SL IB Physics

Your notes

Kinematics

Contents

Your notes

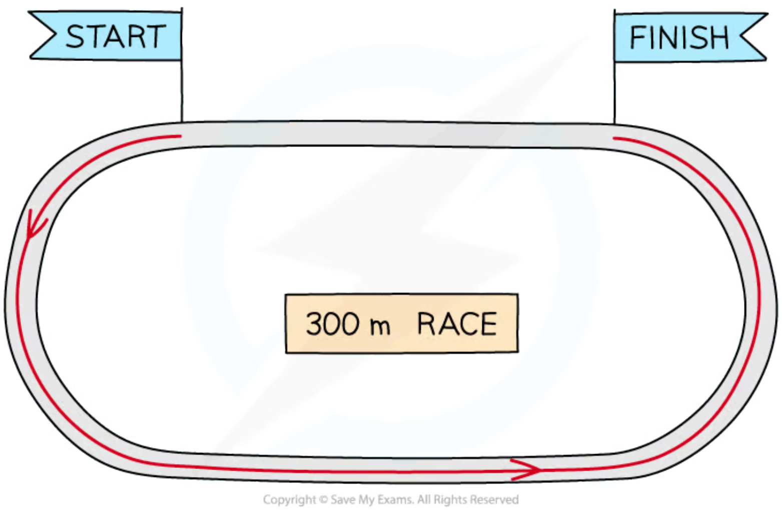
Distance & Displacement

Distance & Displacement

Distance

- **Distance** is a measure of how far an object travels
- It is a scalar quantity - in other words, the direction is not important

Total running distance



The athletes run a total distance of 300 m

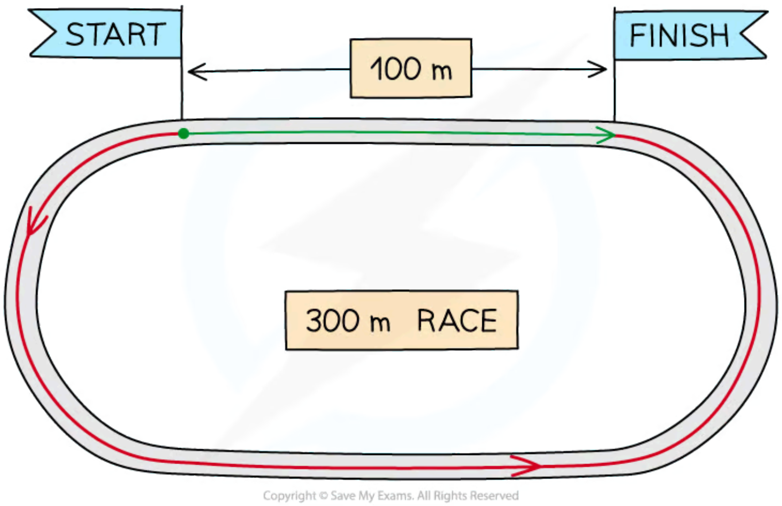
- Consider some athletes running a 300 m race on a 400 m track
- The **distance** travelled by the athletes is **300 m**

Displacement

- **Displacement** is a measure of how far something is from its starting position, along with its direction
  - In other words, it is the **change** in position
- It is a vector quantity - it describes both magnitude and direction

Total distance vs total displacement

Your notes



The athletes run a total distance of 300 m, but end up 100 m from where they started

- Distance is a scalar quantity because...

  It describes how far an object has travelled overall, but not the direction it has travelled in

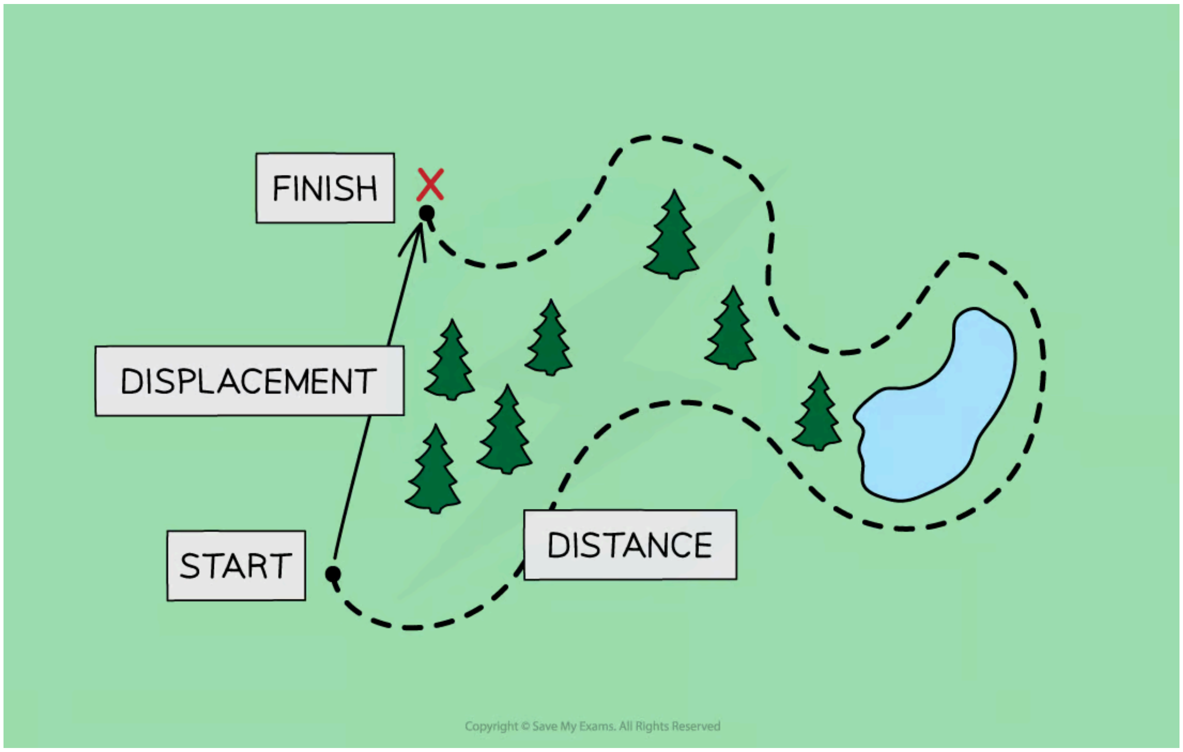
- Displacement is a vector quantity because...

  It describes how far an object is from where it started and in what direction

- When a student travels to school, there will probably be a **difference** in the distance they travel and their displacement
  - The **overall distance** they travel includes the total lengths of all the roads, including any twists and turns
  - The **overall displacement** of the student would be a straight line between their home and school, regardless of any obstacles, such as buildings, lakes or motorways, along the way

What is the difference between distance and displacement?

Your notes



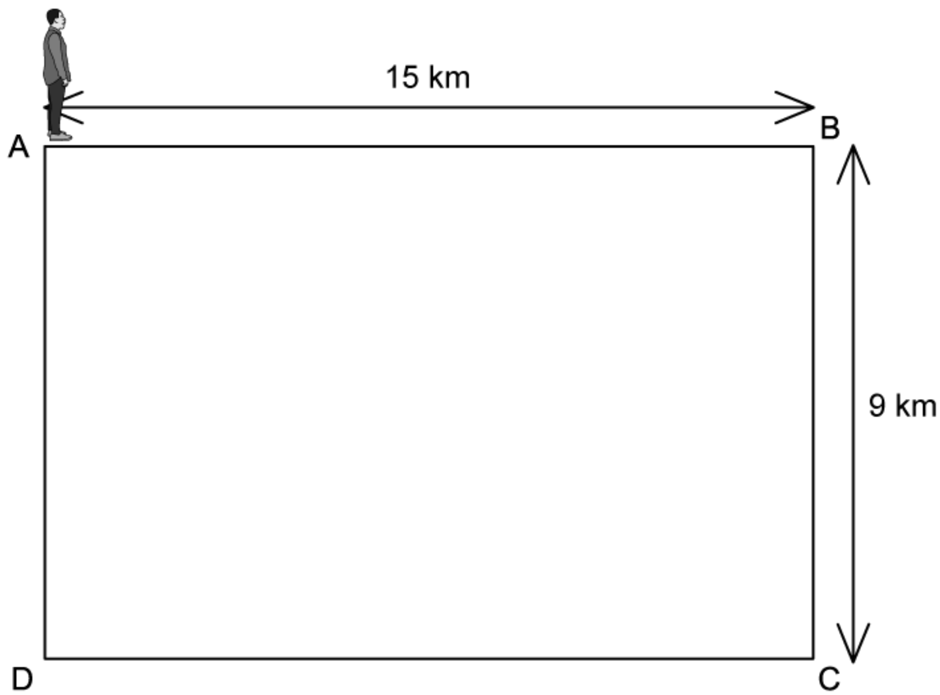
Displacement is a vector while distance is a scalar quantity

- Consider the same 300 m race again
  - The athletes have still run a total **distance** of **300 m** (this is indicated by the arrow in red)
  - However, their **displacement** at the end of the race is **100 m to the right** (this is indicated by the arrow in green)
  - If they ran the full 400 m, their final displacement would be **zero**

Your notes

Worked example

A professor walks around her garden following the path ABCDA.



Calculate, at the end of their walk

(a) the distance the professor travels.
(b) the displacement of the professor.

Answer:

(a) The distance the professor travels is:

- The total distance of each side of the rectangle

$$15 + 9 + 15 + 9 = 48 \text{ km}$$

(b) The displacement of the professor is:

- The displacement is how far the professor is from their **original** position
- As they travel back to point A, the total displacement = **0 km**

Speed & Velocity

Your notes

Speed & Velocity

Speed

- The **speed** of an object is the distance it travels every second
- Speed is a **scalar** quantity
  - This is because it only contains a magnitude (without a direction)
- The **average speed** of an object is given by the equation:

$$average\ speed = \frac{total\ distance}{time\ taken}$$

- The SI units for speed are **meters per second (m s^{-1})** but speed can often be measured in alternative units e.g. km h^{-1} or mph, when it is more appropriate for the situation

Velocity

- The **velocity** of a moving object is similar to its speed and also describes the direction of the velocity
- Velocity is defined as:

The rate of change of displacement

- Velocity is, therefore, a vector quantity because it describes both **magnitude** and **direction**

The difference between speed and velocity

- Speed is a **scalar** quantity whilst velocity is **vector**
  - Velocity is the **speed** in a given **direction**

The cars in the diagram above have the same speed (a scalar quantity) but different velocities (a vector quantity). Fear not, they are in different lanes!

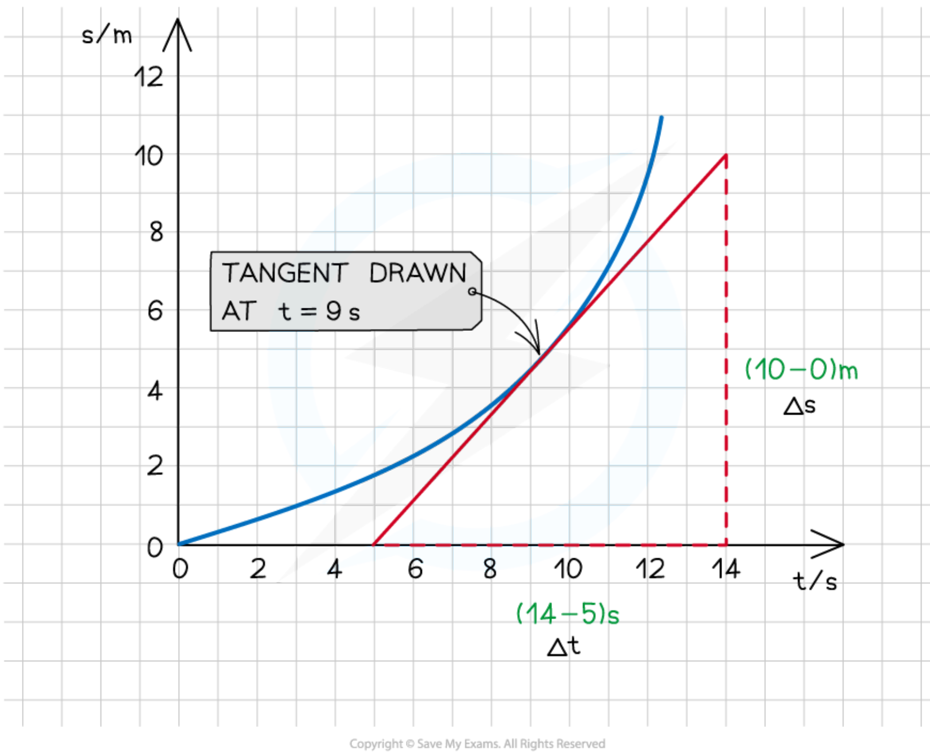
- This means velocity can also have a **negative** value
  - E.g. a ball thrown upwards at a velocity of 3 m s^{-1} comes down at a velocity –5 m s^{-1}, if upwards is considered positive
  - However, their **speeds** are still 3 m s^{-1} and 5 m s^{-1} respectively

Your notes

Instantaneous Speed & Velocity

- The **instantaneous** speed (or velocity) is the speed (or velocity) of an object **at any given point in time**
- This could be for an object moving at a constant velocity or **accelerating**
  - An object at constant velocity is shown by a **straight line** on a displacement – time graph
  - An object accelerating is shown by a **curved line** on a displacement – time graph
  - An accelerating object will have a **changing velocity**

- To find the instantaneous velocity on a displacement-time graph:
  - Draw a **tangent** at the required time
  - Calculate the **gradient** of that tangent



The instantaneous velocity is found by drawing a tangent on the displacement time graph

- In the graph above, at *t* = 9 s, the velocity is:

$$gradient = \frac{10 - 0}{14 - 5} = 1.11 \text{ m s}^{-1}$$

Average Speed & Velocity

- The average velocity $\overline{V}$ of an object can be calculated using

Your notes

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

- Where:
  - Δx = total displacement, or change in position (m)
  - Δt = total time taken (s)
- If the initial velocity *u* and final velocity *v* are known, the average velocity can also be calculated from

$$\bar{v} = \frac{(u + v)}{2}$$

- To find the average velocity on a displacement-time graph, divide the **total displacement** (on the y-axis) by the **total time** (on the x-axis)
  - This method can be used for both a curved or a straight line on a displacement-time graph

Worked example

Florence Griffith Joyner set the women's 100 m world record in 1988, with a time of 10.49 s.

Calculate her average speed during the race.

