HL IB Physics

Your notes

Scalars & Vectors

Contents

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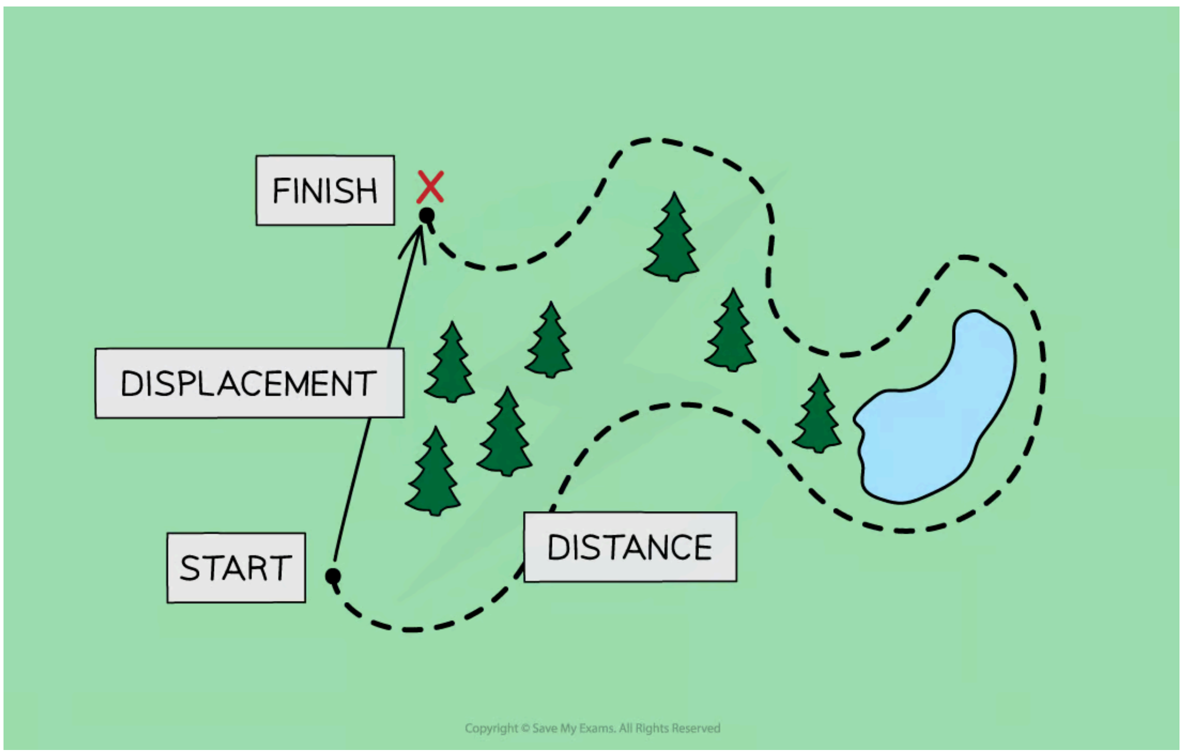
Scalar & Vector Quantities

Scalar & Vector Quantities

- A **scalar** is a quantity which **only** has a magnitude (size)
- A **vector** is a quantity which has **both** a **magnitude** and a **direction**
- For example, if a person goes on a hike in the woods to a location which is a couple of miles from their starting point
  - As the crow flies, their **displacement** will only be a few miles but the **distance** they walked will be much longer



Displacement is a vector while distance is a scalar quantity

- **Distance** is a **scalar** quantity
  - This is because it describes how an object has travelled overall, but not the direction it has travelled in

- **Displacement** is a **vector** quantity
  - This is because it describes how far an object is from where it started and in what direction

- Some common scalar and vector quantities are shown in the table below:

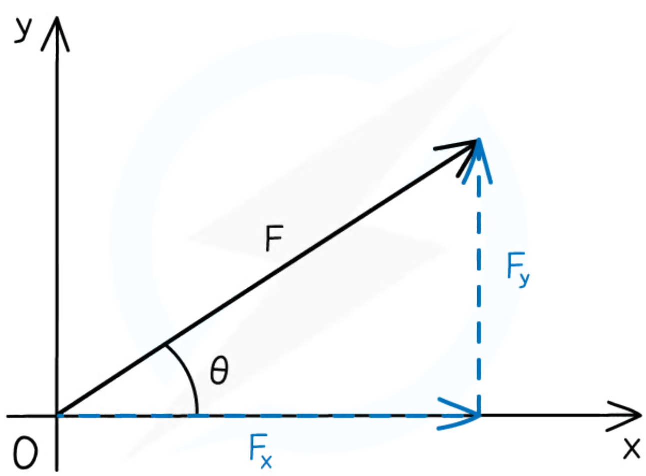
Scalars and Vectors Table

Your notes

SCALARS	VECTORS
DISTANCE	DISPLACEMENT
SPEED	VELOCITY
MASS	ACCELERATION
TIME	FORCE
ENERGY	MOMENTUM
VOLUME	
DENSITY	
PRESSURE	
ELECTRIC CHARGE	
TEMPERATURE	

Representing Vectors

- **Vectors** are represented by an arrow
  - The **arrowhead** indicates the **direction** of the vector
  - The **length** of the arrow represents the **magnitude**



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The force vector F has both a direction and a magnitude

- **Component** vectors are sometimes drawn with a dotted line and a **subscript** indicating horizontal or vertical
  - For example, F_x is the horizontal component and F_y is the vertical component of the force F

Examiner Tip

Do you have trouble figuring out if a quantity is a vector or a scalar? Just think - can this quantity have a minus sign? For example - can you have negative energy? No. Can you have negative displacement? Yes!

