IB · DP · Maths

1 hour 14 questions

Practice Paper 1

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Total Marks /80

1 (a) A store manager wanted to get an idea of how much shoppers were spending. The manager conducted a survey asking shoppers how much they had spent.

The data is shown in the following table.

Amount spend in pounds (£p)	Number of shoppers
$£0 \leq p < £2$	5
$£2 \leq p < £5$	14
$£5 \leq p < £10$	20
$£10 \leq p < £20$	s
$£20 \leq p \leq £50$	3

Explain why only an ***estimate*** of the mean amount spent by shoppers can be found using the data in the table (even when the value of s is known).

(1 mark)

(b) An estimate of the mean amount spent by the shoppers is £8.58.

Find the value of s.

(4 marks)

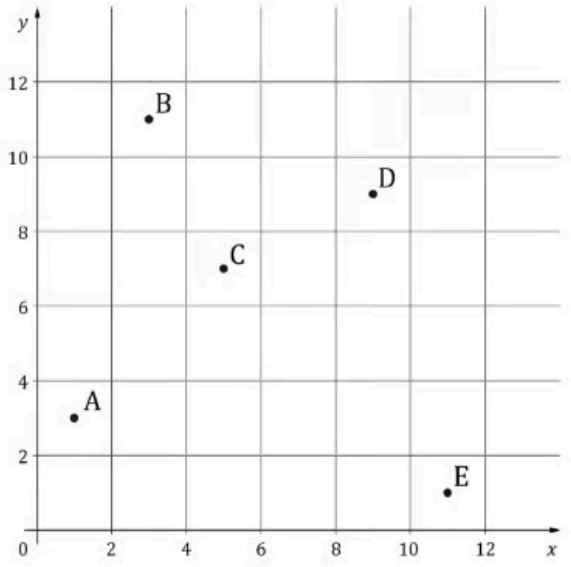
(c) It was not practical to ask every shopper in store on a particular day, so the manager stood at the shop exit for an hour and asked some, but not all, of the shoppers how much they had spent.

Identify the sampling technique used in the survey.

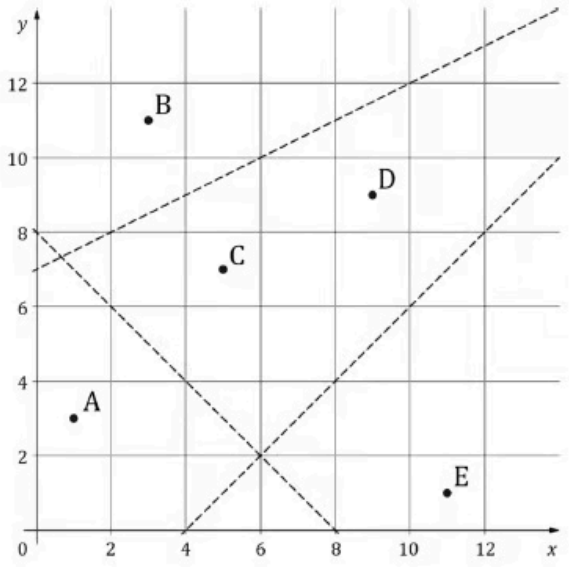
(1 mark)

2 (a) Rangers use aerial imagery to help locate big cats on the savannah. This week the plane is not available so they must use last week's image which shows the last known locations of five male cats at points $A(1, 3)$, $B(3, 11)$, $C(5, 7)$, $D(9, 9)$ and $E(11,1)$ as illustrated on the following coordinate axes.

Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



Male cats stick to very rigid territories keeping their distance from other males to avoid confrontation. Using the image above, rangers draw three straight lines to form an incomplete Voronoi diagram.



Calculate the gradient of the line segment CD.

