

DP IB Maths: AI HL



Your notes

2.1 Linear Functions & Graphs

Contents

* 2.1.1 Equations of a Straight Line



Your notes

2.1.1 Equations of a Straight Line

Equations of a Straight Line

How do I find the gradient of a straight line?

- Find two points that the line passes through with coordinates (x_1, y_1) and (x_2, y_2)
- The gradient between these two points is calculated by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- This is given in the **formula booklet**
- The gradient of a straight line measures its **slope**
 - A line with gradient 1 will go up 1 unit for every unit it goes to the right
 - A line with gradient -2 will go down two units for every unit it goes to the right

What are the equations of a straight line?

- $y = mx + c$
 - This is the **gradient-intercept form**
 - It clearly shows the gradient m and the y -intercept $(0, c)$
- $y - y_1 = m(x - x_1)$
 - This is the **point-gradient form**
 - It clearly shows the gradient m and a point on the line (x_1, y_1)
- $ax + by + d = 0$
 - This is the **general form**
 - You can quickly get the x -intercept $\left(-\frac{d}{a}, 0\right)$ and y -intercept $\left(0, -\frac{d}{b}\right)$

How do I find an equation of a straight line?

- You will need the gradient
 - If you are given two points then first find the gradient
- It is easiest to start with the **point-gradient form**
 - then rearrange into whatever form is required
 - multiplying both sides by any denominators will get rid of fractions
- You can check your answer by using your GDC
 - Graph your answer and check it goes through the point(s)
 - If you have two points then you can enter these in the **statistics mode** and find the regression line
$$y = ax + b$$

Examiner Tip

- A sketch of the graph of the straight line(s) can be helpful, even if not demanded by the question
 - Use your GDC to plot them
- Ensure you state equations of straight lines in the format required
 - Usually $y = mx + c$ or $ax + by + d = 0$
 - Check whether coefficients need to be integers (they usually are for $ax + by + d = 0$)



Your notes



Your notes

Worked example

The line l passes through the points $(-2, 5)$ and $(6, -7)$.

Find the equation of l , giving your answer in the form $ax + by + d = 0$ where a , b and d are integers to be found.

Find the gradient between $(-2, 5)$ and $(6, -7)$

Formula booklet

$$m = \frac{-7 - 5}{6 - -2} = -\frac{3}{2}$$

Gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use the point-gradient formula

Formula booklet

Equations of a straight line

$$y - y_1 = m(x - x_1)$$

$$(x_1, y_1) = (-2, 5) \quad m = -\frac{3}{2}$$

$$y - 5 = -\frac{3}{2}(x - -2) \quad \text{Simplify}$$

$$y - 5 = -\frac{3}{2}(x + 2)$$

$$2(y - 5) = -3(x + 2)$$

$$2y - 10 = -3x - 6$$

$$3x + 2y - 4 = 0$$

Multiply by denominator

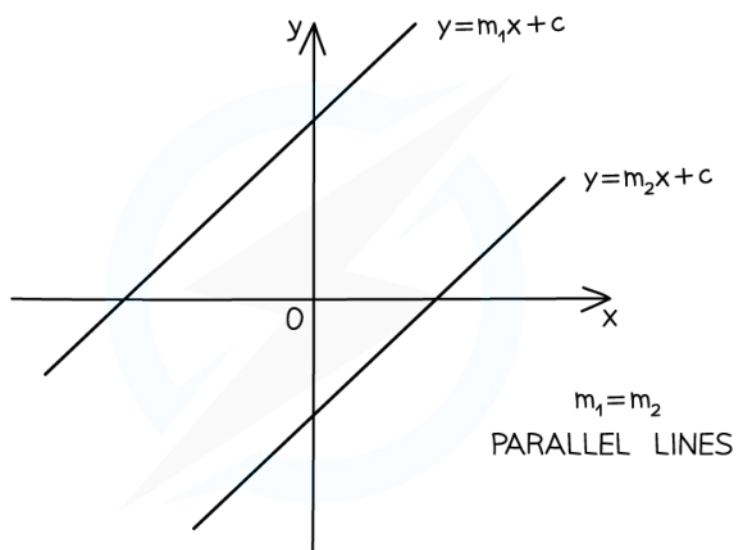
Expand

Rearrange

Parallel Lines

How are the equations of parallel lines connected?

