4, 8)$ and $\text{C}(8, 0)$ represent the locations of first aid responders in the town.



Calculate the area for which the first aid responder at C has responsibility.

(3 marks)

(b) A new station is due to open at $\text{D}(8,2)$ and as such the Voronoi diagram will need to be adjusted.

i) Write down the equation of the perpendicular bisector of $[\text{BD}]$. Give your answer in the form $y = mx + c$.

ii) Write down the equation of the perpendicular bisector of $[\text{CD}]$. Give your answer in the form $y = mx + c$.

(3 marks)

3 (a) Ben and Sam are both cyclists competing in a 22.5 km race at the Herne Hill Velodrome in London, England. One lap of the velodrome is 450 m.

Ben takes a total of 42 minutes to complete the race.

Calculate Ben's mean lap time in seconds.

(2 marks)

(b) Given that each of Ben's laps took him 1% longer to complete than the previous one, calculate how long it took him (in seconds) to complete his first and last laps.

(3 marks)

(c) Sam completes the first lap in 45 seconds and takes 0.2 seconds longer per lap.

Determine who completed the race the first out of Ben and Sam. Justify your answer.

(3 marks)

4 (a) The temperature, T, of a cake, in degrees Celsius, , $°C$ can be modelled by the function

$$T(t) = a \times 1.17^{-\frac{t}{4}} + 18, \qquad t \geq 0,$$

where a is a constant and t is the time, in minutes, since the cake was taken out of the oven.

In the context of this model, state what the value of 18 represents.

(1 mark)

(b) The cake was $180°C$ when it was taken out of the oven.

Find the value of a.

(2 marks)

(c) Find the temperature of the cake half an hour after being taken out of the oven.

(2 marks)

5 (a) ABCD is a parallelogram with vertices $\mathrm{A}(2,\ 3,\ 0)$, $\mathrm{B}(3,\ 9,\ 4)$, $\mathrm{C}(7,\ 4,\ 2)$ and $\mathrm{D}(6,\ -2,\ -2)$.

Find the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$.

(2 marks)

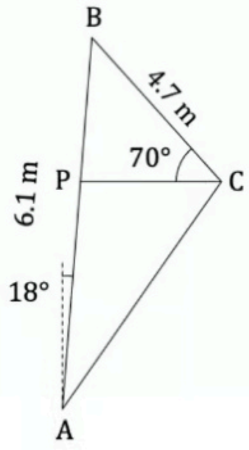
(b) Find the area of the parallelogram.

(3 marks)

(c) By finding the scalar product of $\overrightarrow{\mathrm{BA}}$ and $\overrightarrow{\mathrm{BC}}$, determine if the angle $\mathrm{A\hat{B}C}$ is acute or obtuse.

(4 marks)

6 (a) The diagram below shows the triangular sail of a windsurfing board, ABC, with a horizontal boom PC. AB = 6.1 m and makes an angle of $18°$ to the vertical. BC = 4.7 m and $B\hat{C}P$ = 70°.



Find the area of the whole sail.

(4 marks)

(b) Calculate the length of the boom PC.

(2 marks)

7 (a) The volume of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$

Find $\frac{dV}{dr}$.

(1 mark)

(b) Find the rate of change of the volume with respect to the radius when $r = 5$.

Give your answer in terms of π.

(2 marks)

(c) Show that $\frac{dV}{dr}$ is an increasing function for all relevant values of r.

(3 marks)

8 (a) Frank plays a game involving a biased six-sided die.
The faces of the die are numbered 1 to 6.
The score of the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score , x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{2}p$	$\frac{1}{8}$	$\frac{3}{2}p$	$\frac{1}{12}$	$3p$

Calculate the exact value of p.

(2 marks)

(b) Frank plays the game once.

Calculate the expected score.

(3 marks)

9 (a) Consider the complex numbers $z_1 = 1 - 2\text{i}$ and $z_2 = -3 + 5\text{i}$.

