IB · DP · Maths

1 hour 13 questions

Practice Paper 1

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Total Marks /80

1 (a) Let $Q = \dfrac{30 \sin 2a}{8b}$, where $a = 45°$ and $b = 2$.

Calculate the exact value of Q.

(2 marks)

(b) Give your answer from part (a) correct to

i) two decimal places

ii) two significant figures.

(2 marks)

(c) Nina estimates the value of Q to be 2.

Calculate the percentage error in Nina's estimate.

(2 marks)

2 A pharmacy sells face masks in a variety of sizes. Their sales over a week are recorded in the table below:

	Kids		**Adults**			
Size	Small	Large	S	M	L	XL
Frequency f	29	4	8	24	15	4

i) Write down the mode for this data.

ii) Explain why, in this case, the mode from part (i) would not be particularly helpful to the shop owner when reordering masks.

iii) Given that the shop is open every day of the week, calculate the mean number of masks sold per day.

(4 marks)

3 (a) A coffee scoop is made out of 2 mm thick stainless steel. It is in the shape of a hemisphere and has an outer diameter of 3.8 cm.

Show that the outer surface area of the scoop is $\frac{361}{50}\pi$ cm^2.

(2 marks)

(b) Calculate the volume of coffee that the scoop can hold, in cm^3.

(3 marks)

(c) The density of the coffee when compacted in the scoop is 0.825 g/cm^3. A single bag of ground coffee beans contains 350 g and costs $16.

Calculate the cost per scoop of ground coffee.

(4 marks)

4 (a) The total cost, C, in New Zealand dollars (NZD), of a premium gym membership at Cityfitness can be modelled by the function

$$C = 16.99t + 49, t \geq 0$$

where t is the time in weeks.

Calculate the cost of the gym membership for a year. Give your answer correct to 2 decimal places.

(1 mark)

(b) Find the number of weeks it takes for the total cost to exceed 2000 NZD.

(2 marks)

(c) At Les Mills the initial payment is 20 NZD lower than Cityfitness, however the weekly cost is 8.51 NZD higher than Cityfitness

Write a cost function for a gym membership at Les Mills using an appropriate model.

(1 mark)

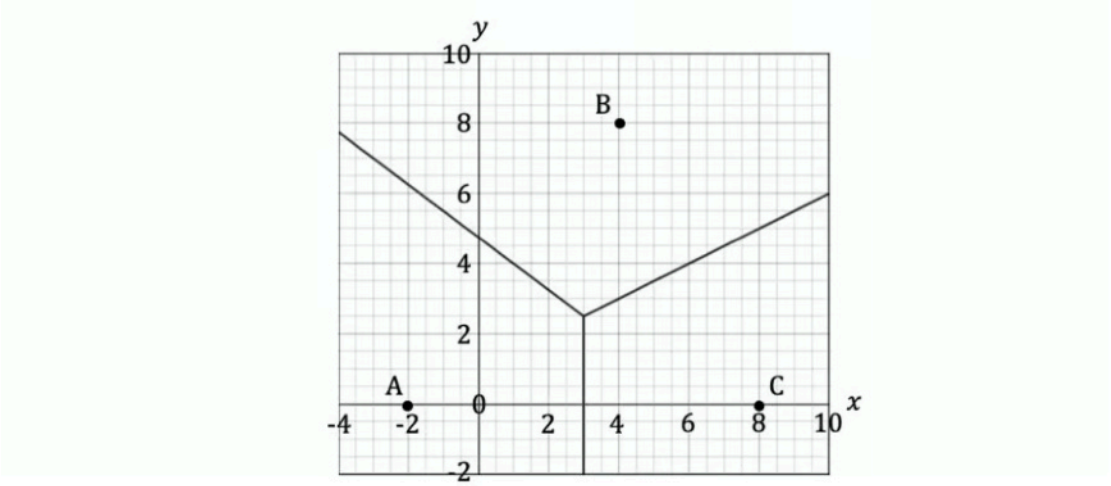
(d) Calculate how many weeks it will take for the cost of a Les Mills gym membership be more than the cost of a Cityfitness gym membership.