- **Parallel lines** are always equidistant meaning they never intersect
- Parallel lines have the same gradient
 - If the gradient of line l_1 is m_1 and gradient of line l_2 is m_2 then...
 - $m_1 = m_2 \Rightarrow l_1 \text{ \& } l_2$ are parallel
 - $l_1 \text{ \& } l_2$ are parallel $\Rightarrow m_1 = m_2$
- To determine if two lines are parallel:
 - Rearrange into the gradient-intercept form $y = mx + c$
 - Compare the coefficients of x
 - If they are equal then the lines are parallel



Copyright © Save My Exams. All Rights Reserved



Your notes



Your notes

Worked example

The line l passes through the point $(4, -1)$ and is parallel to the line with equation $2x - 5y = 3$.

Find the equation of l , giving your answer in the form $y = mx + c$.

Rearrange into $y = mx + c$ to find the gradient

$$5y = 2x - 3 \Rightarrow y = \frac{2}{5}x - \frac{3}{5} \therefore \text{gradient} = \frac{2}{5}$$

Parallel lines $\Rightarrow m_1 = m_2$

$$m = \frac{2}{5}$$

Use the point-gradient formula

Formula booklet

Equations of a straight line	$y - y_1 = m(x - x_1)$
------------------------------	------------------------

$$(x_1, y_1) = (4, -1) \quad m = \frac{2}{5}$$

$$y + 1 = \frac{2}{5}(x - 4)$$

$$y + 1 = \frac{2}{5}x - \frac{8}{5}$$

$$y = \frac{2}{5}x - \frac{13}{5}$$

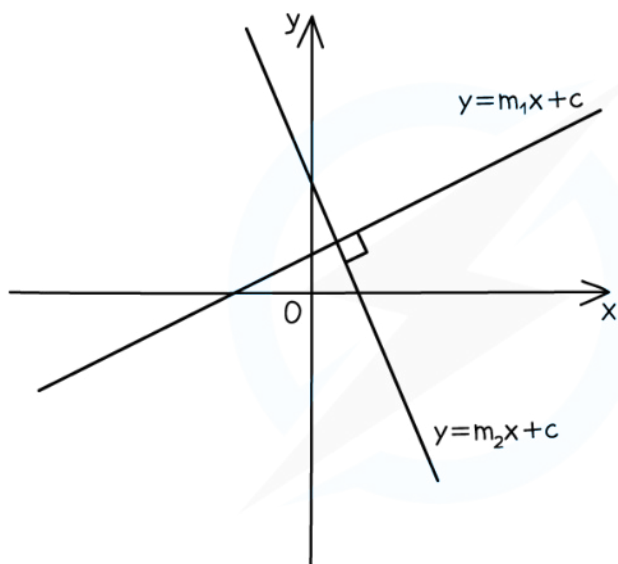


Your notes

Perpendicular Lines

How are the equations of perpendicular lines connected?

- **Perpendicular lines** intersect at right angles
- The gradients of two perpendicular lines are negative reciprocals
 - If the gradient of line l_1 is m_1 and gradient of line l_2 is m_2 then...
 - $m_1 \times m_2 = -1 \Rightarrow l_1 \text{ \& } l_2$ are perpendicular
 - $l_1 \text{ \& } l_2$ are perpendicular $\Rightarrow m_1 \times m_2 = -1$
- To determine if two lines are perpendicular:
 - Rearrange into the gradient-intercept form $y = mx + c$
 - Compare the coefficients of x
 - If their product is -1 then they are perpendicular
- Be careful with horizontal and vertical lines
 - $x = p$ and $y = q$ are perpendicular where p and q are constants



$$m_1 \times m_2 = -1$$

PERPENDICULAR LINES

Copyright © Save My Exams. All Rights Reserved



Your notes

Worked example

The line l_1 is given by the equation $3x - 5y = 7$.

The line l_2 is given by the equation $y = \frac{1}{4} - \frac{5}{3}x$.

Determine whether l_1 and l_2 are perpendicular. Give a reason for your answer.

Rearrange l_1 into $y = mx + c$ form

$$5y = 3x - 7 \Rightarrow y = \frac{3}{5}x - \frac{7}{5}$$

Identify gradients

$$m_1 = \frac{3}{5} \quad m_2 = -\frac{5}{3}$$

$m_1 \times m_2 = -1 \Rightarrow$ Perpendicular lines

$$\frac{3}{5} \times -\frac{5}{3} = -1$$

l_1 and l_2 are perpendicular as $m_1 \times m_2 = -1$