(2 marks)

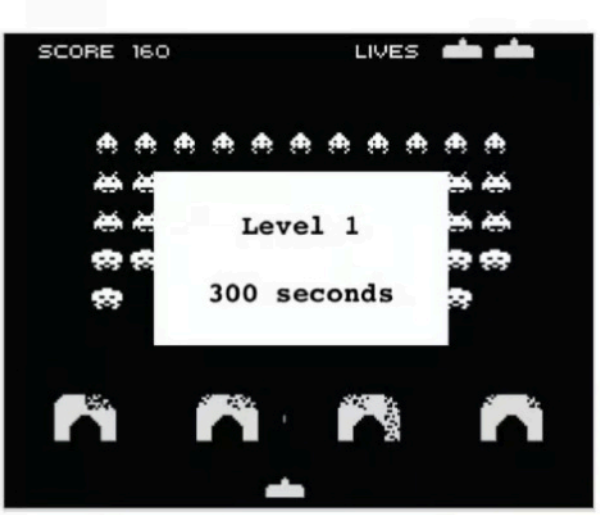
(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

(3 marks)

(c) In the context of the question, explain the significance of the Voronoi cell containing site C.

(1 mark)

3 (a) In a version of the computer game Space Invaders a player has a set time to complete each level. On Level 1 a player has a time limit of 300 seconds to complete it.



The times allowed for each level form an arithmetic sequence. On Level 4 the time limit is 255 seconds.

Work out the value of the common difference, d.

(2 marks)

(b) The final level has a time limit of 45 seconds. Find the number of levels in the game.

(2 marks)

(c) Find the maximum time allowed to complete the whole game.

(2 marks)

4 (a) Sirona has been watching the birds that visit her backyard birdfeeders, and recording which of the food options they go to first when they arrive at the feeding station. The results of her observations are shown in the table below.

	First food option chosen			
	sunflower seeds	**buggy bites**	**table mix**	**mealworms**
robins	1	4	2	5
blackbirds	2	10	1	7
dunnocks	7	5	7	1

Sirona conducted a χ^2 test for independence at a 5 % level of significance.

State the null hypothesis.

(1 mark)

(b) Calculate the p-value for this test.

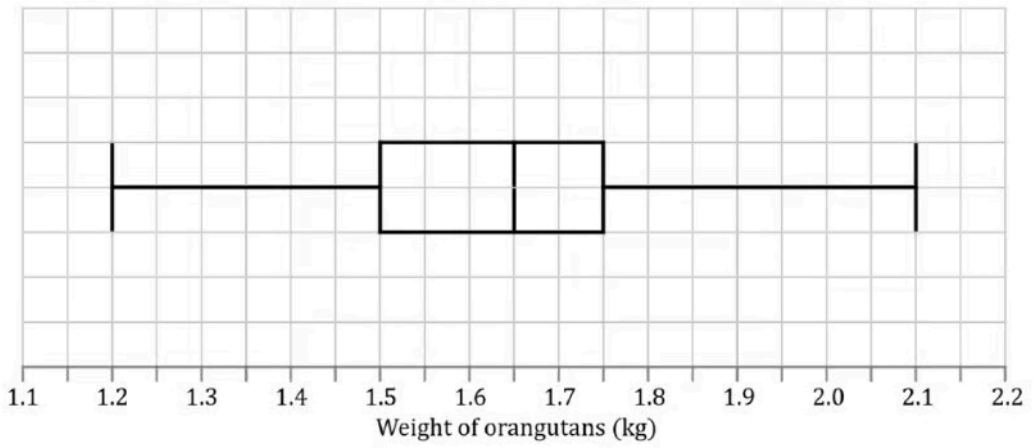
(2 marks)

(c) State, giving a reason, whether the null hypothesis should be accepted.

(2 marks)

5 (a) Sasha works for a conservation charity who rescue orphaned orangutans. Over many years she records the weight (kg) of the orangutans when they first arrive.

The data is illustrated in the following box and whisker diagram.



Write down the median weight of the orangutans.

(1 mark)

(b) Write down the lower quartile.

(1 mark)

(c) Find the interquartile range.

(2 marks)

(d) The weights of these orangutans are normally distributed.

Show that the data does not contain any outliers.

(2 marks)

6 (a) Doctor Scotpop is investigating the population of otters in Scotland.

The doctor found, based on historical data, that the population of otters, P, could be modelled by $P = 7500 + A(1.09)^t$ where A is a constant and t is the number of years since the start of the year $2000, t \geq 0$.

At the start of the year 2000, the population of otters in Scotland was 8000.
Find the value of the constant A.