Answer:

- Sprinters typically speed up from rest to a maximum speed
- Because Florence's speed changes over the course of the race, we can calculate her average speed using the equation:

average speed = total distance ÷ time taken

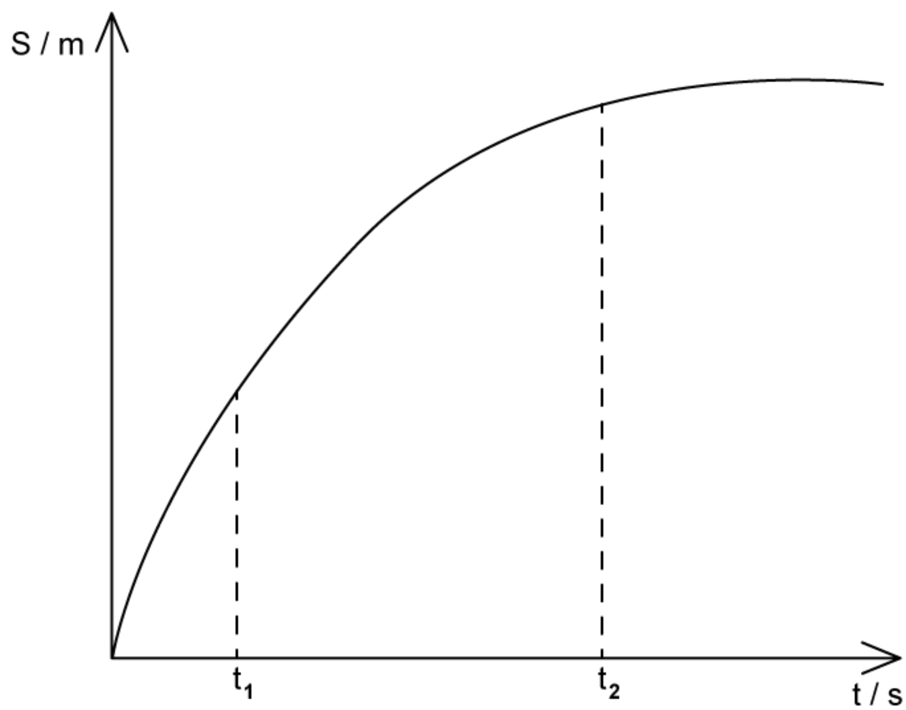
- Where:
  - Total distance, s = 100 m
  - Time taken, t = 10.49 s

average speed = 100 ÷ 10.49 = 9.5328 = **9.53 m s^{-1}**

Your notes

Worked example

The variation of displacement of a box sliding across a rough surface with time *t* is shown on the graph below.



The magnitudes of the instantaneous velocities of the trolley at time t_1 and t_2 are v_1 and v_2 respectively.

List the following velocities in order from fastest to slowest:

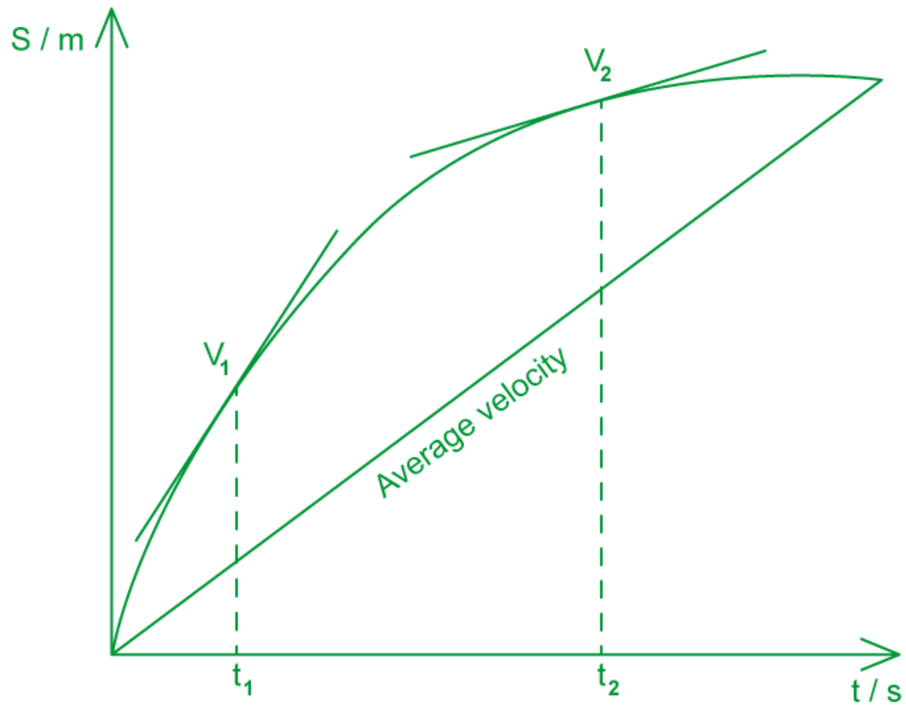
v_1	v_2	average velocity

Answer:

Step 1: Sketch the velocities from the graph

- The instantaneous velocity is the gradient of a tangent at a certain time

Your notes



- The average velocity is the total displacement over the total time

Step 2: Compare the gradients of each velocity

- The fastest velocity will have the **steepest** gradient and the slowest velocity the **shallowest** gradient
- In order from fastest to slowest:

$$v_1 > \text{average velocity} > v_2$$

Examiner Tip

When you draw a tangent to a curve, make sure it **just touches** the point at which you wish to calculate the gradient. The angle between the curve and the tangent line should be roughly equal on both sides of the point.

If you are asked to find the instantaneous velocity from a graph, you will be told the **time** at which they want this velocity for.

Your notes

Acceleration

Acceleration

- Acceleration is defined as:

  The rate of change of velocity

- Acceleration is a vector quantity and is measured in metres per second squared (m s^{-2})
  - It describes how much an object's velocity **changes** every **second**
- The **average acceleration** of an object can be calculated using:

$$average\ acceleration = \frac{change\ in\ velocity}{time\ taken}$$

$$a = \frac{\Delta v}{\Delta t}$$

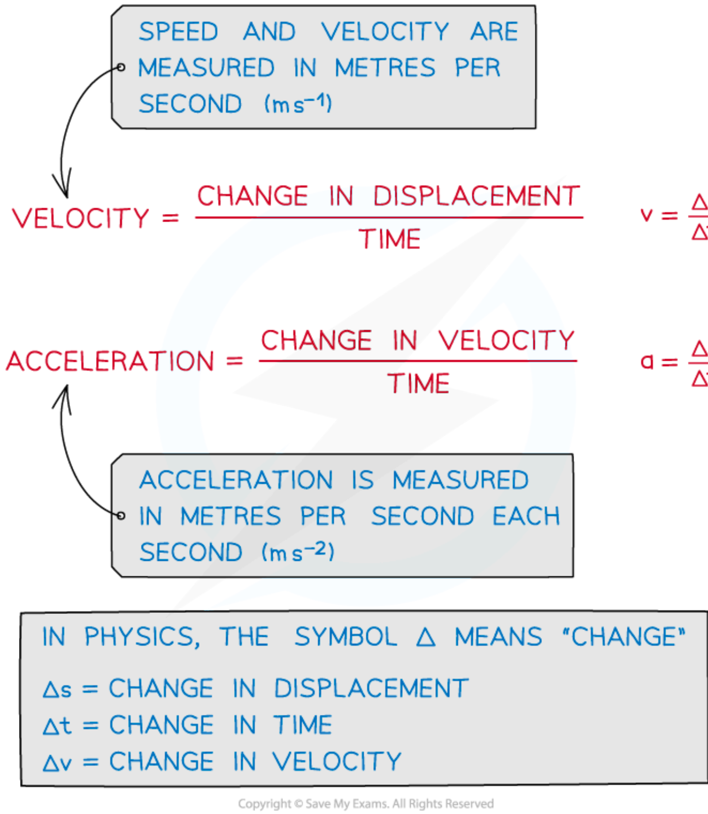
- Where:
  - a = average acceleration (m s^{-2})
  - Δv = change in velocity (m s^{-1})
  - Δt = total time taken (s)
- The **change in velocity** is the **difference** between the initial and final velocity, as written below:

  change in velocity = final velocity – initial velocity

$$\Delta v = (v - u)$$

Equations linking displacement, velocity, and acceleration

Your notes

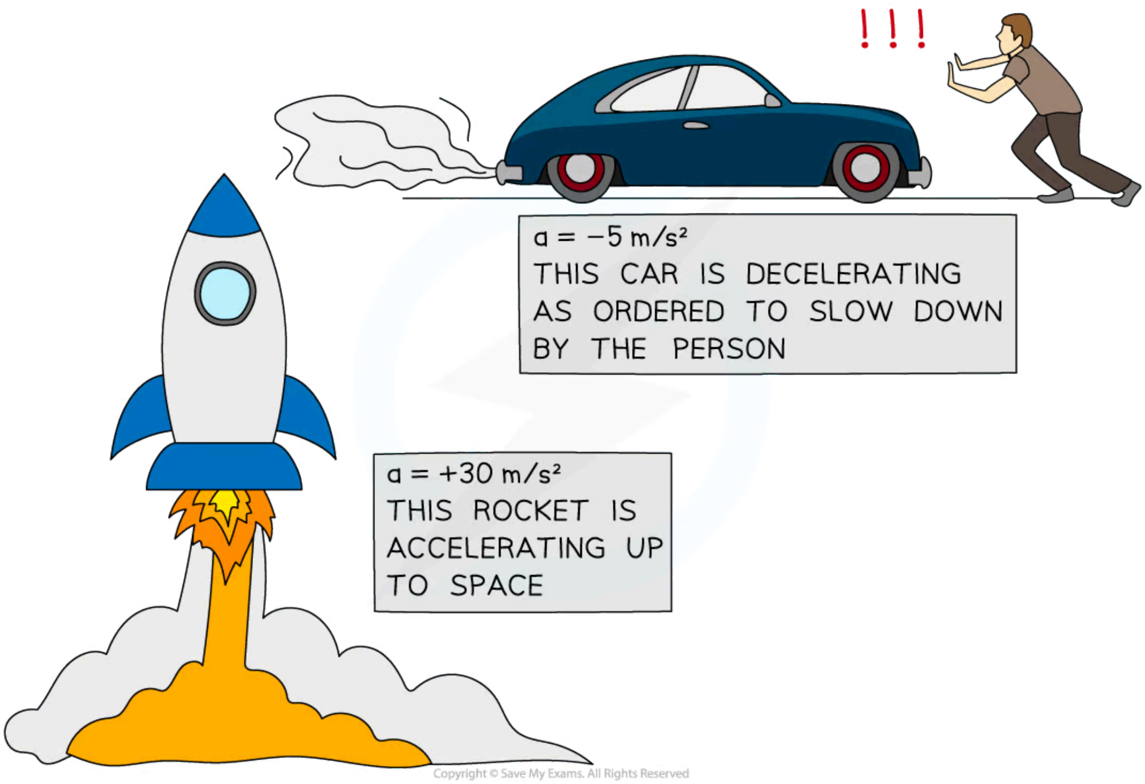


Instantaneous Acceleration

- The **instantaneous** acceleration is the acceleration of an object **at any given point in time**
- This could be for an object with a constantly changing acceleration
  - An object accelerating is shown by a **curved line** on a velocity-time graph

What is a negative acceleration called?

- The **acceleration** of an object can be **positive** or **negative**, depending on whether the object is speeding up or slowing down
  - If an object is **speeding up**, its acceleration is **positive**
  - If an object is **slowing down**, its acceleration is **negative** (deceleration)
- However, acceleration can also be negative if it is accelerating in the **negative direction**



A rocket speeding up (accelerating) and a car slowing down (decelerating)

Your notes

Your notes

Worked example

A Japanese bullet train decelerates at a constant rate in a straight line.

The velocity of the train decreases from an initial velocity of 50 m s^{-1} to a final velocity of 42 m s^{-1} in 30 seconds.

(a) Calculate the change in velocity of the train.
(b) Calculate the deceleration of the train, and explain how your answer shows the train is slowing down.

Answer:

(a)

- The change in velocity is equal to

$$\Delta v = v - u$$

- Where:
  - Initial velocity, u = 50 m s^{-1}
  - Final velocity, v = 42 m s^{-1}

$$\Delta v = 42 - 50 = -8 \text{ m s}^{-1}$$

(b)

- Acceleration is equal to

$$a = \frac{\Delta v}{\Delta t}$$

- Where the time taken is Δt = 30 s

$$a = \frac{-8}{30} = -0.27 \text{ m s}^{-1}$$

- The answer is **negative**, which indicates the train is **slowing down**

Examiner Tip

Remember the units for acceleration are **metres per second squared**, m s^{-2}. In other words, acceleration measures how much the velocity (in m s^{-1}) changes every second, (m s^{-1}) s^{-1}

Kinematic Equations

Your notes

Kinematic Equations

- The kinematic equations of motion are a set of four equations that can describe any object moving with **constant** or **uniform** acceleration
- They relate the five variables:
  - *s* = **displacement**
  - *u* = **initial velocity**
  - *v* = **final velocity**
  - *a* = **acceleration**
  - *t* = **time interval**

Kinematic Equations of Motion

- There are four kinematic equations:

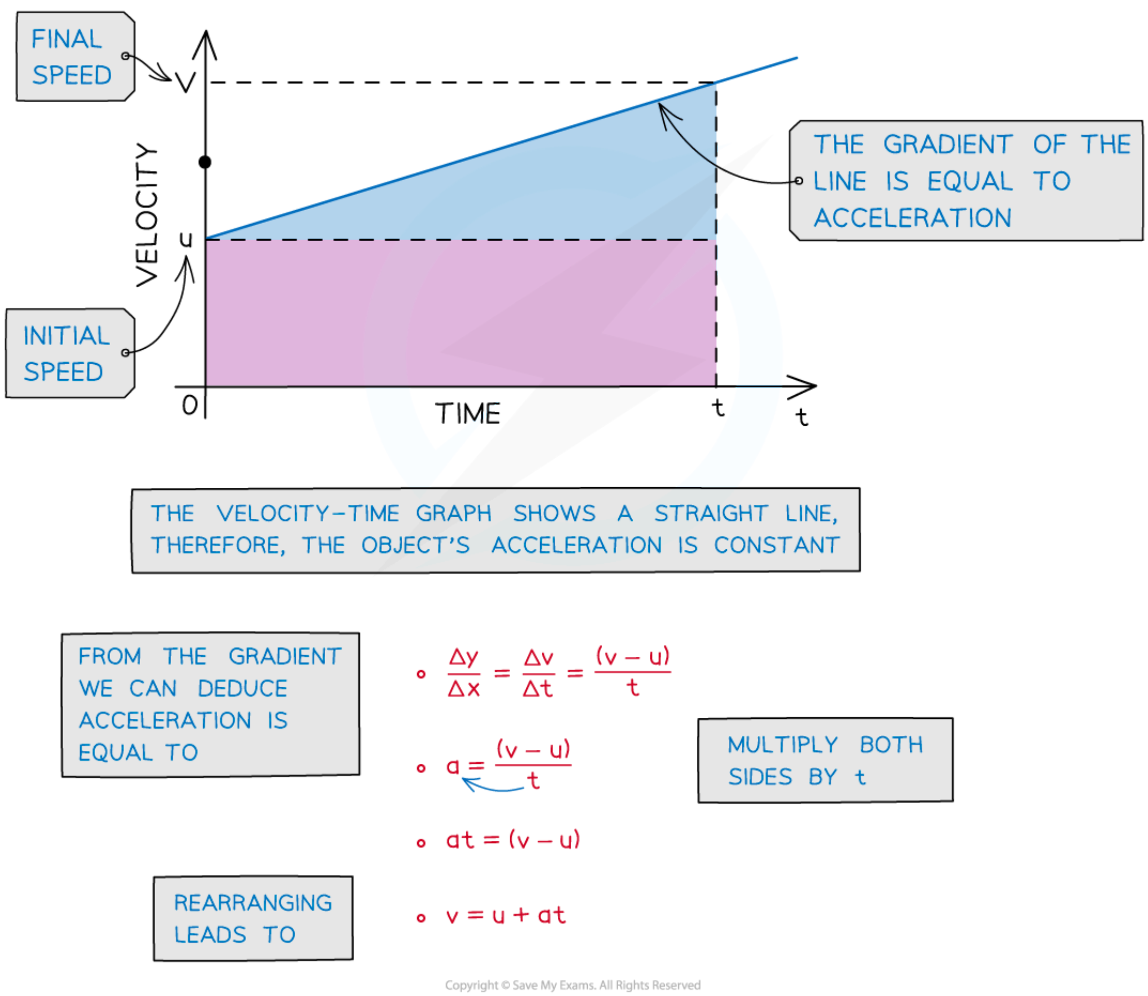
SUMMARY

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{(v + u)}{2}t \qquad v^2 = u^2 + 2as$$