Combining & Resolving Vectors

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Combining & Resolving Vectors

- Vectors can be changed in a variety of ways, such as
  - Combining through vector **addition** or **subtraction**
  - Combining through vector **multiplication**
  - Resolving into **components** through trigonometry

Combining Vectors

- Vectors can be combined by **adding** or **subtracting** them to produce the **resultant vector**
  - The **resultant vector** is sometimes known as the 'net' vector (e.g. the net force)
- There are two methods that can be used to combine vectors: the **triangle method** and the **parallelogram method**

Triangle method

- To combine vectors using the triangle method:
  - **Step 1:** link the vectors head-to-tail
  - **Step 2:** the resultant vector is formed by connecting the tail of the first vector to the head of the second vector

Parallelogram method

- To combine vectors using the parallelogram method:
  - **Step 1:** link the vectors tail-to-tail
  - **Step 2:** complete the resulting parallelogram
  - **Step 3:** the resultant vector is the diagonal of the parallelogram

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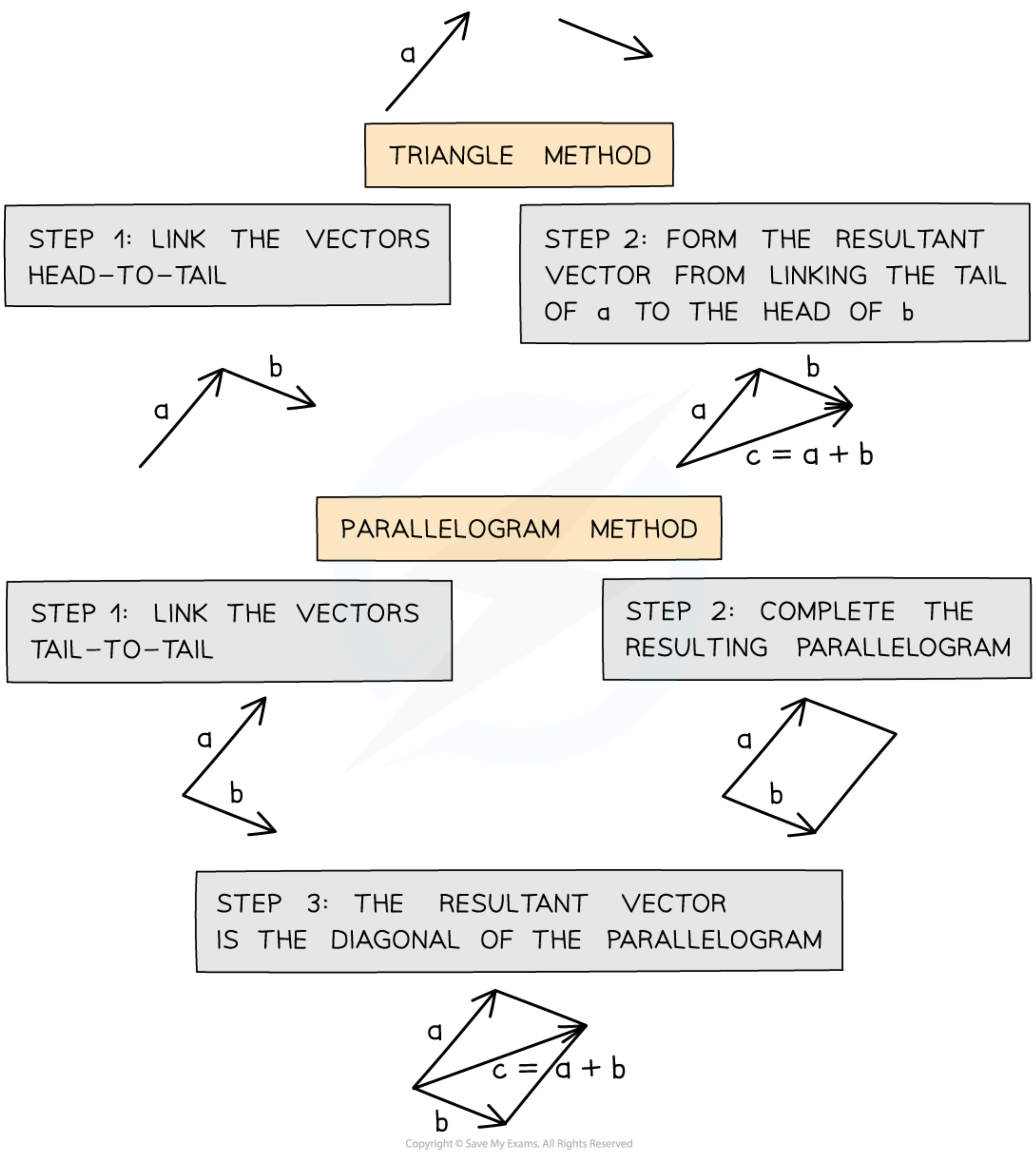
Worked example

Draw the vector c = a + b.