(2 marks)

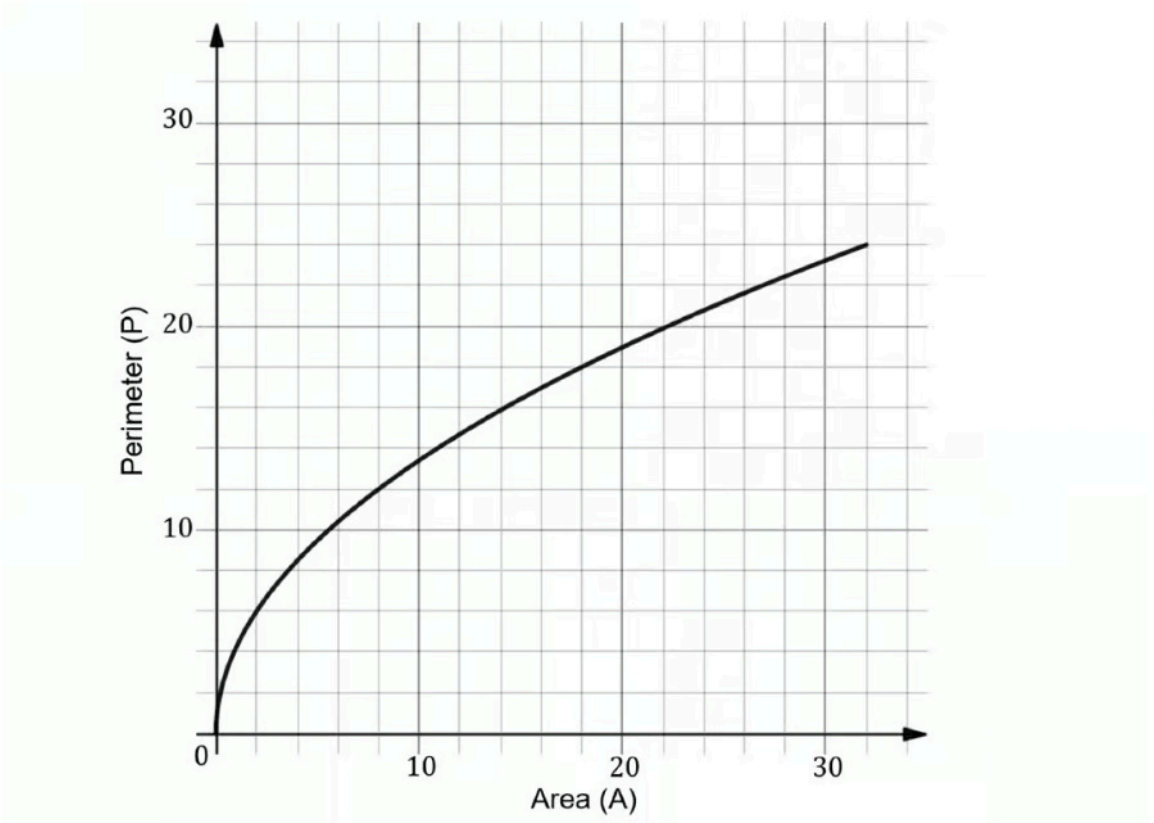
(b) Find the population of otters in Scotland at the start of the year 2020.

(2 marks)

(c) Doctor Scotpop estimates the peak population of otters Scotland can sustain is $15\,500$.
Work out the year in which Doctor Scotpop expects the population of otters to peak.

(2 marks)

7 (a) The perimeter of a rectangle P, whose width is double its height, can be represented by the function $P(A) = 6\sqrt{\frac{A}{2}}$, $A \geq 0$, where A is the area of the rectangle. The graph of the function P is shown for $0 \leq A \leq 32$.



Write down the value of $P(32)$.

(1 mark)

(b) The range of $P(A)$ is $0 \leq P(A) \leq n$

Hence write down the value of n.