How to derive the kinematic equations

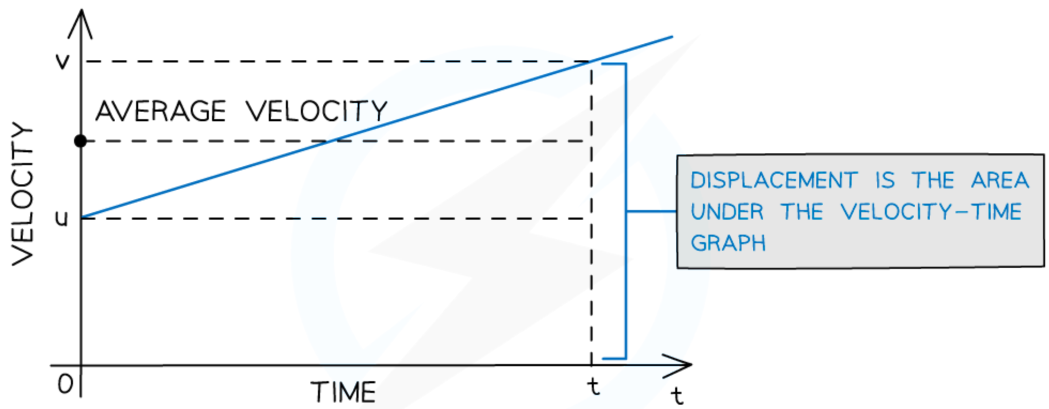
Deriving $v = u + at$



Derivation of v = u + at

Your notes

Deriving $s = \frac{(u + v)}{2} t$



THE OBJECT'S AVERAGE VELOCITY IS HALF-WAY BETWEEN u AND v:

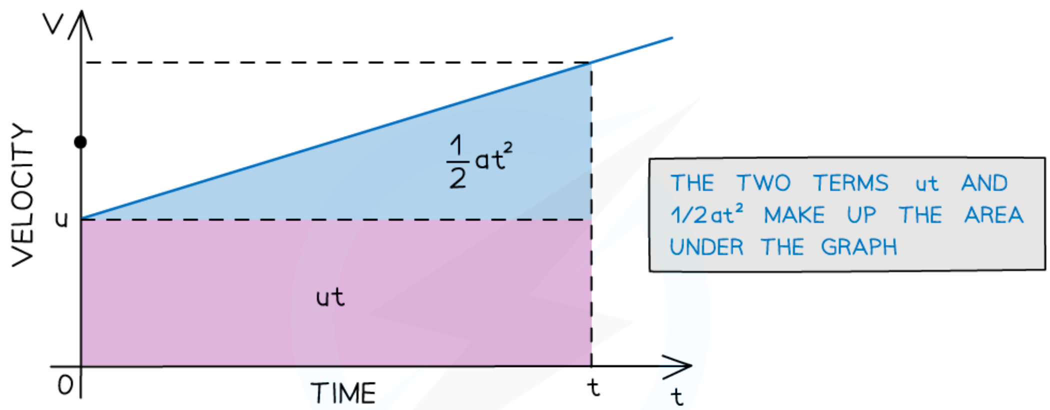
$$\frac{(v + u)}{2}$$

DISPLACEMENT IS EQUAL TO AVERAGE VELOCITY × TIME SO:

$$s = \frac{(v + u)}{2} t$$

Your notes

Deriving $s = ut + \frac{1}{2}at^2$



TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at$ (1)
- $s = \frac{(v + u)}{2}t$ (2)

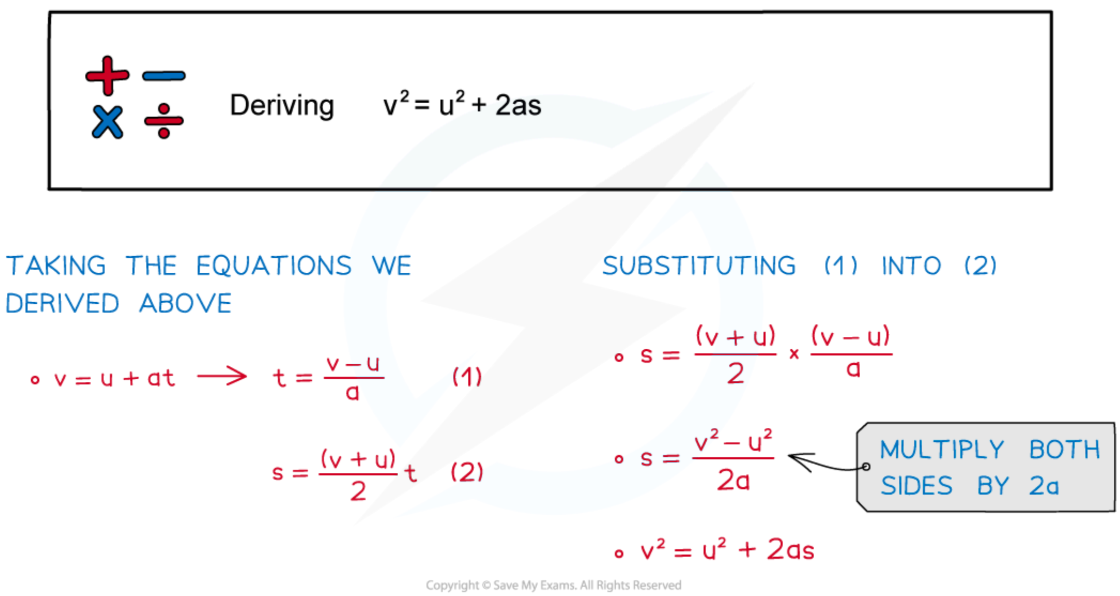
SUBSTITUTING EQUATION (1) AND (2)

MULTIPLY EVERYTHING IN THE BRACKET BY t

- $s = \frac{(u + u + at)}{2}t$
- $s = \frac{2ut}{2} + \frac{at^2}{2}$
- $s = ut + \frac{1}{2}at^2$

SEPARATE THE t AND t² TERMS

Your notes



This final equation can be derived from two of the others

Key Takeaways

- The key terms to look out for are:

  'Starts from rest'

- This means ***u*** **= 0** and ***t*** **= 0**
- This can also be assumed if the initial velocity is not mentioned

  'Falling due to gravity'

- This means ***a*** **=** ***g*** **= 9.8 m s^{-2}**
  - It doesn't matter which way is positive or negative for the scenario, as long as it is consistent for all the vector quantities
  - If downwards is considered positive, this is 9.81 m s^{-2} , otherwise, it is –9.81 m s^{-2}
- For example, if downwards is negative then for a ball travelling upwards, ***s*** must be positive and ***a*** must be negative

  'Constant acceleration in a straight line'

- This is a key indication for the kinematic equations are intended to be used
  - For example, an object **falling** in a **uniform** gravitational field **without** air resistance

How to use the kinematic formulae

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion $a = \pm 9.81$ m s^{-2}, an object which starts or finishes at rest will have $u = 0$ or $v = 0$

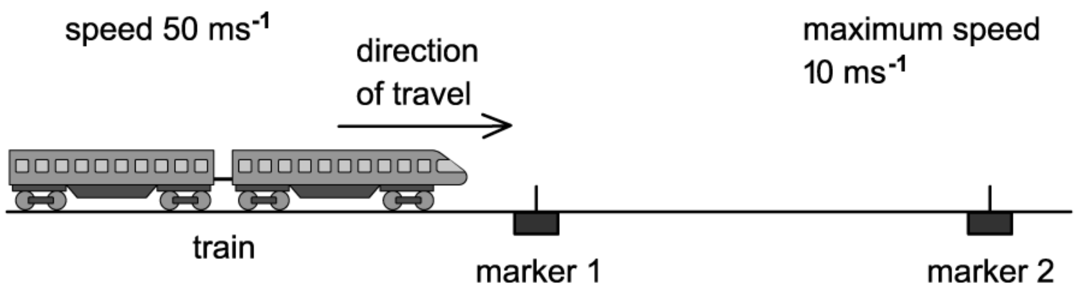
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links *s*, *u*, *a* and *t* is $s = ut + ½at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

Your notes

Your notes

Worked example

The diagram shows an arrangement to stop trains that are travelling too fast.



At marker 1, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker 2 at no more than 10 m s^{-1}.

The train carries a detector that notes the times when the train passes each marker and will apply an emergency brake if the time between passing marker 1 and marker 2 is less than 20 s.

Trains coming from the left travel at a speed of 50 m s^{-1}.

Determine how far marker 1 should be placed from marker 2.

Answer:

STEP 1

OUR KNOWN VARIABLES ARE

- $u = 50$ ms^{-1}
- $v = 10$ ms^{-1}
- $t = 20$ s

AND WE ARE ASKED TO FIND DISTANCE, s.

STEP 2

SO THE EQUATION THAT LINKS u, v, t AND s IS

$$s = \frac{(u + v)}{2}t$$

STEP 3

NO REARRANGING IS REQUIRED SO WE SIMPLY PLUG IN THE VARIABLES:

$$s = \frac{(50 + 10)}{2} \times 20 = 30 \times 20 = 600 \text{ m}$$

Your notes

Worked example

A cyclist is travelling directly east through a village, which is completely flat, at a velocity of 6 m s^{-1}. They then start to constantly accelerate at 2 m s^{-2} for 4 seconds.

(a) Calculate the distance that the cyclist covers in the 4 s acceleration period.
(b) Calculate the cyclist's final velocity after the 4 s interval of acceleration.

Later on in their journey, cyclist **A** is now cycling through a different village, at a constant velocity of 18 m s^{-1}. Cyclist **A** passes a friend, Cyclist **B** who begins accelerating from rest at a constant acceleration of 1.5 m s^{-2} in the same direction as Cyclist **A** at the moment they pass.

(c) Calculate how long it takes for Cyclist **B** to catch up to Cyclist **A**.

Answer:

(a) Calculate the displacement, *s*

Step 1: List the known quantities

- Initial velocity, u = 6 m s^{-1}
- Acceleration, a = 2 m s^{-2}
- Time, t = 4 s
- Displacement = s (this needs to be calculated)

Step 2: Identify the best SUVAT equation to use

- Since the question states **constant acceleration**, the kinematic equations can be used
- In this problem, the equation that links *s*, *u*, *a*, and *t* is

$$s = ut + \frac{1}{2}at^2$$

Step 3: Substitute the known quantities into the equation

$$s = (6 \times 4) + (0.5 \times 2 \times 4^2) = 24 + 16$$

Displacement: $\boldsymbol{s}$ **= 40 m**

(b) Calculate the final velocity, *v*

Step 1: List the known quantities

- Initial velocity, u = 6 m s^{-1}
- Acceleration, a = 2 m s^{-2}
- Time, t = 4 s
- Final velocity = v (this needs to be calculated)

Step 2: Identify and write down the equation to use

- Since the question states constant acceleration - SUVAT equation(s) - can be used

Your notes

- In this problem, the equation that links *v*, *u*, *a*, and *t* is:

$$v = u + at$$

Step 3: Substitute the known quantities into the equation

$$v = 6 + (2 \times 4)$$

Final velocity: **v = 14 m s^{-1}**

(c) Calculate the time *t* for **B** to catch up to **A**

Step 1: List the known quantities for cyclist A

- Initial velocity, u_A = 18 m s^{-1}
- Acceleration, a_A = 0 m s^{-2}
- Time = *t*
- Displacement = s_A

Step 2: List the known quantities for cyclist B

- Initial velocity, u = 0 m s^{-1}
- Acceleration, a = 1.5 m s^{-2}
- Time = *t*
- Displacement = s_B

Step 3: Write expressions for Cyclist A and Cyclist B in terms of their displacement

- Cyclist **A**'s motion can be expressed by:

$$s_A = u_A t + \frac{1}{2} a_A t^2$$

$$s_A = 18t + 0 = 18t$$

- Cyclist **B**'s motion can be expressed by:

$$s_B = u_B t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + \left(\frac{1}{2} \times 1.5 \times t^2\right) = \frac{3}{4} t^2$$

Step 4: Equate the two equations and solve for *t*

- The two equations describe the displacement of each cyclist respectively
- When equating them, this will find the time when the cyclists are at the same location

$$s_A = s_B$$

Your notes

$$18t = \frac{3}{4}t^2 \quad \Rightarrow \quad \frac{3}{4}t^2 - 18t = 0$$

$$(t^2 - 24t) = 0$$

- Therefore, solving for *t*, it can be two possible answers:

  t = 0 s **or** t = 24 s

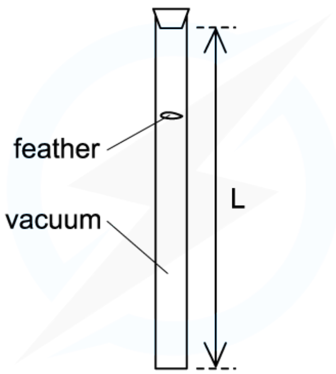
- Since the question is seeking the time when the two cyclists meet after first passing each other, the final answer is **24 s**

Your notes

Worked example

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.



Use an appropriate SUVAT equation to calculate the length *L* of the tube.

Answer:

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81\ ms^{-2}$ $\quad u = 0$ $\quad t = 0.5\ s$ $\quad L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2\ m$$

Your notes

Examiner Tip

This is one of the most important sections of this topic - usually, there will be one, or more, questions in the exam about solving problems with the kinematic equations equations. The best way to master this section is to practice as many questions as possible!

Watch out for the direction of vectors: displacement, acceleration and velocity. Take a single direction as positive (and hence the opposite direction is negative) and **stick with it** throughout the question, this is the most common pitfall.

Don't worry, you won't have to memorise these, they are give in your data booklet in the exam.

You may sometimes see these equations referred to as 'SUVAT' equations.