(3 marks)

5 (a) A town is built in a rectangular area bounded by the lines y=-2, y=10, x = —4 and x = 10 as shown on the Voronoi diagram below.

Each horizontal and vertical unit on the grid below represents 1 km.

Points A(—2, 0), B(4, 8) and C(8, 0) represent the locations of first aid responders in the town.



Calculate the area for which the first aid responder at C has responsibility.

(3 marks)

(b) A new station is due to open at D(8, 2) and as such the Voronoi diagram will need to be adjusted.

i) Write down the equation of the perpendicular bisector of [BD]. Give your answer in the form $y = mx + c$.

ii) Write down the equation of the perpendicular bisector of [CD]. Give your answer in the form $y = mx + c$.

(3 marks)

6 (a) An environmental organisation is trying to establish if altitude affects the growth of pine needles. A number of needles have been taken from trees at both high and low altitudes and their lengths, in inches, recorded. The results are shown in the table below.

Low altitude	6.1	8.2	7.7	8.0	11.9	6.9	7.5	7.1	8.1
High altitude	7.4	7.9	8.3	6.6	9.5	7.9	8.2	8.1	8.5

Perform a t-test to compare the mean lengths of the pine needles.

Write down the null and alternative hypotheses.

(2 marks)

(b) State whether this is a one-tailed test or a two-tailed test.

(1 mark)

(c) Perform a t-test at the 10% significance level. Write down the p-value.

(2 marks)

(d) Write down the conclusion of the test. Give a reason for your answer.

(2 marks)

7 (a) The temperature, T, of a cake, in degrees Celsius, $°\mathrm{C}$, can be modelled by the function

$$T(t) = a \times 1.17^{-\frac{t}{4}} + 18, \qquad t \geq 0,$$

where a is a constant and t is the time, in minutes, since the cake was taken out of the oven.

In the context of this model, state what the value of 18 represents.

(1 mark)

(b) The cake was 180°C when it was taken out of the oven.

Find the value of a.

(2 marks)

(c) Find the temperature of the cake half an hour after being taken out of the oven.

(2 marks)

8 (a) Ben and Sam are both cyclists competing in a 22.5 km race at the Herne Hill Velodrome in London, England. One lap of the velodrome is 450 m.

Ben takes a total of 42 minutes to complete the race.

Calculate Ben's mean lap time in seconds.

(2 marks)

(b) Given that each of Ben's laps took him 1% longer to complete than the previous one, calculate how long it took him (in seconds) to complete his first and last laps.

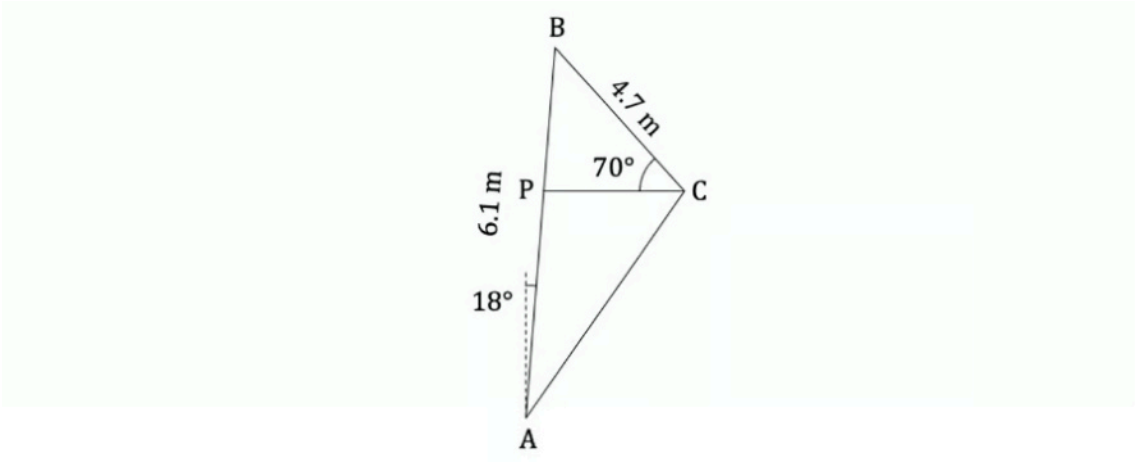
(3 marks)

(c) Sam completes the first lap in 45 seconds and takes 0.2 seconds longer per lap.