Answer:



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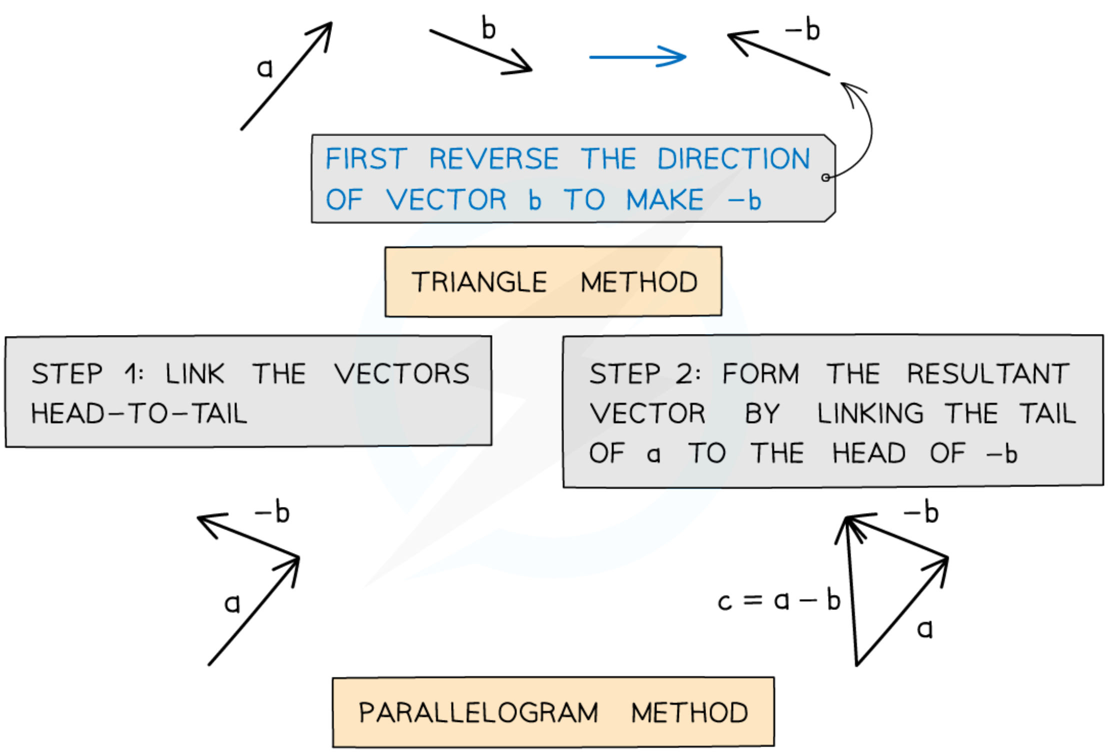
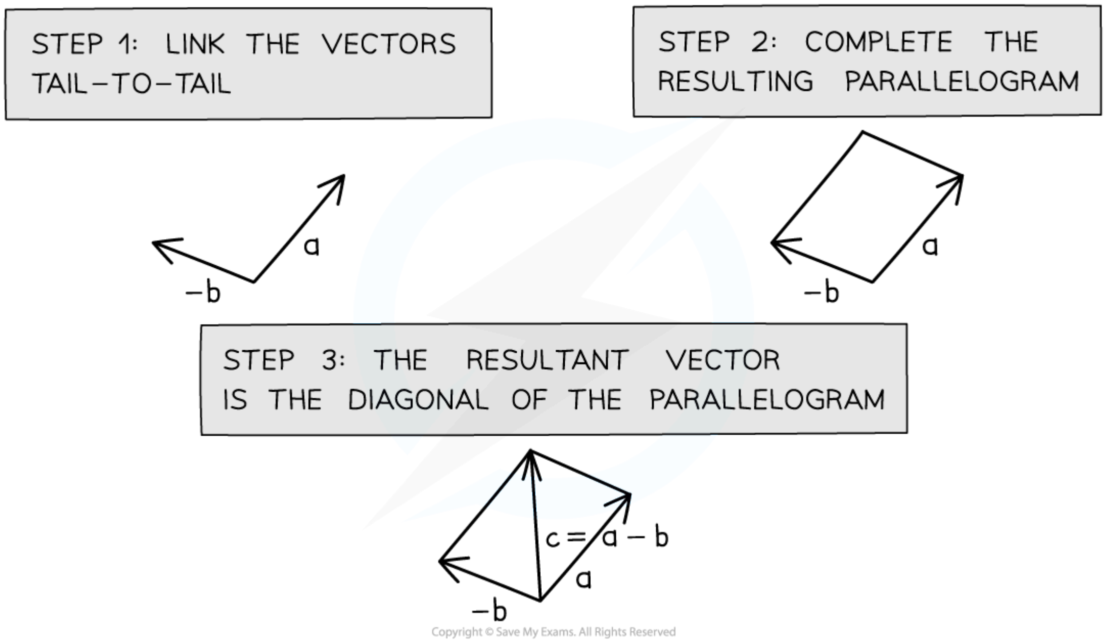
Worked example

Draw the vector c = a – b.