Motion Graphs

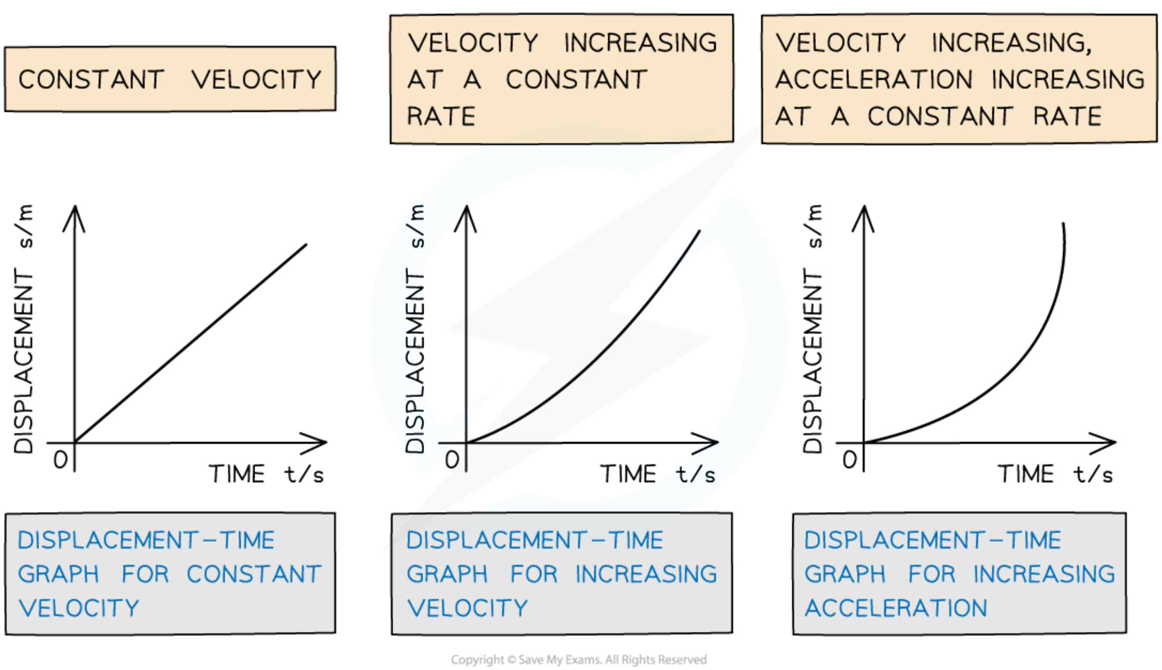
Your notes

Motion Graphs

- The motion of objects can be analysed in terms of position, velocity and acceleration
  - These are all related to each other by **gradients** and **areas** under curves
- Three types of graphs that can represent the motion of an object are:
  - **Displacement-time** graphs
  - **Velocity-time** graphs
  - **Acceleration-time** graphs

Displacement-Time Graphs

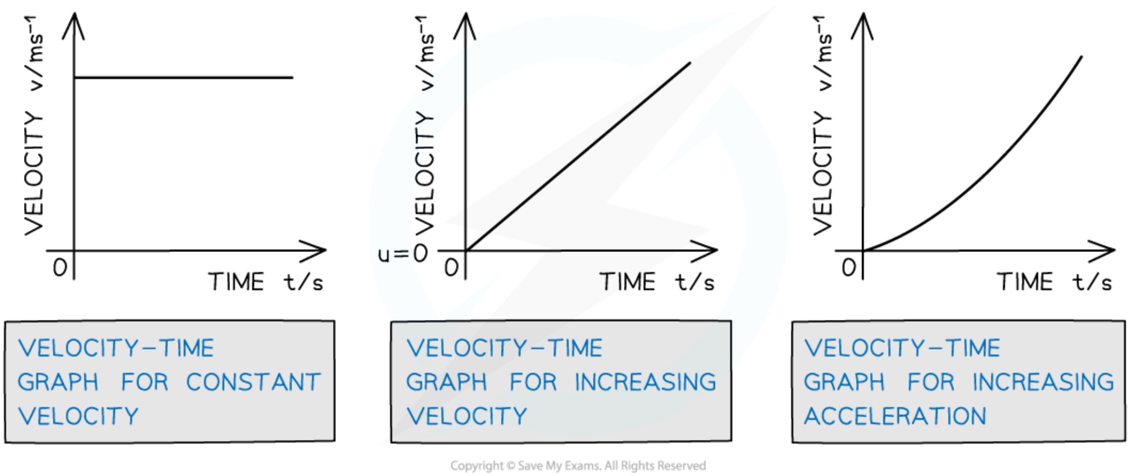
- On a **displacement-time** graph:
  - **Slope** equals **velocity**
  - The **y-intercept** equals the **initial displacement**
  - A **straight** (diagonal) line represents a **constant** velocity
  - A **curved** line represents an **acceleration**
  - A **positive slope** represents motion in the **positive direction**
  - A **negative slope** represents motion in the **negative direction**
  - A **zero** slope (horizontal line) represents a state of **rest**
  - The area under the curve is meaningless



Displacement-time graphs displacing different values of velocity

Velocity-Time Graphs

- On a **velocity-time** graph:
  - **Slope** equals **acceleration**
  - The **y-intercept** equals the **initial velocity**
  - A **straight** (diagonal) line represents **uniform acceleration**
  - A **curved** line represents **non-uniform acceleration**
  - A **positive** slope represents **acceleration** in the **positive direction**
  - A **negative** slope represents **acceleration** in the **negative direction**
  - A **zero** slope (horizontal line) represents motion with **constant velocity**
  - The **area** under the curve equals the **change** in **displacement**

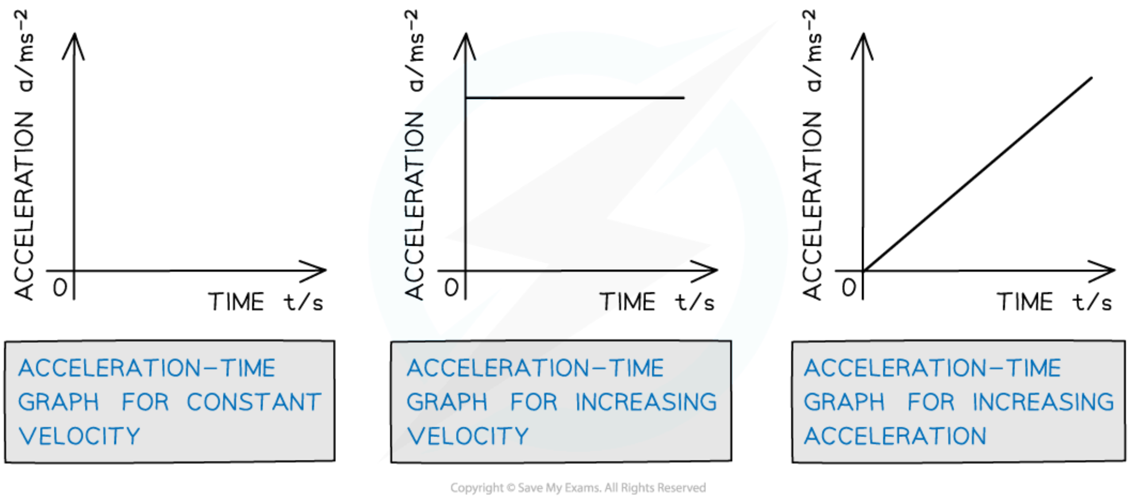


Velocity-time graphs displacing different values of acceleration

Acceleration-Time Graphs

- On an acceleration-time graph:
  - Slope is meaningless
  - The **y-intercept** equals the **initial acceleration**
  - A **zero slope** (horizontal line) represents an object undergoing **constant acceleration**
  - The **area** under the curve equals the **change** in **velocity**

Your notes



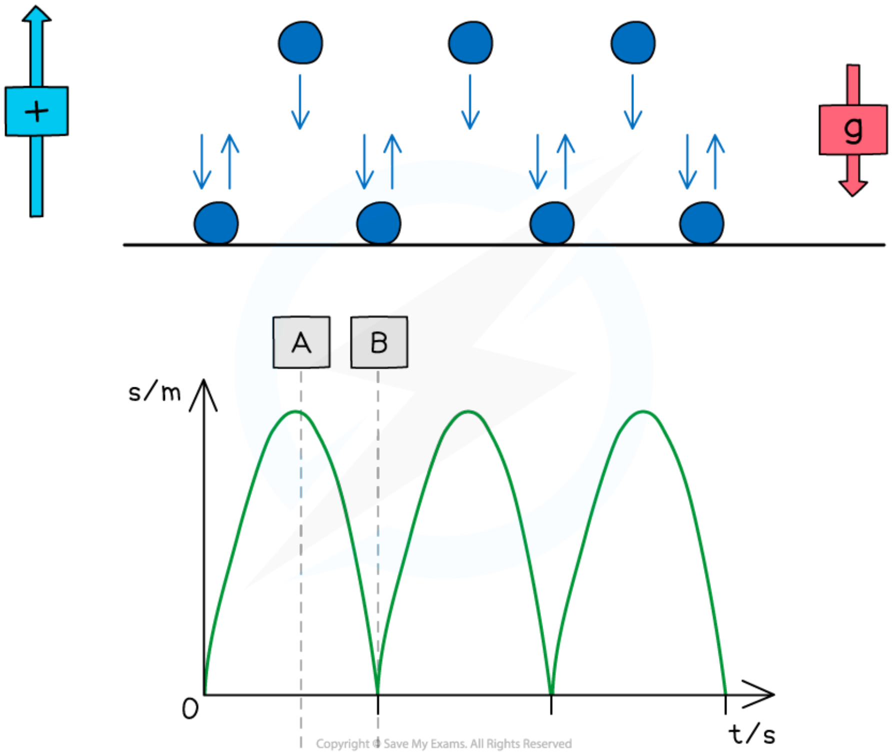
How displacement, velocity and acceleration graphs relate to each other

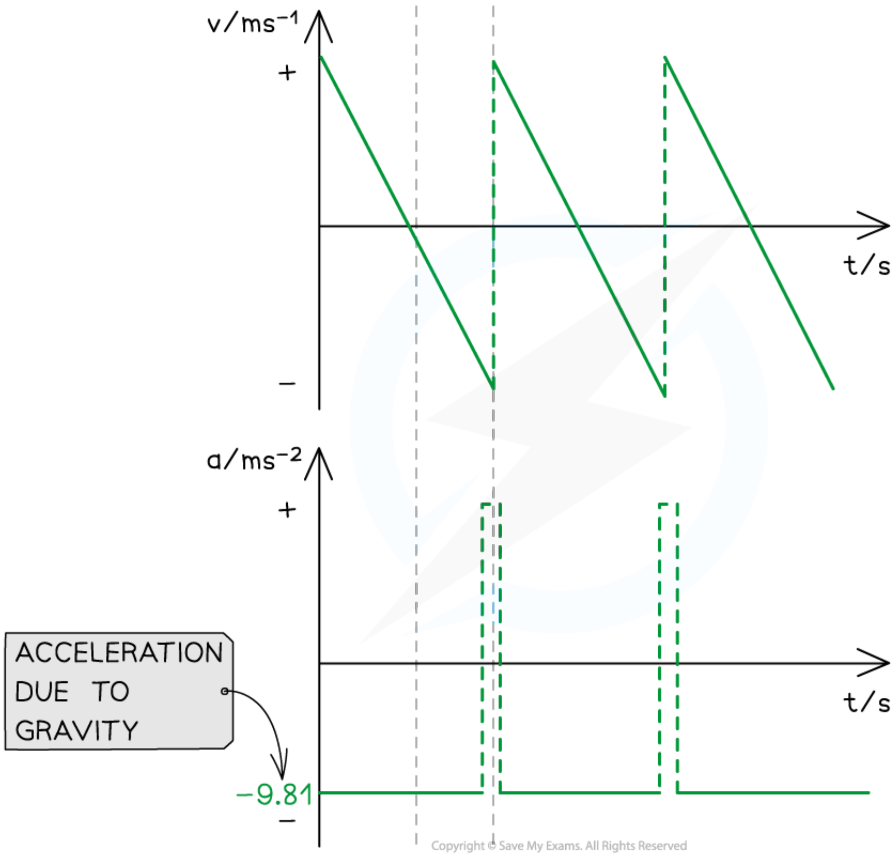
- Acceleration can either be
  - **Uniform** i.e. a constant value. For example, acceleration due to gravity on Earth
  - **Non-uniform** i.e. a changing value. For example, an object with increasing acceleration

Motion of a Bouncing Ball

- For a bouncing ball, the acceleration due to gravity is **always** in the same direction (in a uniform gravitational field such as the Earth's surface)
  - This is assuming there are **no** other forces on the ball, such as **air resistance**
- Since the ball changes its direction when it reaches its highest and lowest point, the direction of the velocity will change at these points
- The vector nature of velocity means the ball will sometimes have a:
  - **Positive velocity** if it is travelling in the positive direction
  - **Negative velocity** if it is traveling in the negative direction
- An example could be a ball bouncing from the ground back upwards and back down again
  - The positive direction is taken as upwards
  - This will be either stated in the question or can be chosen, as long as the direction is consistent throughout
- Ignoring the effect of air resistance, the ball will reach the same height every time before bouncing from the ground again
- When the ball is traveling upwards, it has a positive velocity which slowly decreases (decelerates) until it reaches its highest point

Your notes





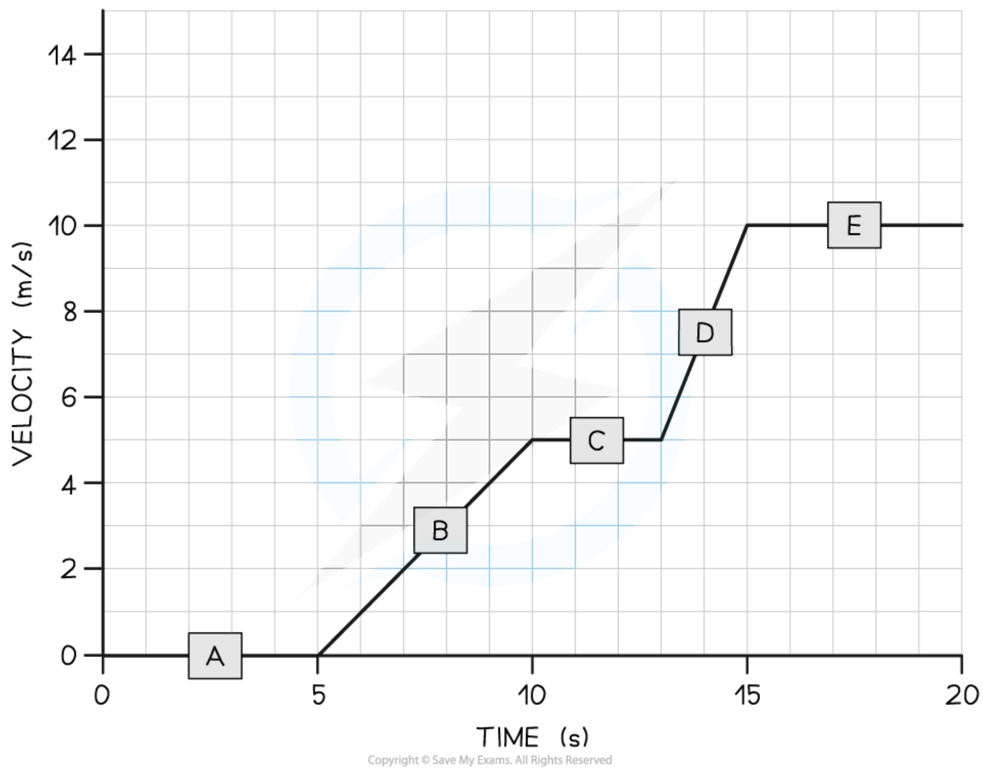
- At point **A** (the highest point):
  - The ball is at its **maximum displacement**
  - The ball momentarily has **zero velocity**
  - The **velocity** changes from **positive** to **negative** as the ball changes direction
  - The **acceleration**, *g*, is still **constant** and directed vertically downwards
- At point **B** (the lowest point):
  - The ball is at its **minimum displacement** (on the ground)
  - Its **velocity** changes instantaneously from **negative** to **positive**, but its **speed** (magnitude) **remains the same**
  - The **change** in direction causes a **momentary acceleration** (since acceleration = change in velocity / time)

Your notes

Worked example

Tora is training for a cycling tournament.