(1 mark)

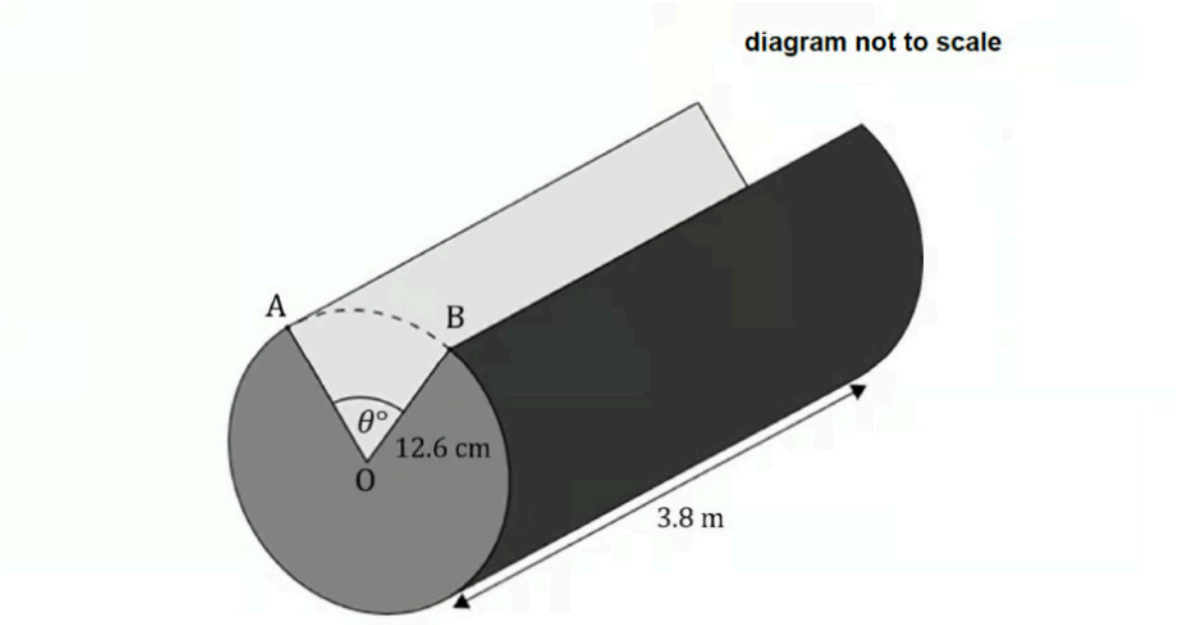
(c) On the axes above, draw the graph of the inverse function, P^{-1}

(3 marks)

(d) In the context of the question, explain the meaning of $P^{-1}(12) = 8$.

(1 mark)

8 Alex has been commissioned to create an art sculpture using 50 cylindrical bars of metal with a length-ways wedge cut out. Each piece will have length 3.8 m and radius 12.6 cm as illustrated in the following diagram, where O indicates the centre of the circular cross-section.



The whole sculpture will use 7.15 m^3 of metal.

Find the angle $\theta°$ defining the size of the wedge that each bar must have cut out of it.

(4 marks)

9 (a) Astronomers classify the brightness of stars according to a scale of magnitudes. The difference in magnitude between two stars is defined by the formula

$$m_1 - m_2 = 2.5\log_{10}\left(\frac{b_2}{b_1}\right)$$

where m_1 and m_2 are the magnitudes of the two stars, and b_1 and b_2 are the corresponding apparent brightnesses measured in watts per metre squared (W m^{-2}). The magnitude of a star is a unitless measure, and its value can be positive or negative.

The star Sirius has a magnitude of —1.4 and an apparent brightness of $1.04 \times 10^{-7}\ \text{W m}^{-2}$

The star Polaris has an apparent brightness of $4.62 \times 10^{-9}\ \text{W m}^{-2}$. Calculate the magnitude of Polaris.

(2 marks)

(b) The Sun has a magnitude of -26.7. Calculate the apparent brightness of the Sun.

(2 marks)

10 (a) Jethro exercises every day, but he is terrible at making decisions. Therefore each day he chooses one of his friends at random, and rings them to ask what exercise he should do.

Of Jethro's 18 friends, 13 of them are keen cyclists and will always tell him to go cycling. The other 5 are tai chi fanatics and will always tell him to do tai chi. On a given day, Jethro always does the exercise that his randomly chosen friend tells him to do.

Find the probability that on any given day Jethro will do tai chi.

(1 mark)

(b) The month of June has 30 days.

Find the probability that Jethro will do tai chi 10 times in June.

(2 marks)

(c) Find the probability that Jethro will go cycling at least 24 times in June.

(3 marks)

11 (a) Leo Forest is a keen golfer and would like to improve the distances he achieves with his tee shots (his first shot). In a bid to improve his tee shot distance Leo hires a golf coach. He records the distances, in yards, of his tee shots both before and after coaching, with the results shown in the table below.