Answer:





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Vector Multiplication

- The product of a scalar and a vector is **always** a vector
- For example, consider the scalar quantity **mass** m and the vector quantity **acceleration** $\vec{a}$
- The product of mass m and acceleration $\vec{a}$ gives rise to a vector quantity **force** $\vec{F}$

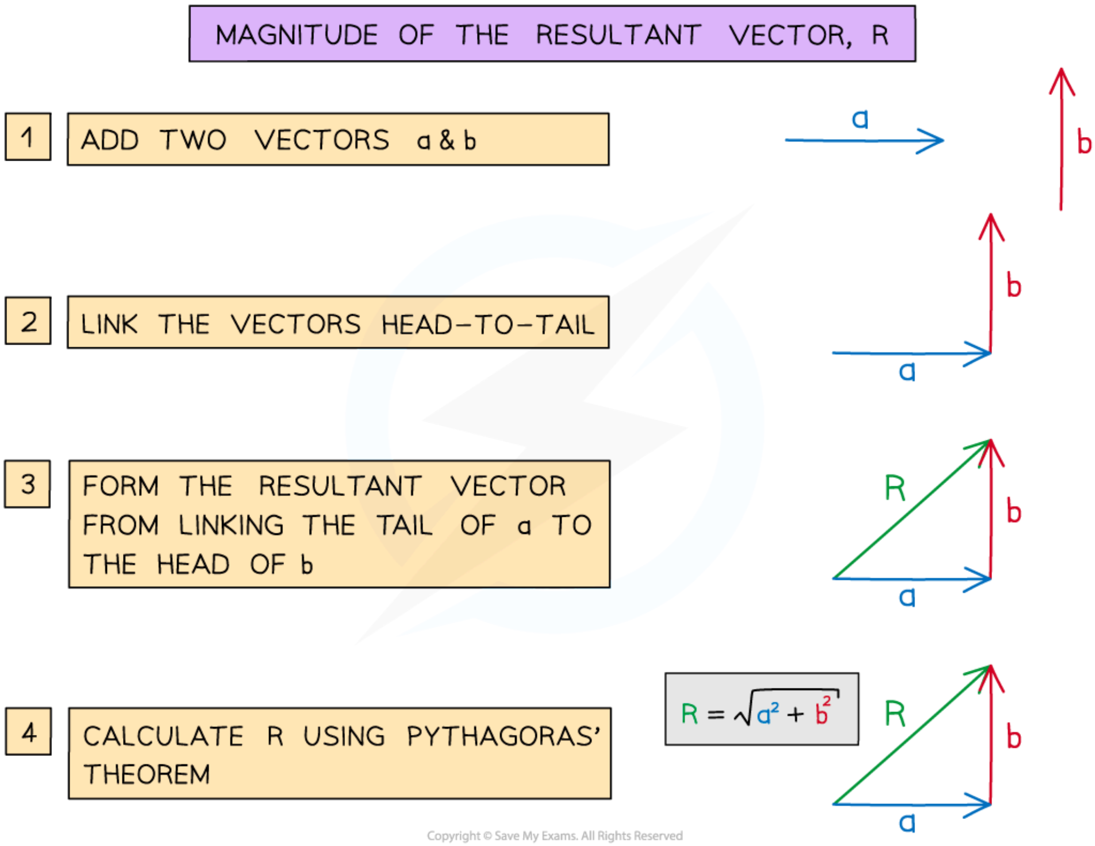
$$\vec{F} = m \times \vec{a}$$

- For another example, consider the scalar quantity **mass** m and the vector quantity **velocity** $\vec{v}$
- The product of mass m and velocity $\vec{v}$ gives rise to a vector quantity **momentum** $\vec{p}$

$$\vec{p} = m \times \vec{v}$$

Resolving Vectors

- Two vectors can be represented by a single **resultant vector**
  - Resolving a vector is the opposite of adding vectors
- A single resultant vector can be resolved
  - This means it can be represented by **two** vectors, which in combination have the same effect as the original one



The magnitude of the resultant vector is found by using Pythagoras' Theorem

- When a single resultant vector is broken down into its **parts**, those parts are called **components**
- For example, a force vector of magnitude F_R and an angle of θ to the horizontal is shown below