The velocity-time graph below shows her motion as she cycles along a flat, straight road.



(a) In which section (A, B, C, D, or E) of the velocity-time graph is Tora's acceleration the largest?
(b) Calculate Tora's acceleration between 5 and 10 seconds.

Answer:

(a)

Step 1: Recall that the slope of a velocity-time graph represents the magnitude of acceleration

- The slope of a velocity-time graph indicates the magnitude of acceleration
  Therefore, the only sections of the graph where Tora is accelerating is section B and section D
- Sections A, C, and E are flat – in other words, Tora is moving at a constant velocity (i.e. not accelerating)

Step 2: Identify the section with the steepest slope

- Section D of the graph has the steepest slope
- Hence, the largest acceleration is shown in **section D**

Your notes

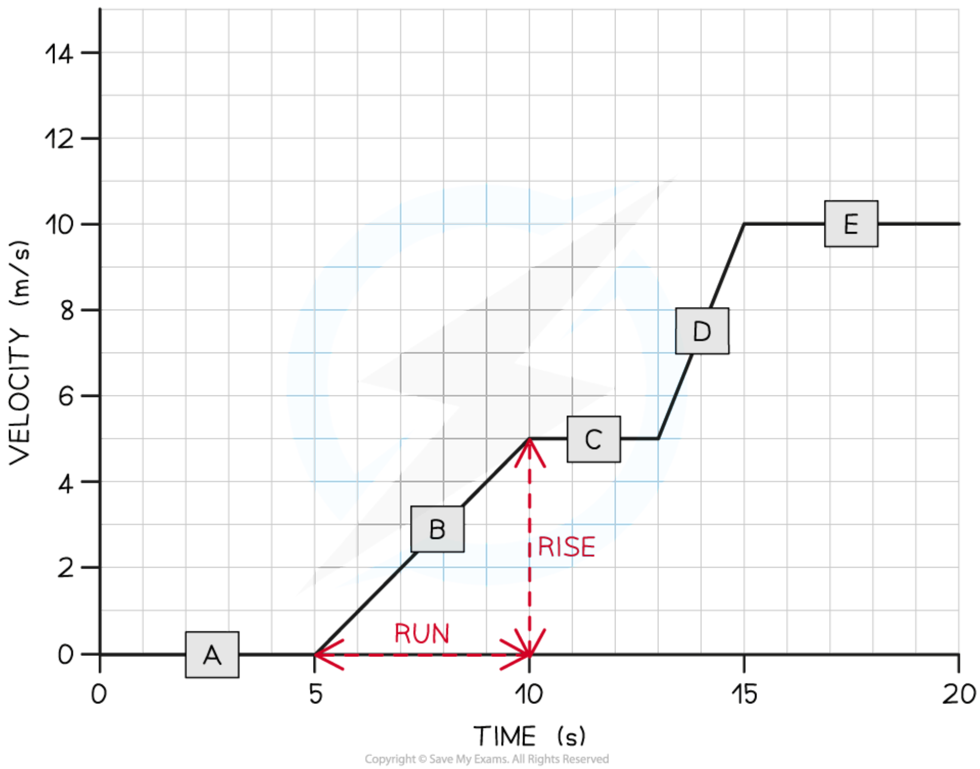
(b)

Step 1: Recall that the gradient of a velocity-time graph gives the acceleration

- Calculating the gradient of a slope on a velocity-time graph gives the acceleration for that time period

Step 2: Draw a large gradient triangle at the appropriate section of the graph

- A gradient triangle is drawn for the time period between 5 and 10 seconds below:



Step 3: Calculate the size of the gradient and state this as the acceleration

- The acceleration is given by the gradient, which can be calculated using:

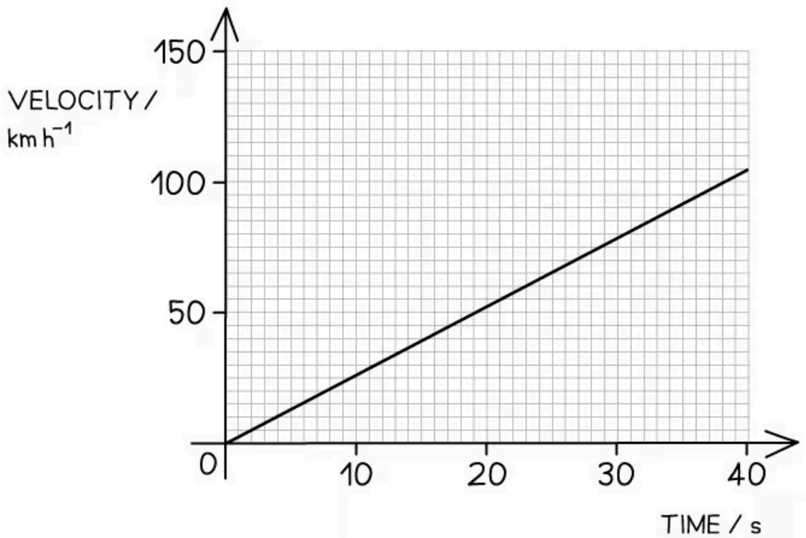
$$acceleration = gradient = \frac{5}{5} = 1 \text{ m s}^{-2}$$

- Therefore, Tora accelerated at 1 m s^{-2} between 5 and 10 seconds

Your notes

Worked example

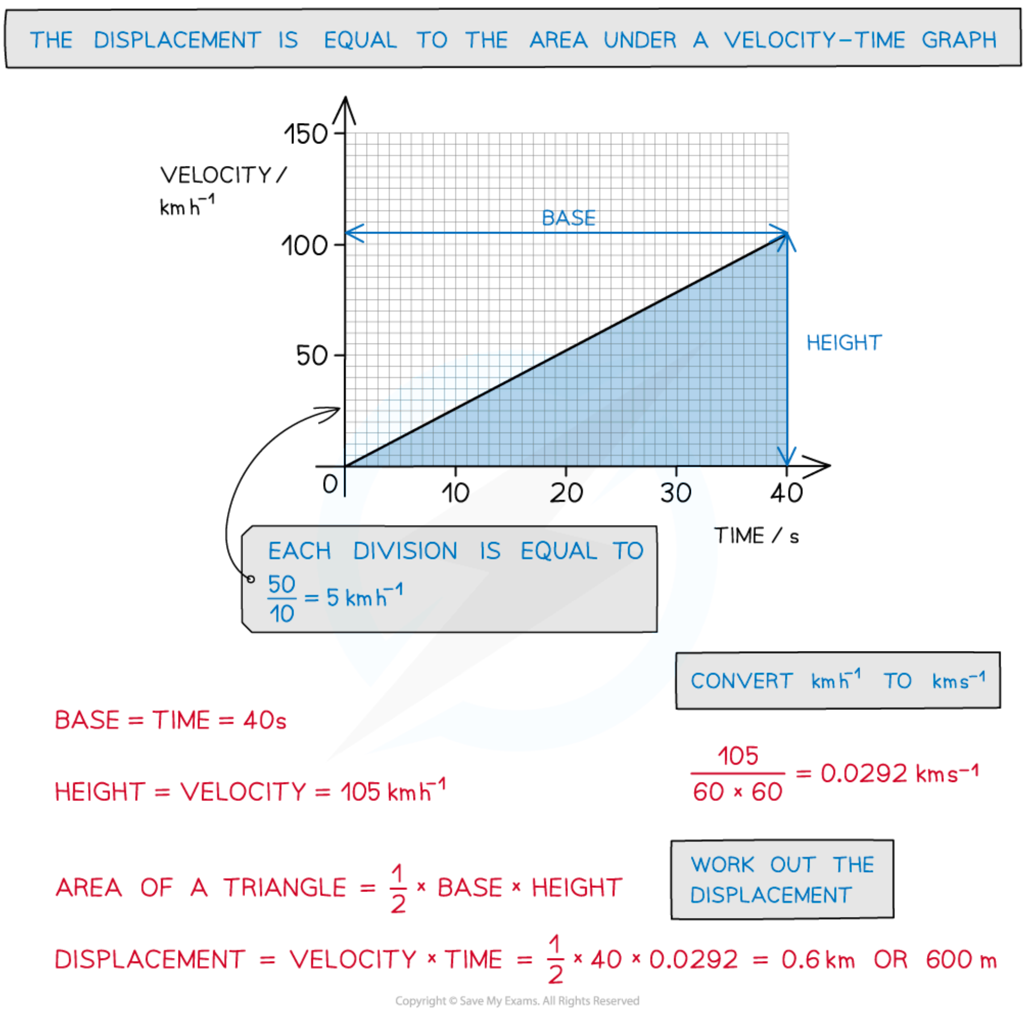
The velocity-time graph of a vehicle travelling with uniform acceleration is shown in the diagram below.



Calculate the displacement of the vehicle at 40 s.

Answer:

Your notes


THE DISPLACEMENT IS EQUAL TO THE AREA UNDER A VELOCITY–TIME GRAPH
VELOCITY / km h⁻¹
150
100
50
0
10
20
30
40
TIME / s
BASE
HEIGHT
EACH DIVISION IS EQUAL TO 50/10 = 5 km h⁻¹
CONVERT km h⁻¹ TO km s⁻¹
BASE = TIME = 40s
HEIGHT = VELOCITY = 105 km h⁻¹
105 / (60 × 60) = 0.0292 km s⁻¹
WORK OUT THE DISPLACEMENT
AREA OF A TRIANGLE = 1/2 × BASE × HEIGHT
DISPLACEMENT = VELOCITY × TIME = 1/2 × 40 × 0.0292 = 0.6 km OR 600 m
Copyright © Save My Exams. All Rights Reserved

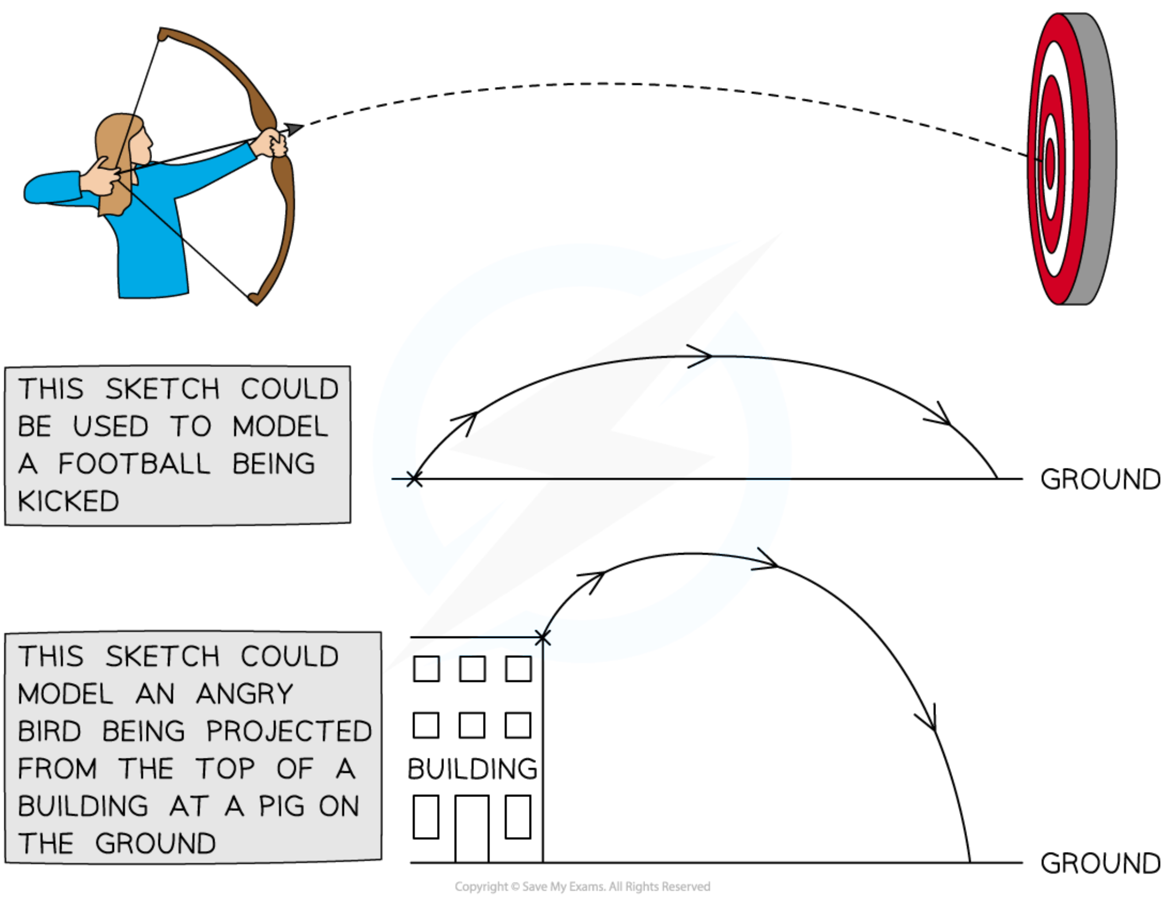
Projectile Motion

Your notes

Projectile Motion

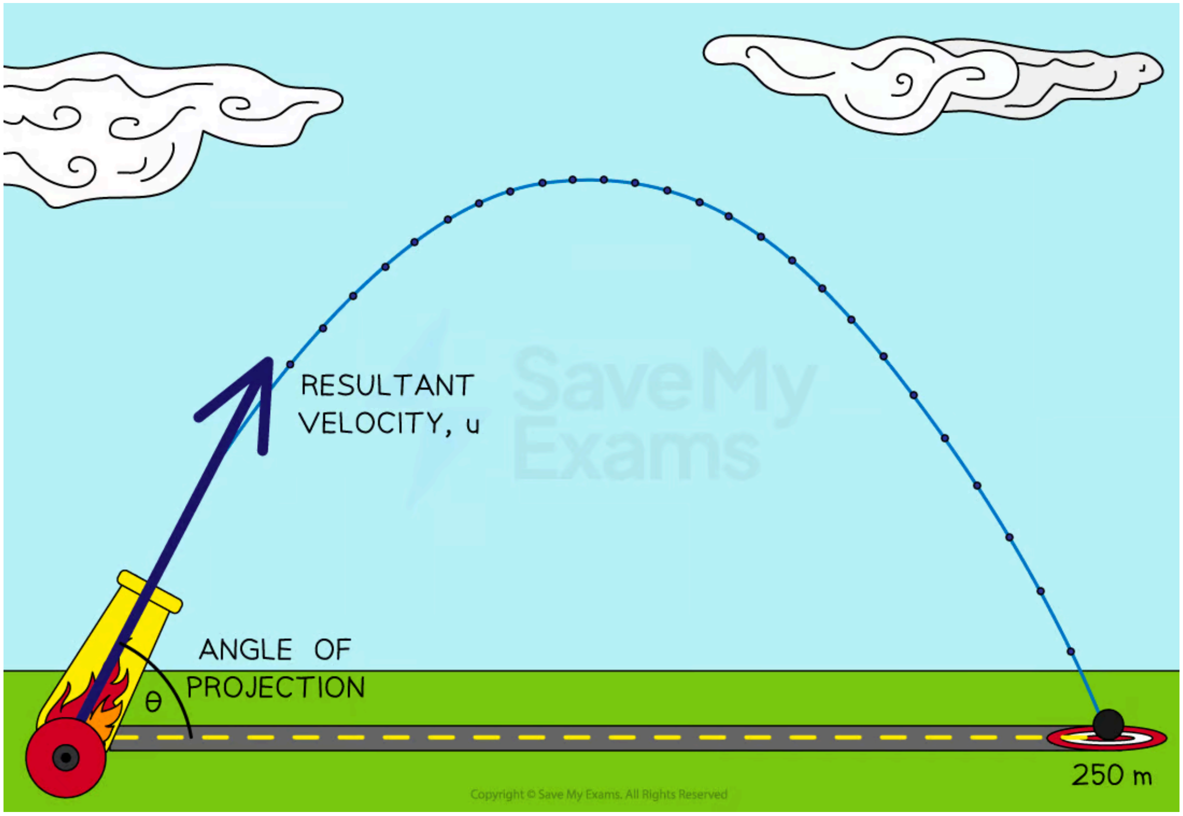
What is a projectile?