Determine who completed the race the first out of Ben and Sam. Justify your answer.

(3 marks)

9 (a) The diagram below shows the triangular sail of a windsurfing board, ABC, with a horizontal boom PC. $AB = 6.1$ m and makes an angle of 18° to the vertical. $BC = 4.7$ m and $B\widehat{C}P = 70°$.



Find the area of the whole sail.

(4 marks)

(b) Calculate the length of the boom PC.

(2 marks)

10 (a) Frank plays a game involving a biased six-sided die.
The faces of the die are numbered 1 to 6.
The score of the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score , x	1	2	3	4	5	6
$\mathbf{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{2}p$	$\frac{1}{8}$	$\frac{3}{2}p$	$\frac{1}{12}$	$3p$

Calculate the exact value of p.

(2 marks)

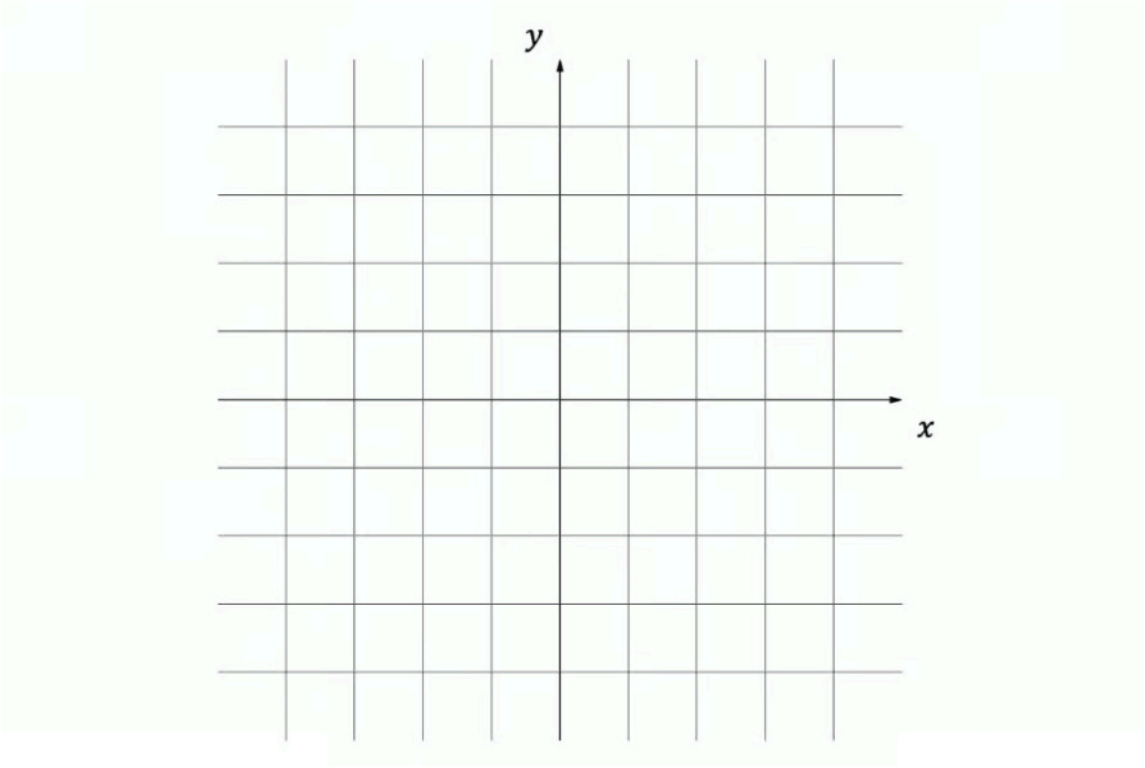
(b) Frank plays the game once.