Tee shot distance *before* **coaching (yards)**	190	196	208	216	201	223	220	230	243
Tee shot distance *after* **coaching (yards)**	218	213	231	224	239	253	246	242	

Leo is interested to see whether the mean distance of his tee shots after coaching has increased or not, and decides to use a t-test at the 5% significance level to compare the means of his tee shot distances before and after coaching.

Write down an assumption about the distribution of the data so that a t-test can be conducted.

(1 mark)

(b) (i) State the null hypothesis.

(ii) State whether this is a one-tailed or two-tailed test.

(2 marks)

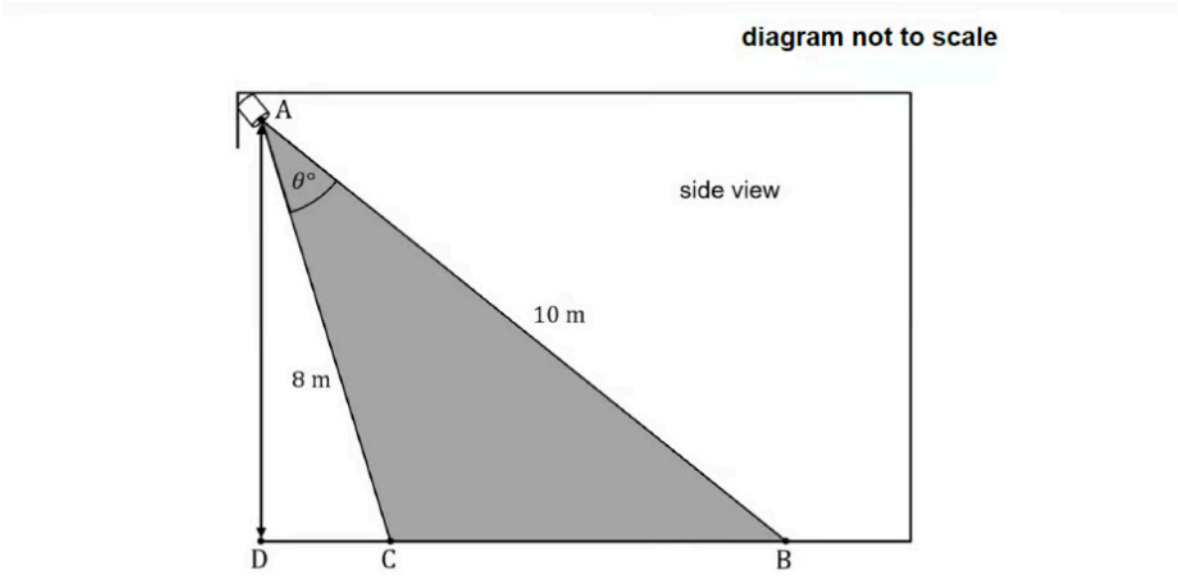
(c) Perform a t-test, writing down the p-value for the test.

(2 marks)

(d) Justifying your decision, what conclusion should Leo draw about the mean of his tee shot distances before and after coaching?

(1 mark)

12 (a) A new spotlight is being installed on a theatre lighting rig. The lighting crew must adjust the angle of the beam and mark out where actors can stand to ensure they are properly lit. The spotlight is located at point A directly above point D at the front of the stage. The area covered by the light is shown by the shaded region enclosed by triangle ABC in the following diagram and can be adjusted by changing the angle $C\widehat{A}B$.



The lighting crew have adjusted the light so that the distance from A to B is 10 m, the distance from A to C is 8 m, and the length of the stage floor covered between points B and C is 5.2 m.

Find the angle the lighting crew have adjusted $C\widehat{A}B$ to.