- A **projectile** is a particle moving freely (non-powered), under gravity, in a two-dimensional plane
- Examples of projectile motion include throwing a ball, jumping off a diving board and hitting a baseball with a baseball bat
- In these examples, it is assumed that:
  - **Resistance** from the air or liquid (known as fluid resistance) the object is travelling through is **negligible**
  - **Acceleration** due to free-fall, ***g*** is constant as the object is moving close to the surface of the Earth



Examples of objects in a projectile motion trajectory

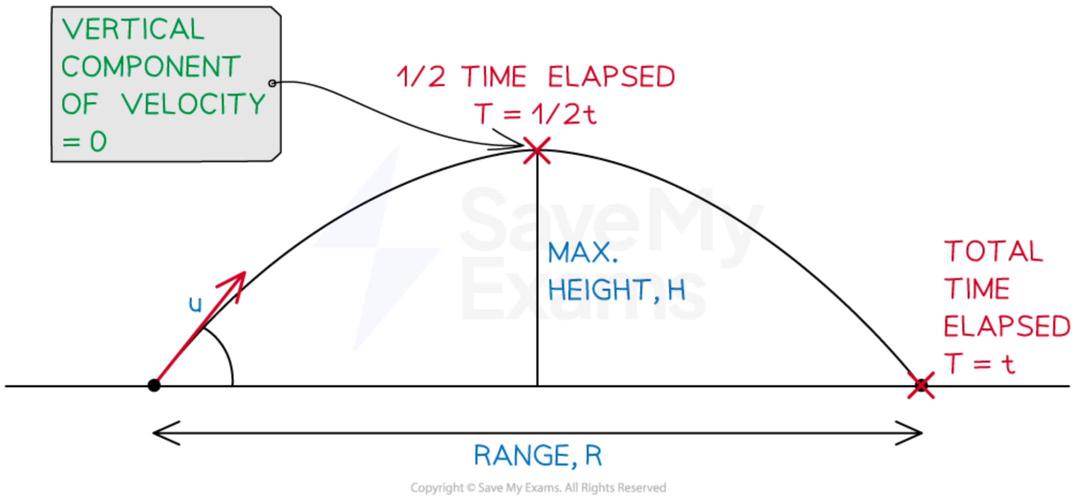
- An object is sent into a projectile motion trajectory with a **resultant velocity**, *u* at an **angle**, θ to the horizontal
  - Examples of this include a ball thrown from a height and a cannonball launched from a cannon



An object in a projectile motion trajectory has a resultant velocity at a given angle to the horizontal ground

- Some key terms to know, and how to calculate them, are:
  - **Time of flight** (total time): how long the projectile is in the air.
    - For typical projectile motion, the time to the maximum height is half of the total time
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
    - This is when the vertical velocity component = 0
    - When the projectile is released and lands on the ground the projectile is at its maximum height when half of its total time has elapsed
  - **Range:** the horizontal distance travelled by the projectile

Your notes



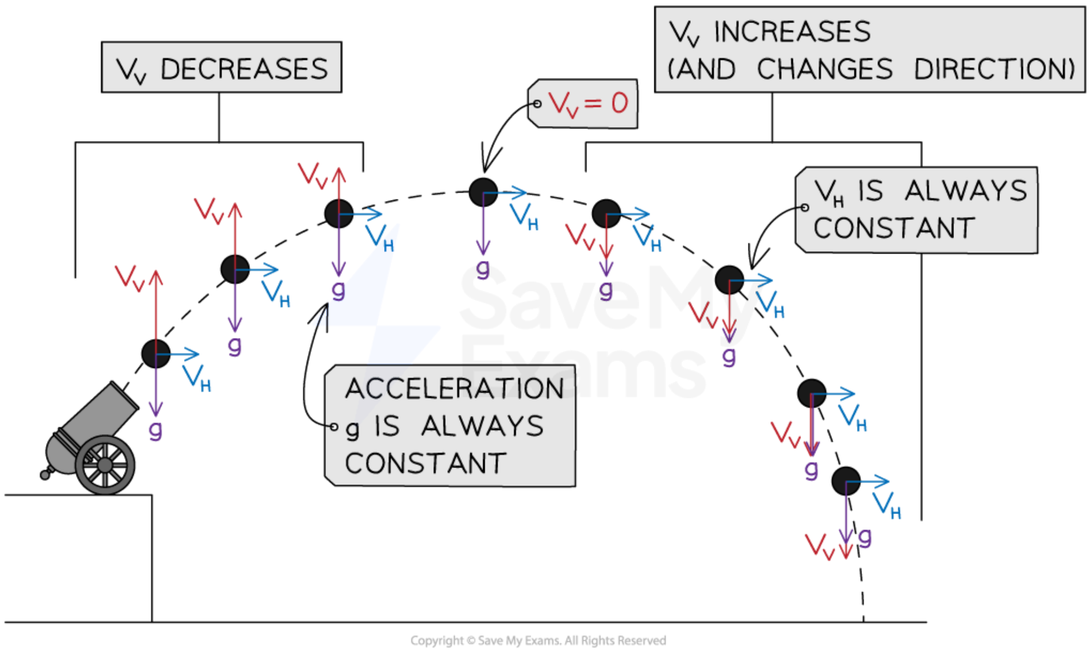
An object in projectile motion will have a vertical velocity of zero at maximum height when half the time has elapsed

Horizontal and Vertical Components

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These quantities are **independent** of each other
  - Displacement, velocity and acceleration are all vector quantities that are different in both components
  - They need to be evaluated separately using the SUVAT Equations

	Horizontal Component	Vertical Component
Displacement	▪ **Maximum range** at the end of the motion when the **total time** has elapsed ▪ **Half the range** at the **maximum height** when half the time has elapsed	**Maximum height** is at the top of the motion when **half the time** has elapsed
Velocity	**Constant**	**Zero** at **maximum height**
Acceleration	**Zero** (because velocity remains constant)	Acceleration of free fall, $\boldsymbol{g}$ = 9.8 ms^{-2} ▪ **Positive** when an object is **falling** towards Earth ▪ **Negative** when an object is **moving away** from Earth

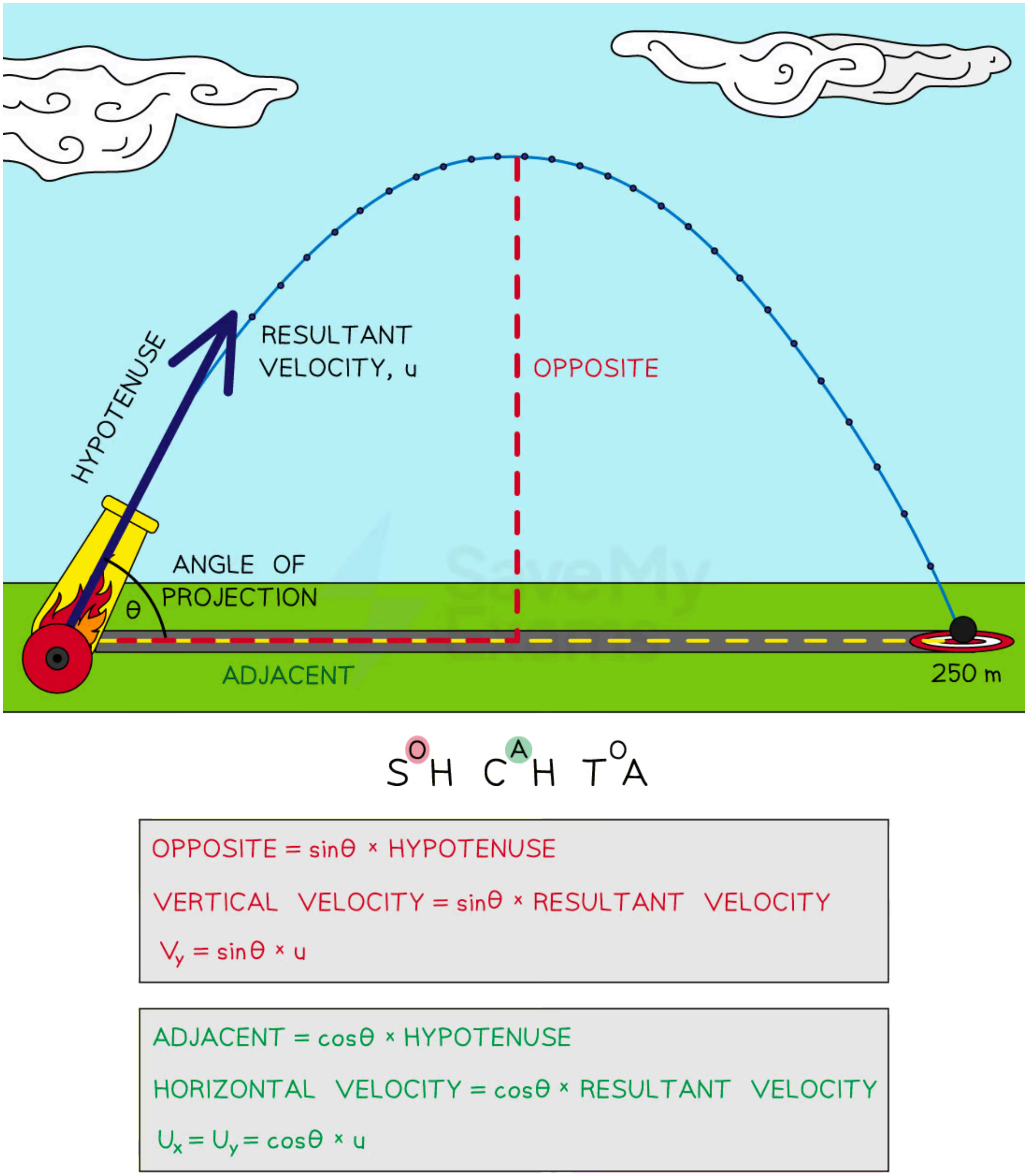
Your notes



Acceleration and horizontal velocity are always constant whilst vertical velocity changes

- The **resultant velocity** of an object in projectile motion can be split into its **horizontal** and **vertical** vector components using trigonometry where:
  - Vertical component = opposite side of the projectile triangle
    - opposite = sinθ × hyp = $\boldsymbol{u \sin\theta}$
  - Horizontal component = adjacent side of the projectile triangle
    - adjacent = cosθ × hyp = $\boldsymbol{u \cos\theta}$

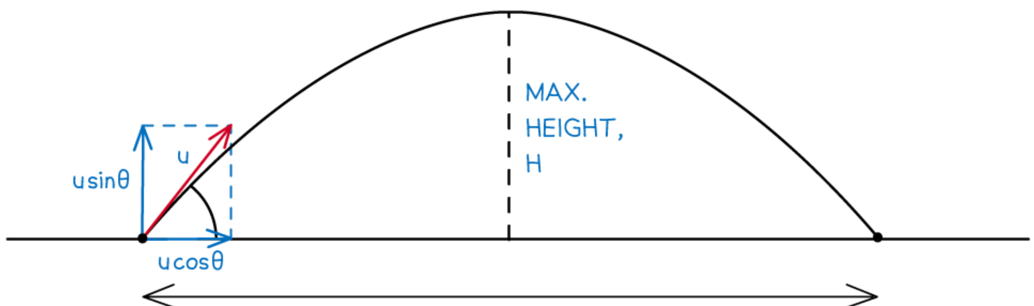
Your notes



The resultant velocity at an angle to the horizontal can be resolved using trigonometry into the horizontal and vertical components

- It can be helpful to see how different equations calculate different quantities using SUVAT equations
- Examples of obtaining the equations for total time, maximum height and range are shown below

Your notes



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ \text{ms}^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \dfrac{u\sin\theta}{g}$

$2t = \dfrac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

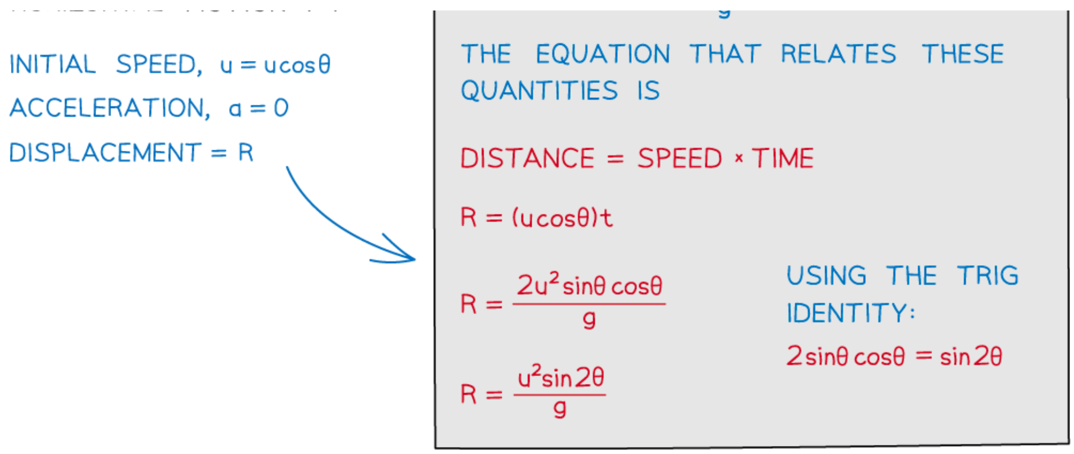
$2gH = (u\sin\theta)^2$

$H = \dfrac{(u\sin\theta)^2}{2g}$

HORIZONTAL MOTION (→)