Calculate the expected score.

(3 marks)

11 (a) Consider the function $g(x) = \sqrt{4 - x}$.

Sketch the graph of the function $g(x)$, labelling the x and y intercepts.



(3 marks)

(b) Find

i) $g(-5)$

ii) x when $g(x) = \frac{1}{2}$.

(2 marks)

12 (a) The volume of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$.

Find $\frac{dV}{dr}$.

(1 mark)

(b) Find the rate of change of the volume with respect to the radius when $r = 5$.

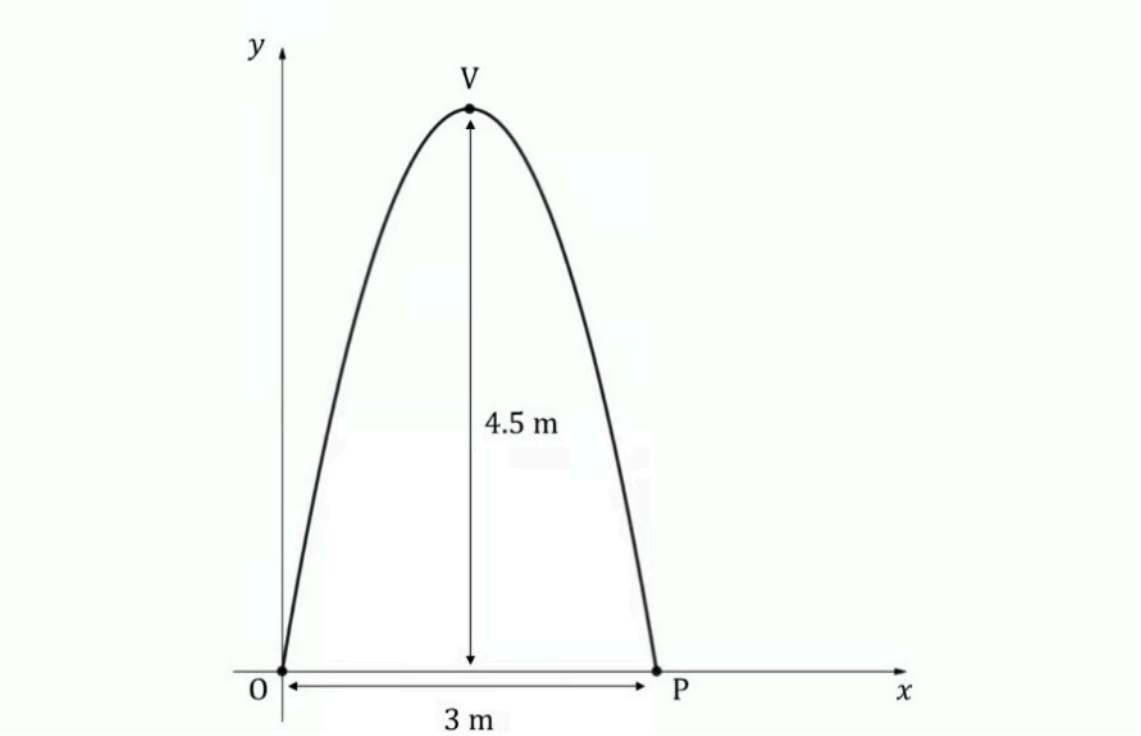
Give your answer in terms of π.

(2 marks)

(c) Show that $\frac{dV}{dr}$ is an increasing function for all relevant values of r.

(3 marks)

13 (a) The following diagram shows an arch that is 4.5 m tall and 3 m wide. The arch crosses the x-axis at the origin, O, and at point P, and its vertex is at point V. The arch may be represented by a curve with an equation of the form $y = x(ax + 6)$, where all units are measured in metres.



Find

i) the coordinates of P

ii) the coordinates of V

iii) the value of a.

(4 marks)

(b) Find the cross-sectional area under the arch.

(2 marks)