(2 marks)

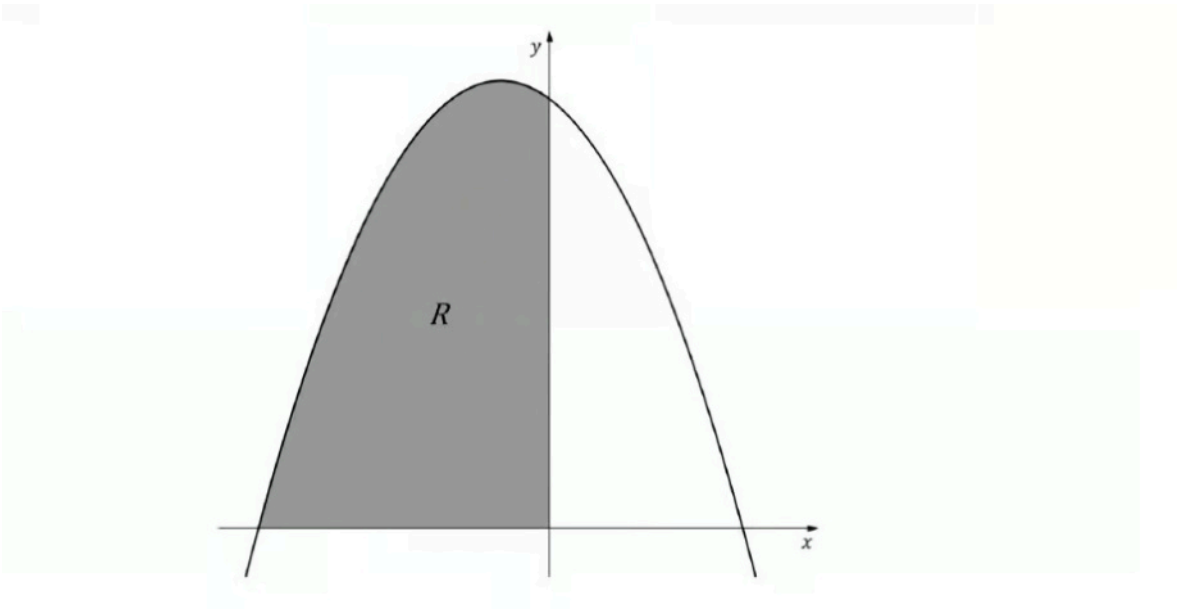
(b) Point C is 1.2 m from the front of the stage at point D. To ensure actors know where to stop when walking towards point B from the direction of point C, the lighting crew mark a point on the floor at which any actor under 1.9 m tall can stand and remain fully lit.

Find the furthest distance **from the front of the stage** that the lighting crew should place their mark.

(6 marks)

13 (a) The following diagram shows part of the graph of $g(x) = (2 - x)(3x + 9)$, $x \in \mathbb{R}$

The shaded region R is bounded by the x-axis, y-axis and the graph of g.



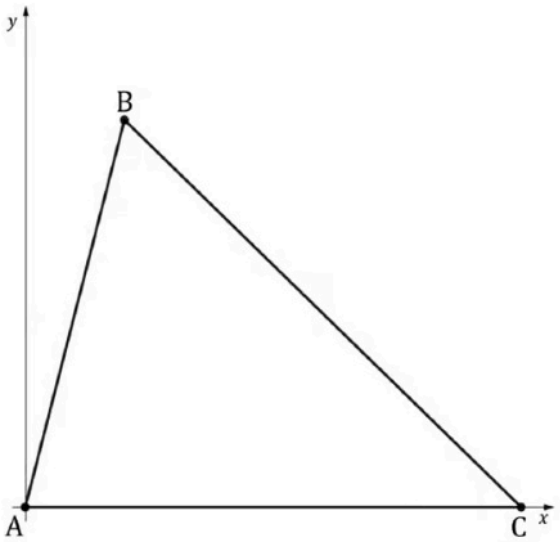
Write down an integral for the area of region R.

(2 marks)

(b) Find the area of region R.

(1 mark)

(c) The three points $\mathrm{A}(0, 0)$, $\mathrm{B}(3, a)$ and $\mathrm{C}(15, 0)$ define the vertices of a triangle.



Find the value of a, the y-coordinate of B, such that the area of the triangle is twice the area of region R.

(2 marks)

14 (a) Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5.

The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

Find the exact value of p.

(1 mark)

(b) Leonardo is playing a game with his biased spinner. The score for the game is the number which the spinner lands on after being spun.

Leonardo plays the game once.

Calculate the expected score.

(2 marks)

(c) Leonardo plays the game twice and adds the two scores together.

Find the probability Leonardo has a **total** score of 2.

(3 marks)