RANGE (R)

$u = u\cos\theta \quad t = \dfrac{2u\sin\theta}{g} \quad a = 0 \quad R = ?$



Examples of using SUVAT equations to determine the time of flight, maximum height and range of a projectile

Solving problems with projectiles

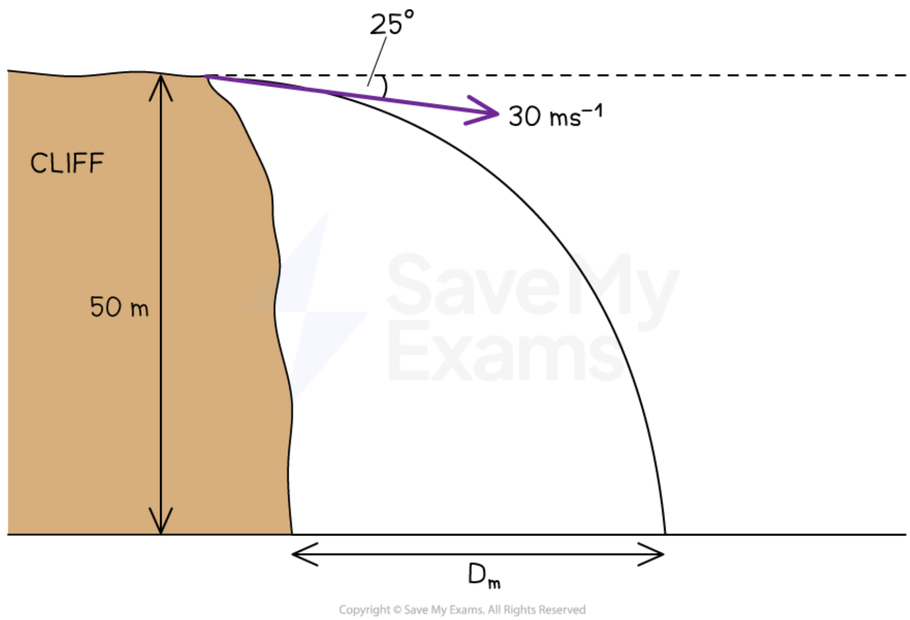
- You may be required to calculate the missing quantities from the following projectile motion scenarios:
  - **Vertical** projection above the horizontal
  - **Vertical** projection below the horizontal
  - **Horizontal** projection
  - **Projection** at an **angle**, the most common scenario

Your notes

Your notes

Worked example

A stone is dropped from the top of a cliff 50.0 m high at an angle of 25.0° below the horizontal. The stone has an initial speed of 30.0 ms^{-1} and follows a curved trajectory. The stone hits the ground at a horizontal distance *D* from the base of the cliff with a vertical velocity of 33.8 ms^{-1}.



Calculate the distance *D*.

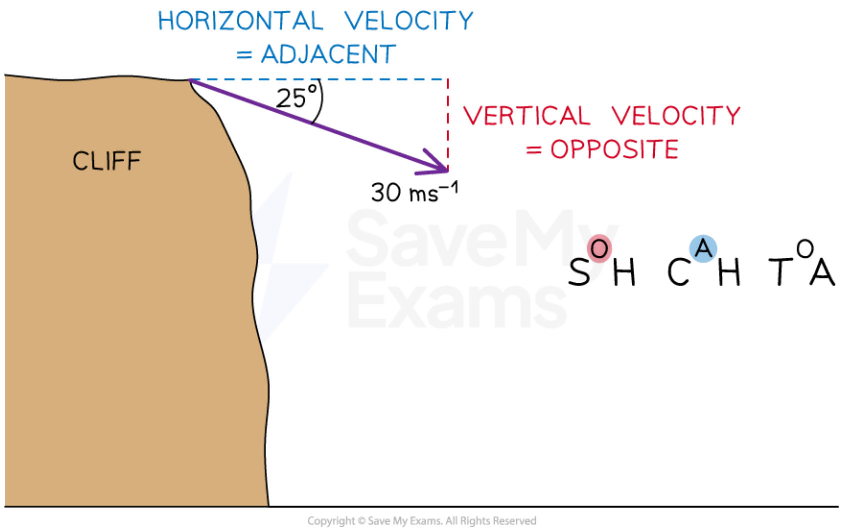
Answer:

Step 1: Understand the information given in the question

- The final vertical velocity of the stone, v = 33.8 ms^{-1}
- The horizontal velocity will remain constant throughout the motion
- The question wants us to calculate the range of the stone
- The vertical acceleration is +g ms^{-2}

Step 2: Resolve velocity into the vertical and horizontal components

Draw a triangle on the diagram to show the vertical and horizontal velocity components



Your notes

Calculate the initial vertical component of velocity, u_V using trigonometry:

- u_V = opposite side
- $u_V = \sin\theta \times$ hypotenuse side
- $u_V = \sin(25) \times 30 = 12.68$ ms^{-1}

Calculate the initial horizontal component of velocity, u_H using trigonometry:

- u_H = adjacent side
- $u_H = \cos\theta \times$ hypotenuse side
- $u_H = \cos(25) \times 30 = 27.19$ ms^{-1}

Step 3: Consider the equations of motion in the vertical and horizontal directions

	Vertical Motion	Horizontal Motion
u	12.68 ms^{-1}	27.19 ms^{-1}
v	33.8 ms^{-1}	27.19 ms^{-1}
a	+9.81 ms^{-2}	0 ms^{-2}
t		
s	50 m	*D* m ?

Step 4: Calculate the time of flight from the vertical motion

$$v = u + at$$

$$33.8 = 12.68 + 9.81t$$

$$33.8 - 12.68 = 9.81t$$

Your notes

$$t = \frac{33.8 - 12.68}{9.81}$$

t = 2.15 s

Step 5: Calculate the range of the stone, *D* using the elapsed time and horizontal motion

$s = ut$

$D = 27.19 \times 2.15$

D = 58.46 m = 58 m (2 s.f.)

Your notes

Worked example

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q? (ignoring air resistance)

Answer:

Step 1: Consider the situation

- In this question, vertical motion only needs to be considered to find the vertical height

Step 2: List the known quantities

- u = 12sin(50) ms^{-1}
- v = 0 ms^{-1}
- a = −9.81 ms^{-2}
- s = ?

Step 3: State the correct kinematic equation

$$v^2 = u^2 + 2as$$

Step 4: Rearrange the equation to make height, *s* the subject

$$\frac{v^2 - u^2}{2a} = s$$

Step 5: Substitute in the known quantities and calculate maximum height, *s*

$$s = \frac{0^2 - (12\sin(50))^2}{2 \times -9.81}$$

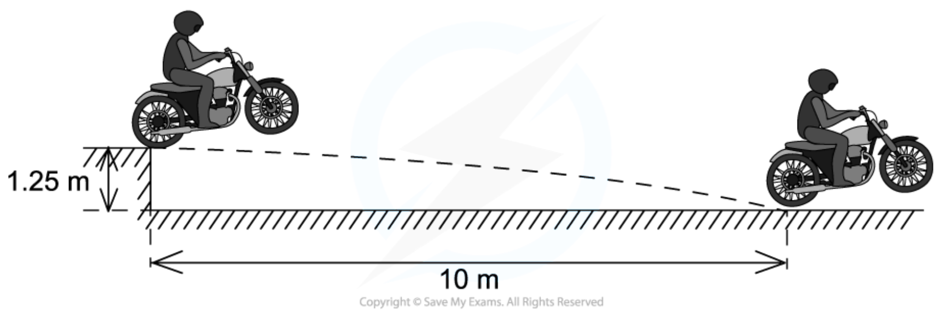
$$s = -4.3 \text{ m}$$

Your notes

Worked example

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off? (ignoring air resistance)



Your notes

IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $a = 9.81$ ms^{-2} $u = 0$ $t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $a = 0$ $t = 0.5$ s $u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20 \text{ ms}^{-1}$$

Examiner Tip

Make sure you don't make these common mistakes:

- Mixing up positive and negative values for vectors
- Mixing up velocities and distances between horizontal and vertical motion
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres

Further, it is worth noting that projectile motion is typically **symmetrical** when air resistance is **ignored** allowing for use of the peak to find the time of total flight or total horizontal distance by doubling the amount to get from the start point to the peak.

In these exam questions, unless specified, fluid resistance can be **ignored**

Your notes

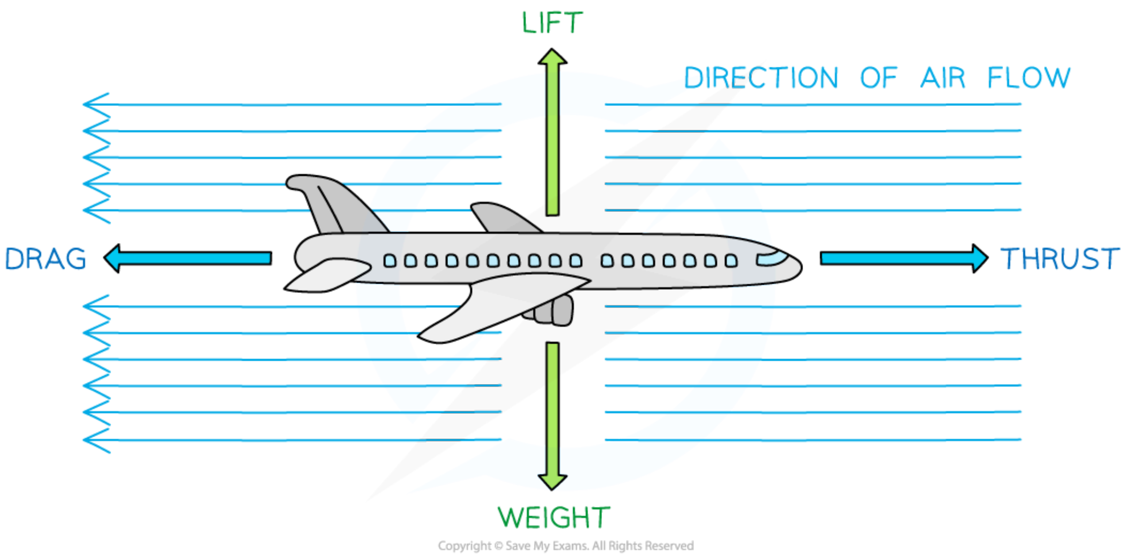
Fluid Resistance

Your notes

Fluid Resistance

- Fluid resistance refers to the effects of gases and liquids on the motion of a body
- When an object moves through a fluid (a gas or a liquid), there are **resistive** forces for that movement
  - These forces are known as **viscous drag**
  - Viscous drag, also known as air resistance, is a type of friction

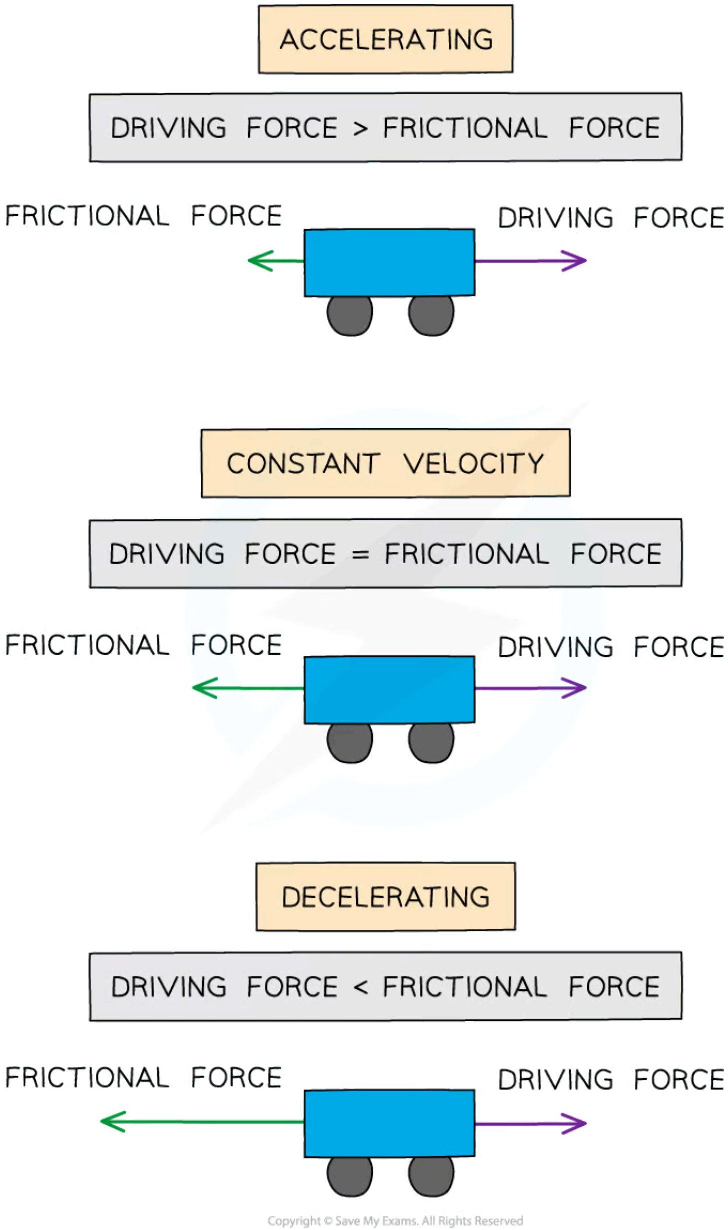
- **Frictional** forces:
  - Always act in the **opposite** direction to the motion of the object
  - Never speed an object up or start them moving
  - Always slow down an object or keep them moving at a constant speed
  - Always transfer energy away from the object to the surroundings
- **Lift** is an **upward** force on an object moving through a fluid. It is **perpendicular** to the fluid flow
  - For example, as an aeroplane moves through the air, the aeroplane pushes down on the air to change its direction
  - This causes an equal and opposite reaction as the air pushes upwards on the wings of the aeroplane (lift) due to Newton's Third Law



Drag forces are always in the opposite direction to the thrust (direction of motion). Lift is always in the opposite direction to the weight

- A key component of drag forces is that they increase with the **speed** of the object
- This is shown in the diagram below:

Your notes



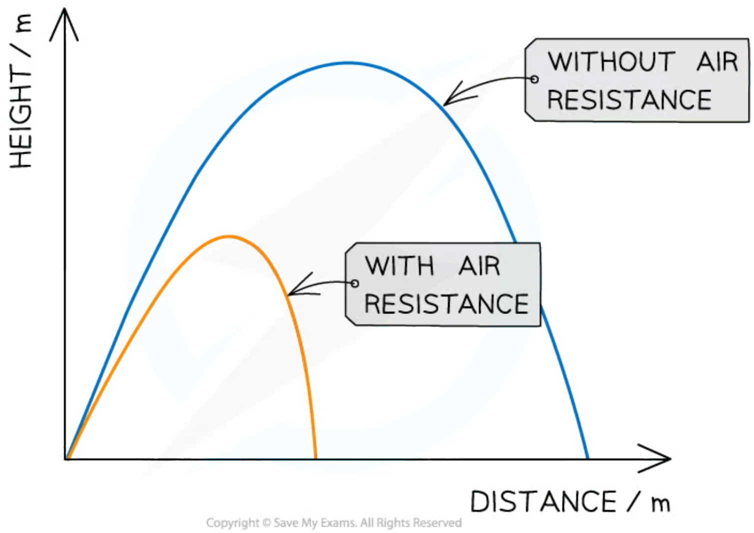
Frictional forces on a car increase with speed

Fluid Resistance in Projectile Motion

- In projectile motion, the factors that are affected by fluid resistance are:
  - Time of flight
  - Horizontal velocity
  - Horizontal acceleration
  - Range

Your notes

- Shape of trajectory
- **Air resistance** is the frictional force which has the most significant effect on a projectile
- Air resistance decreases the **horizontal** component of the velocity of a projectile
  - This means both its **range** and **maximum height** will decrease compared to an identical situation with no air resistance (like a vacuum)



A projectile with air resistance travels a smaller distance and has a lower maximum height than one without air resistance

- When air resistance is applied, the path of the projectile no longer follows a parabola shape
  - Its path is now **steeper** on the way down than it is up
- The flight time will also **decrease** as the projectile is in the air for a **shorter** period of time
  - This is due to having a smaller range and lower maximum height
- In summary:

Air resistance affects	Effect of air resistance
time of flight	decreases
horizontal velocity	decreases
horizontal deceleration	increases
range	decreases
shape of trajectory	no longer a parabola

- The angle and launch speed of a projectile can be varied to **cover a longer range** or reach a **greater maximum height**, depending on the situation
  - For sports, such as the long jump or javelin, an optimum angle against air resistance is used to produce the greatest range (distance)
  - For gymnastics or ski jumper, the initial vertical velocity is made as large as possible to reach a greater maximum height and longer flight path

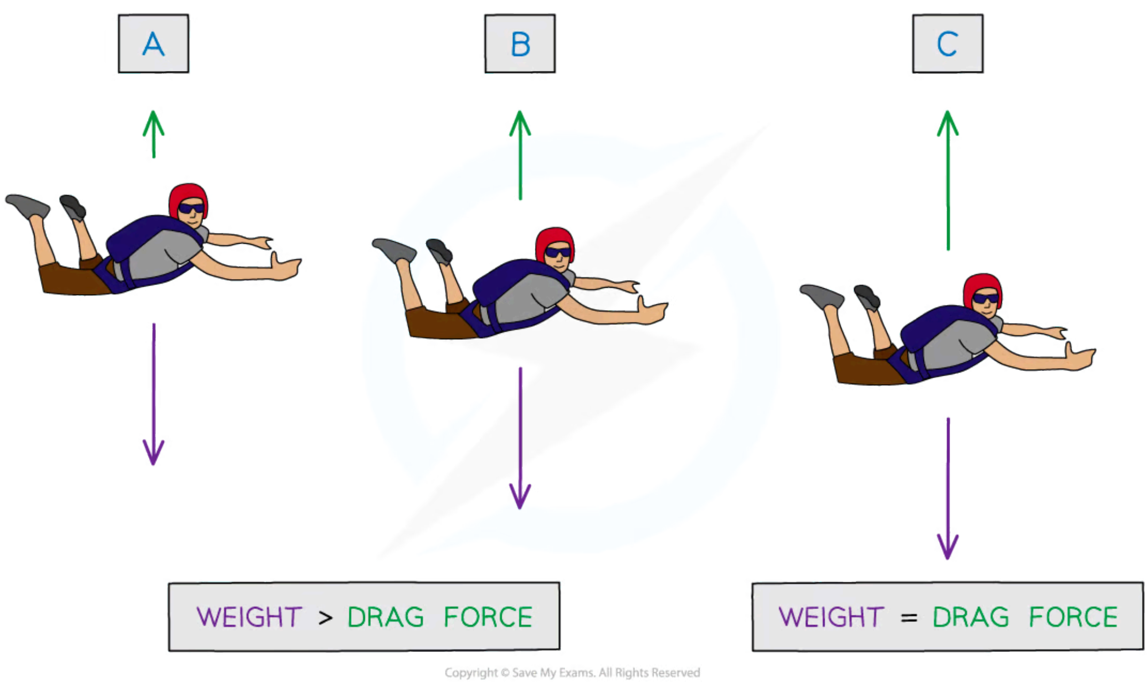
Your notes

Terminal Speed

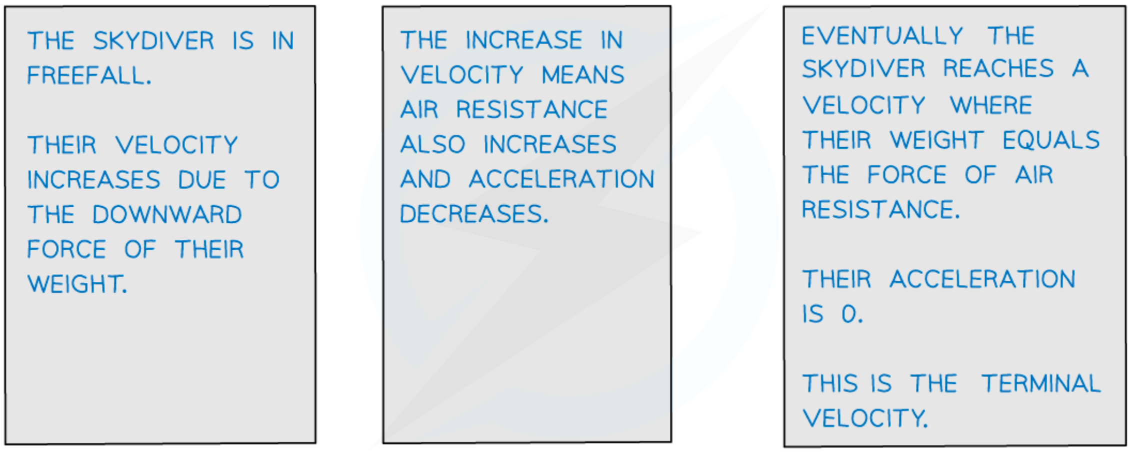
Your notes

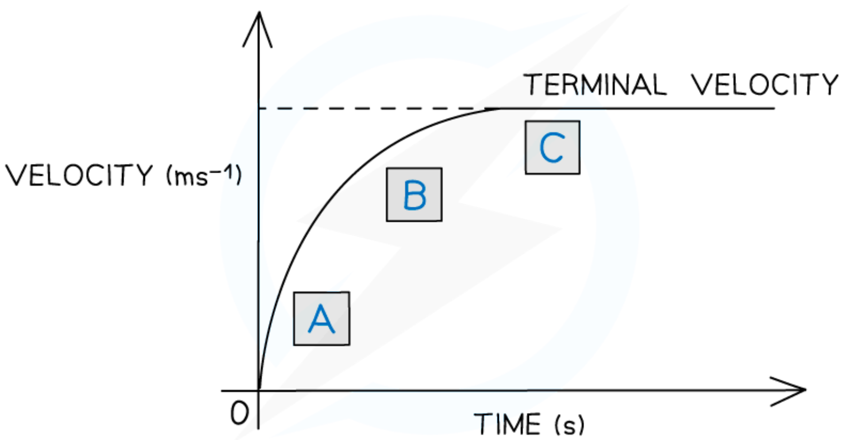
Terminal Speed

- For a body in free fall in a vacuum, the only force acting is weight, and its acceleration *g* is only due to gravity
- The frictional force from fluid resistance **increases** as the body **accelerates**
  - This increase in velocity means the viscous drag force also **increases**
- Due to Newton's Second Law, this means the resultant force and therefore acceleration decreases (recall ***F*** **=** ***ma***)
- When the viscous drag force is equal to the weight on the body, the body will no longer accelerate and will fall at a constant velocity
  - This velocity is called the **terminal velocity**
- Terminal velocity can occur for objects falling through a gas or a liquid



Your notes

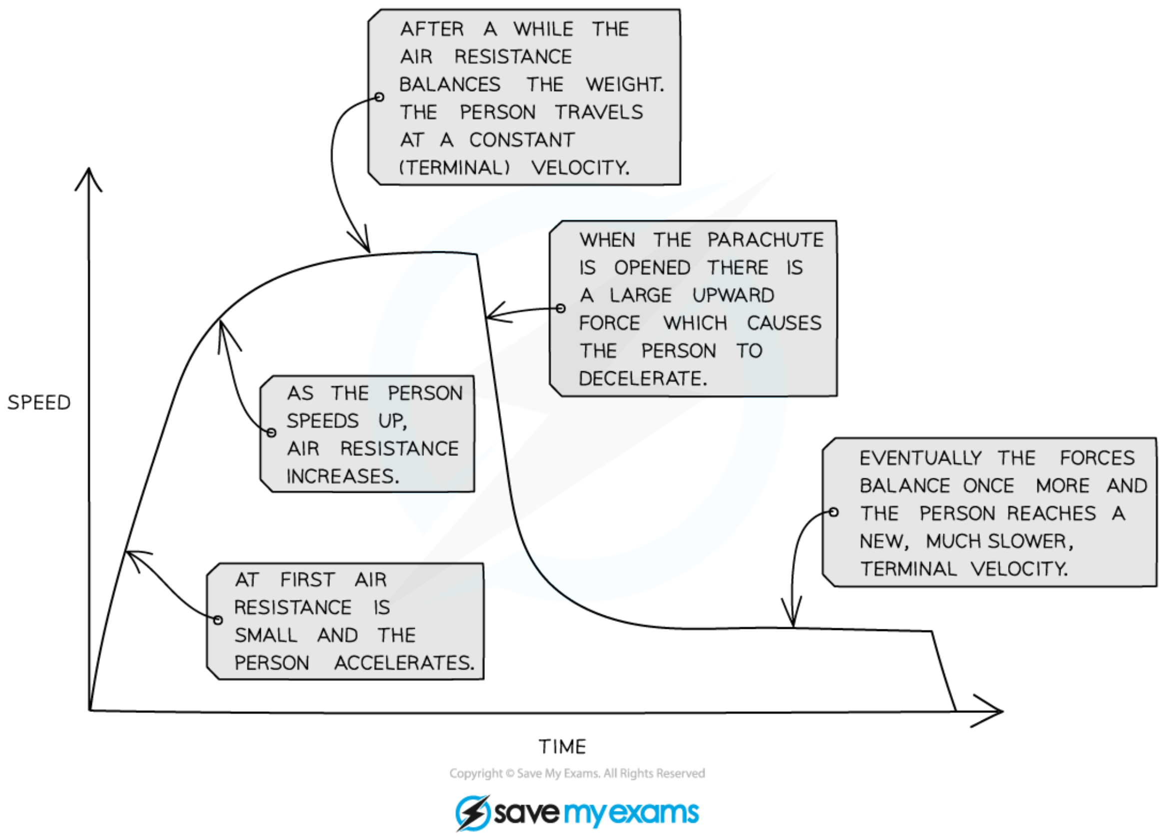




A skydiver in freefall reaching terminal velocity

- The graph shows how the velocity of the skydiver varies with time
- Since acceleration is equal to the **gradient** of a velocity-time graph, it can be seen that
  - The acceleration **decreases**
  - The acceleration eventually becomes **zero** when terminal velocity is reached
- After the skydiver deploys their parachute, they decelerate to a **lower terminal velocity** to reduce the impact on landing
- This is demonstrated by the graph below:

Your notes



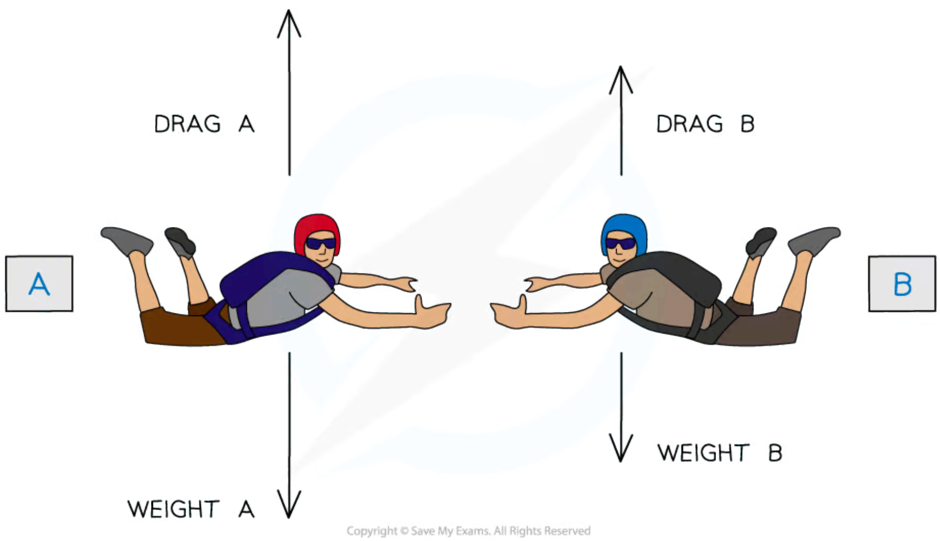
A graph showing the changes in speed of the skydiver throughout their entire journey in freefall

Your notes

Worked example

Skydivers jump out of a plane at intervals of a few seconds.

Skydivers **A** and **B** want to join up as they fall.



Skydiver **A** is heavier than Skydiver **B**, and both skydivers can be assumed to have the same surface area and volume.

If the two skydivers want to reach terminal velocity at the same time, who should jump first?

Answer:

Step 1: Recall the factors that affect terminal velocity during free fall:

- **Weight:** the heavier the person, the greater the weight acting on them
- Therefore, a heavier person will reach a higher terminal velocity

- **Drag:** the heavier the person, the greater the drag force required to balance the extra weight
- Therefore, a heavier person will reach terminal velocity faster

Step 2: Determine which skydiver will reach a higher terminal velocity

- Skydiver **A** has a greater mass (and weight), so he will reach a higher terminal velocity (than B)
- Skydiver **B** has a lower mass (and weight), so he will reach a lower terminal velocity (than A)

Step 3: Determine which skydiver will reach terminal velocity first

- The acceleration due to gravity is initially the same for both skydivers
- However, the drag force increases at a faster rate for the heavier skydiver (**A**), hence they will reach terminal velocity first

Step 4: Determine which skydiver should jump first

- Skydiver **B** should jump first since he will take longer to reach terminal velocity
- This is because skydiver **A** has a higher mass, and hence, weight
- More weight means a greater speed, therefore, **A** will reach terminal velocity faster than **B**

Your notes

Examiner Tip

A common misconception is that skydivers move upwards when their parachutes are deployed - however, this is not the case, they are in fact **decelerating** to a lower terminal velocity.

If a question considers air resistance to be **'negligible'** this means in that question, air resistance is taken to be so small it will not make a difference to the motion of the body. You can take this to mean there are no drag forces acting on the body.