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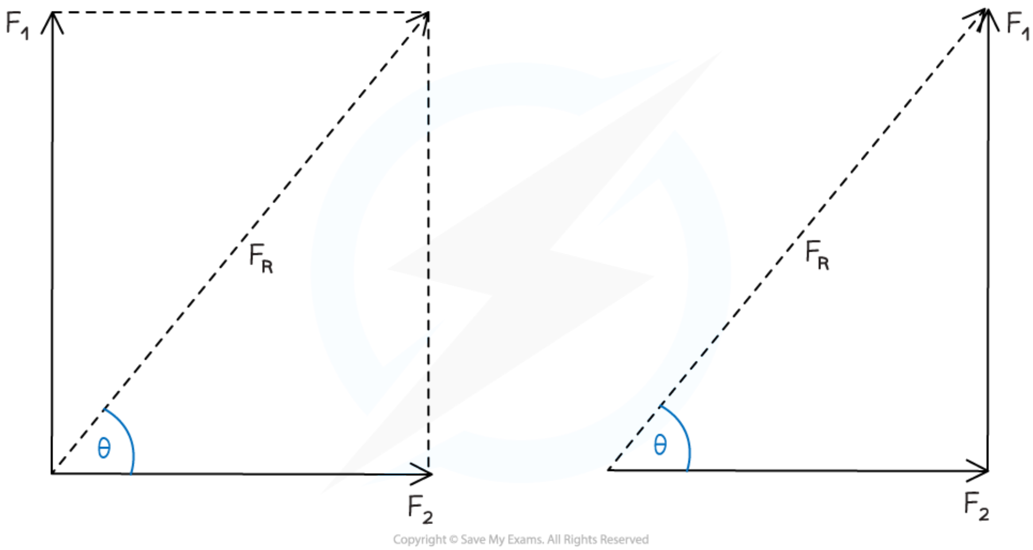
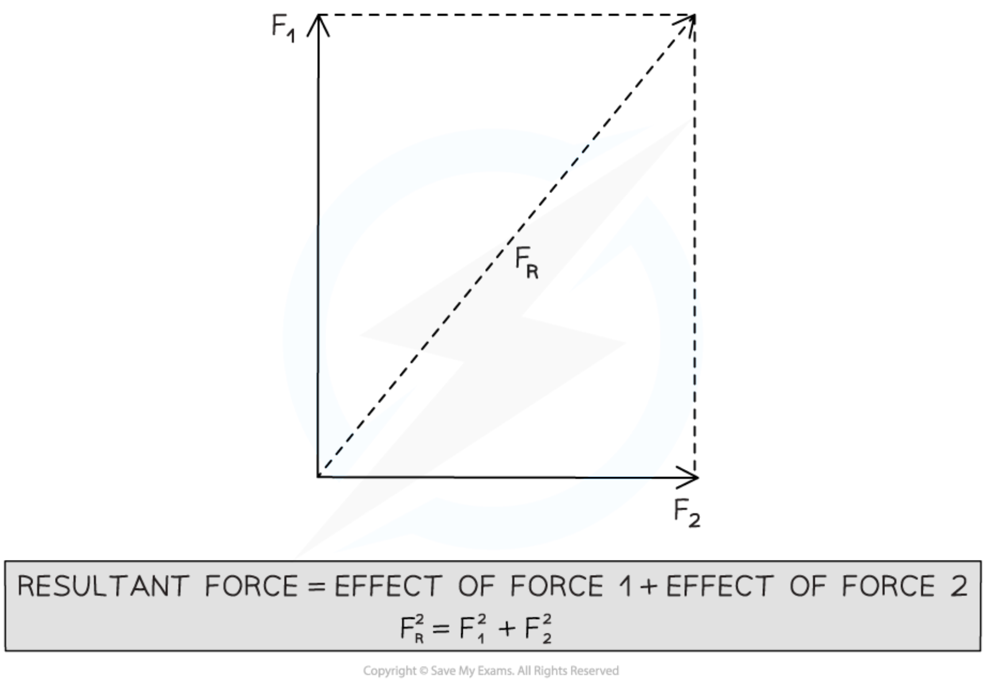


Resolving two force vectors F_1 and F_2 into a resultant force vector F_R

- It is possible to **resolve** this vector into its **horizontal** and **vertical** components using trigonometry



The resultant force F_R can be split into its horizontal and vertical components

- The direction of the resultant vector is found from the angle it makes with the horizontal or vertical
  - The question should imply which angle it is referring to (i.e. calculate the angle from the x-axis)

- Calculating the angle of this resultant vector from the horizontal or vertical can be done using **trigonometry**
  - Either the sine, cosine or tangent formula can be used depending on which vector magnitudes are calculated
- For the **horizontal** component, $F_x = F \cos \theta$
- For the **vertical** component, $F_y = F \sin \theta$

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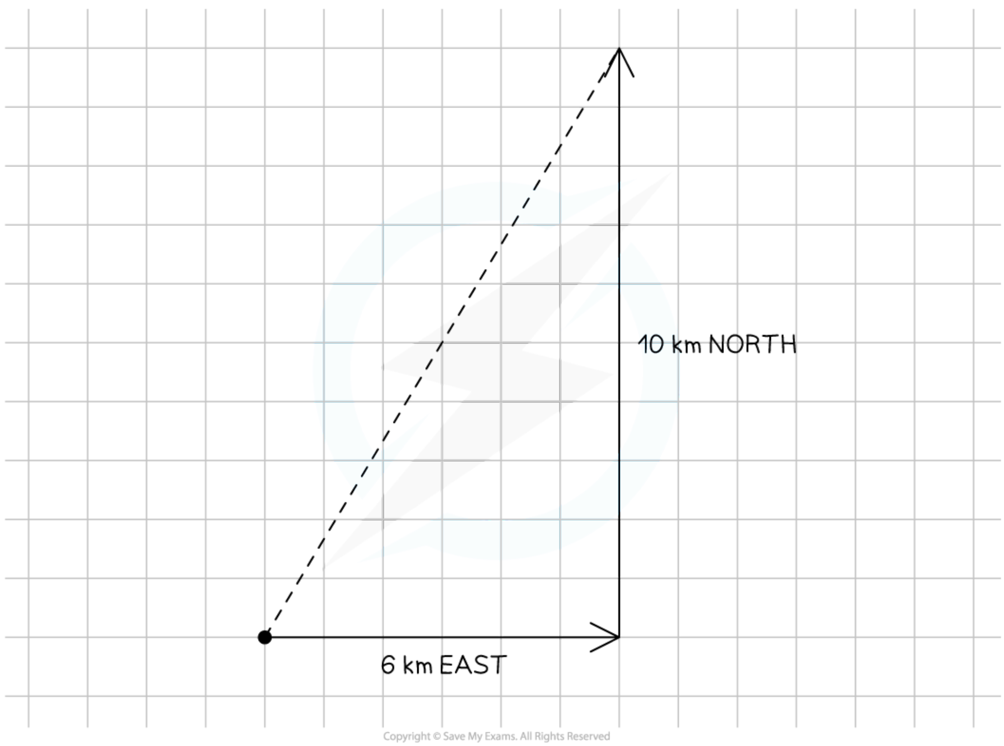
Worked example