Work out the following:

(i) $\text{Re}(z_2 - z_1)$

(ii) $\text{Im}(z_1 z_2)$

(iii) $\left(\dfrac{z_1}{z_2}\right)^*$

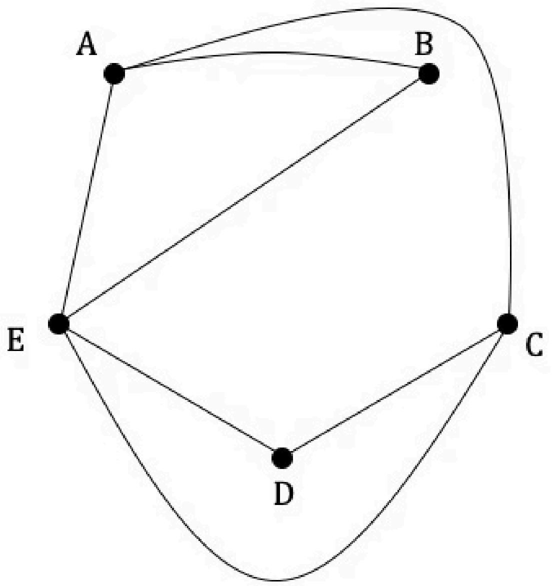
For part (iii) give your answer in the form $a + b\text{i}$, where a and b are real numbers.

(6 marks)

(b) Write down the complex conjugate of z_2 and describe the geometrical relationship between z_2 and $z_2{}^*$.

(2 marks)

10 (a) Let G be the graph below.



Construct the transition matrix for a random walk around G.

(3 marks)

(b) Determine the probability that a random walk of length 3 starting at A will finish at C.

(2 marks)

(c) (i) Find the steady state probabilities for the graph.

(ii) Hence rank the vertices in terms of importance from highest to lowest.

(2 marks)

11 (a) The germination rate of a particular seed is 44%. George sows 25 of these seeds selected at random.

Calculate the expected number of seeds that germinate.

(2 marks)

(b) Find the probability that more than 7 of these seeds will germinate.

(2 marks)

(c) Find the probability that at least 9, but no more than 11 of the seeds germinate.

(3 marks)

12 (a) Scientists are studying a large pond where an invasive plant has been observed growing, and they have begun measuring the area, A m^2, of the pond's surface that is covered by the plant. According to the scientists' model, the rate of change of the area of the pond covered by the plant at any time, t, is proportional to the square root of the area already covered.

Write down a differential equation to represent the scientists' model.

(2 marks)

(b) Solve the differential equation to show that

$$A = \left(\frac{kt + c}{2}\right)^2$$

where k is the constant of proportionality and c is a constant of integration.

(4 marks)

(c) At the time when the scientists begin studying the pond the invasive plant covers an area of 100 m^2. One week later the area has increased to 225 m^2.

Use this information to determine the values of k and c.

(3 marks)

13 (a) A drone travels in a straight line and at a constant speed. It moves from an initial point $A(4, 5, -2)$ to a second point $B(7, -1, 0)$. The person controlling the drone is located at $C(2,3,2)$.

The x, y and z directions are due east, due north and vertically upwards respectively with all distances in metres.

Write down an equation for the line along which the drone travels.

(2 marks)

(b) At some point P on its flight, the drone is vertically level with the person controlling the drone.

Find the coordinates of point P.

(3 marks)

(c) Find the distance between P and the person controlling the drone.

(2 marks)

14 (a) The weight, W, of pumpkins purchased are normally distributed with a mean of 11.3 kg and a standard deviation of 2.1 kg.

In one year, a farmer grows 350 pumpkins on his farm.

Predict the number of pumpkins that weigh between 7.2 kg and 12.5 kg from the farm.

(3 marks)

(b) The heaviest 7% of pumpkins are classified as large and are sold at a higher price.