A hiker walks a distance of 6 km due east and 10 km due north.

Calculate the magnitude of their displacement and its direction from the horizontal.

Answer:

Step 1: Draw a vector diagram



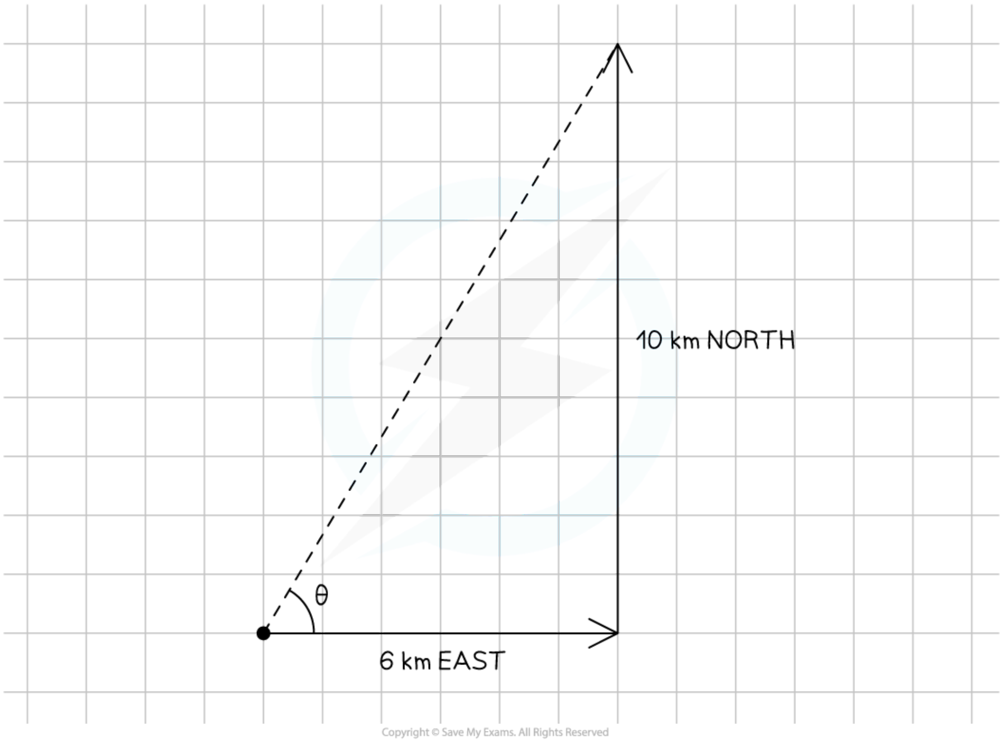
Step 2: Calculate the magnitude of the resultant vector using Pythagoras' Theorem

$$\text{Resultant vector} = \sqrt{6^2 + 10^2}$$

$$\text{Resultant vector} = \sqrt{136}$$

Resultant vector = **11.66**

Step 3: Calculate the direction of the resultant vector using trigonometry



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{6}$$

$$\theta = \tan^{-1}\left(\frac{10}{6}\right) = 59°$$

Step 4: State the final answer complete with direction

- Vector magnitude: 12 km
- Direction: 59° east and upwards from the horizontal

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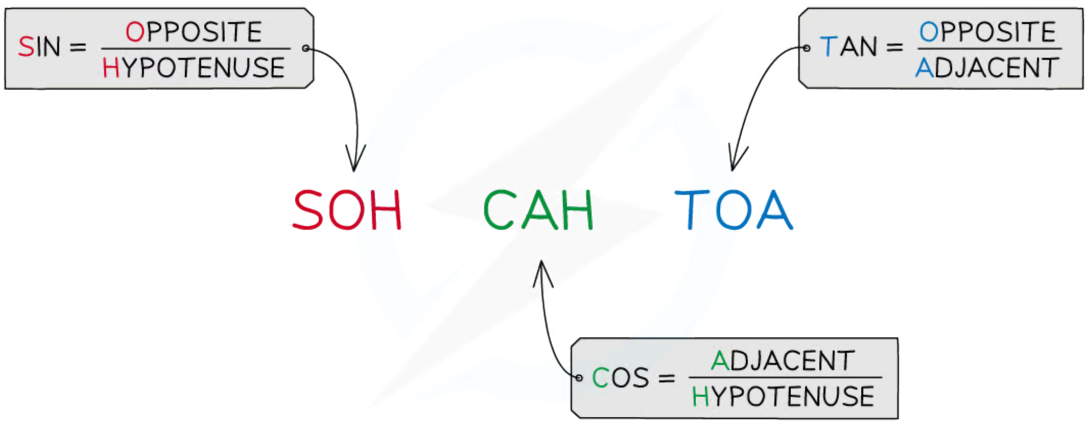
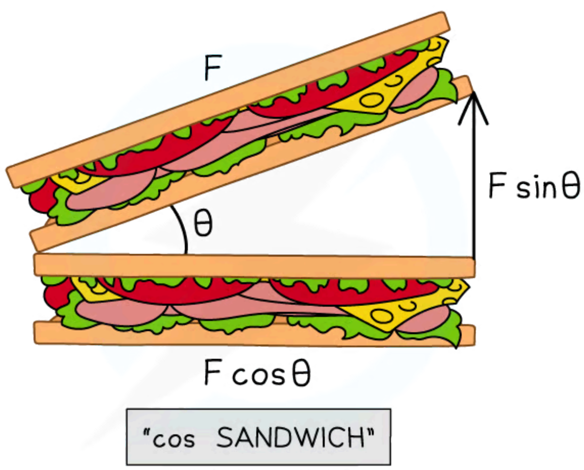
Examiner Tip

Make sure you are confident using trigonometry as it is used a lot in vector calculations!



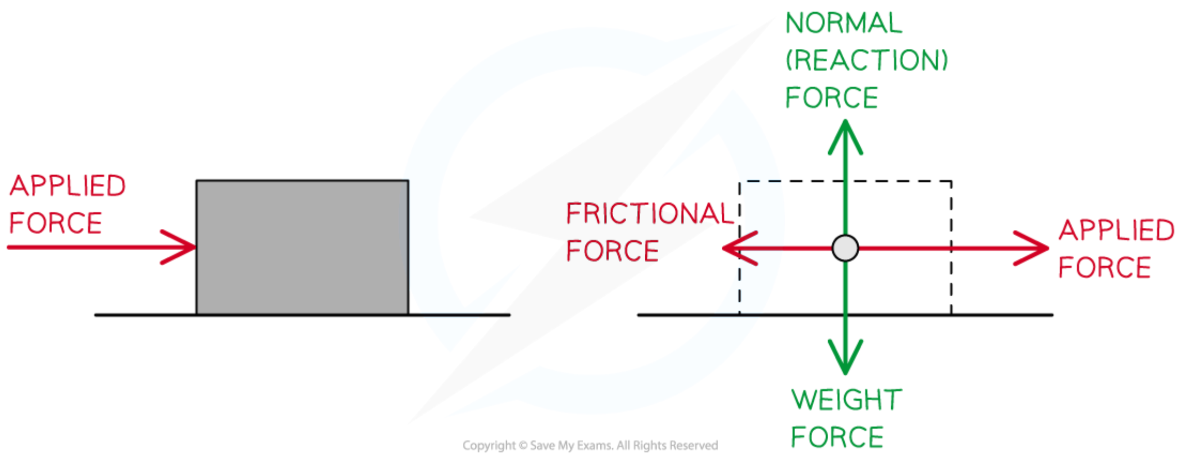
If you're unsure as to which component of the force is cos θ or sin θ, just remember that the cos θ is always the adjacent side of the right-angled triangle AKA, making a 'cos sandwich'



Force as a Vector

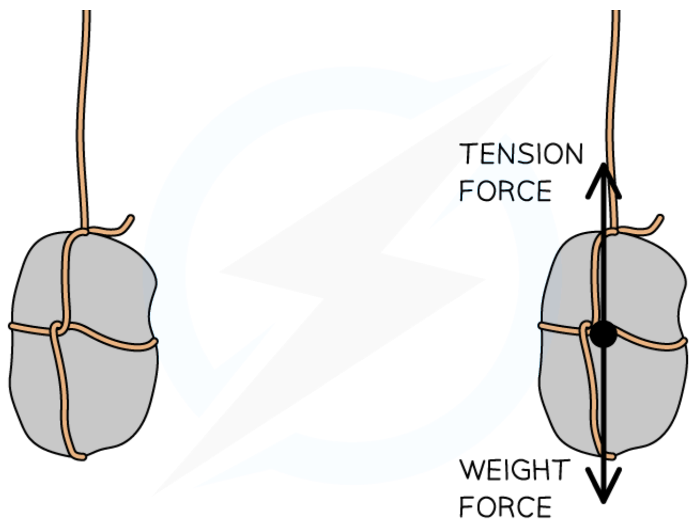
Your notes

- In physics, vectors appear in many different topic areas
  - Specifically, vectors are often **combined** and **resolved** to solve problems when considering motion, forces, and momentum
- Forces vector diagrams are often represented by free-body force diagrams
- The rules for drawing a free-body diagram are the following:
  - **Rule 1:** Draw a point in the centre of mass of the body
  - **Rule 2:** Draw the body free from contact with any other object
  - **Rule 3:** Draw the forces acting on that body using vectors with length in proportion to its magnitude
  - **Rule 4:** Draw the tail of the vector from the centre of mass and use the tip to indicate the direction



Point particle representation of the forces acting on a moving object on a rough horizontal surface

- The below example shows the forces acting on an object suspended from a stationary rope