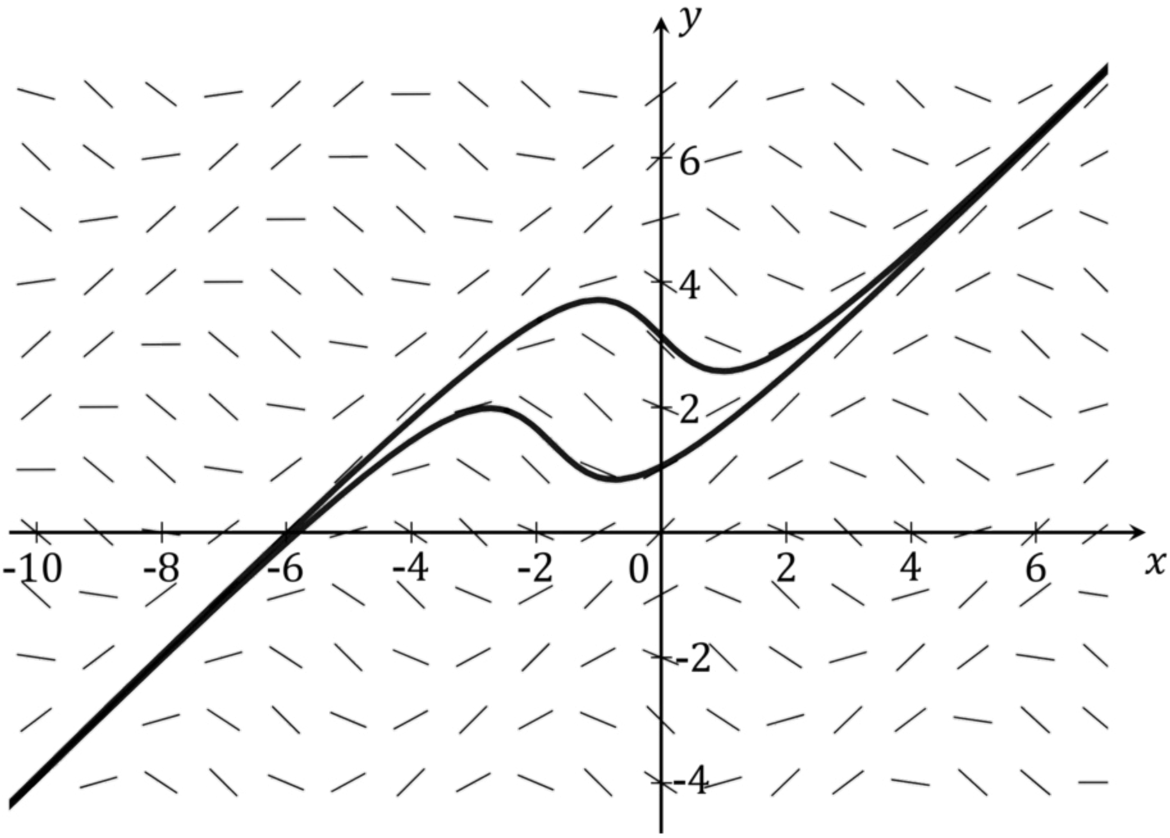
Find the range of weights of pumpkins that can be sold for a higher price.

(3 marks)

15 (a) The diagram below shows the slope field for the differential equation

$$\frac{dy}{dx} = \cos(x - y)$$

The graphs of the two solutions to the differential equation that pass through the points $\left(0, \frac{\pi}{3}\right)$ and $(0, \pi)$ are shown.



Explain the relationship that must exist between x and y for $\frac{dy}{dx}$=0 to be true.

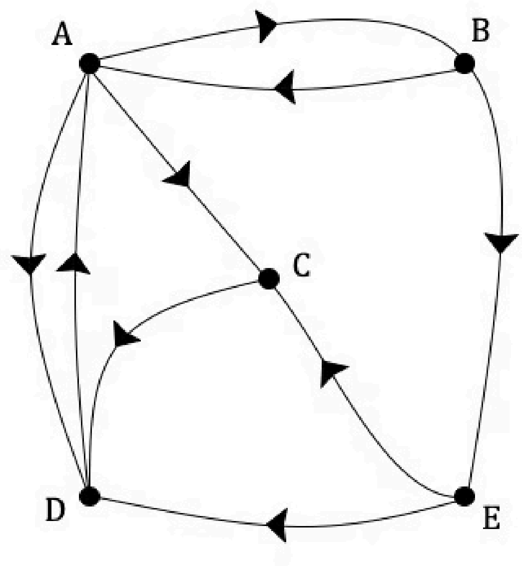
(2 marks)

(b) For the two solutions given, the local minimum points lie on the straight line L_1 and the local maximum points lie on the straight line L_2.

Find the equations of (i) L_1 and (ii) L_2, giving your answers in the form $y = mx + c$.

(3 marks)

16 (a) Consider the graph *G* shown below.



Write down the adjacency matrix for the graph *G*.

(3 marks)

(b) Find the total number of walks of length 7 that start at vertex A and finish at vertex D.

(2 marks)

17 (a) Consider the functions f and g defined by $f(x) = \ln x$ and $g(x) = \ln(2x + 5)$, where each function has the largest possible valid domain.

The graph of f can be transformed onto the graph of g by a single translation and a single stretch, both of which are parallel to one of the coordinate axes.

Describe the sequence of transformations in the case where:

(i) the translation occurs first.

(ii) the stretch occurs first.

(4 marks)

(b) The graph of can be also transformed onto the graph of g by a single translation using the vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

Find the exact values of a and b.

(3 marks)