Free-body diagram of an object suspended from a stationary rope

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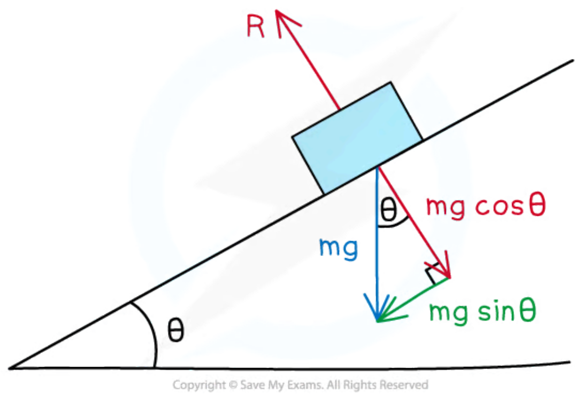
Forces on an Inclined Plane

- A common scenario is an object on an inclined plane
- An inclined plane, or a slope, is a flat surface tilted at an angle, θ



The weight vector of an object on an inclined plane can be split into its components parallel and perpendicular to the slope

- Inclined slope problems can be simplified by considering the components of the forces as **parallel** or **perpendicular** to the slope
- The **weight** ($W = mg$) of the object is always directed **vertically downwards**
- On the inclined slope, weight can be split into the following components:

Perpendicular to the slope: ↙ $W = mg \cos \theta$

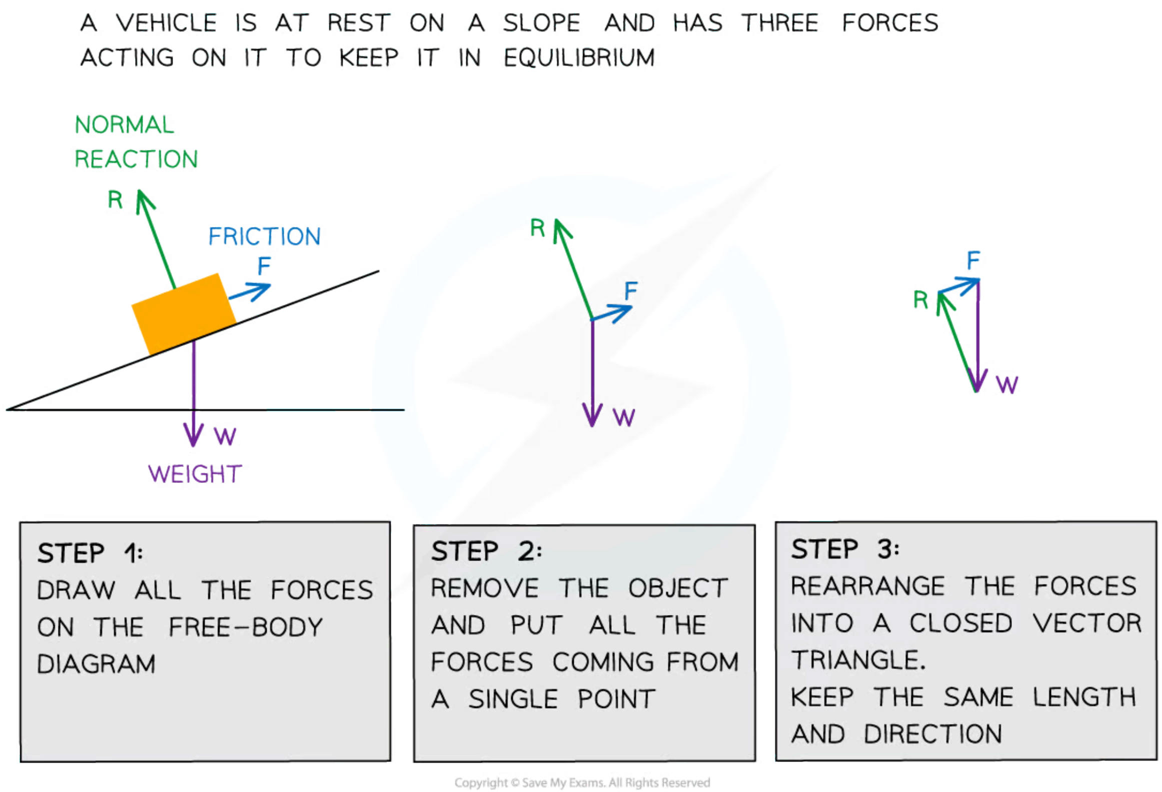
Parallel to the slope: ↘ $W = mg \sin \theta$

- The **normal** (or reaction) force R is always **vertically upwards**, or perpendicular to the surface
- If there is **no friction**, the parallel component of weight, $mg \sin \theta$, causes the object to move down the slope
- If the object is **not moving** perpendicular to the slope, the normal force is $R = mg \cos \theta$

Equilibrium

- Coplanar forces can be represented by vector triangles
- Forces are in equilibrium if an object is either
  - At rest
  - Moving at **constant** velocity
- In equilibrium, coplanar forces are represented by **closed** vector triangles
  - The vectors, when joined together, form a closed path
- The most common forces on objects are
  - Weight
  - Normal reaction force
  - Tension (from cords and strings)

- Friction
- The forces on a body in equilibrium are demonstrated below:



Three forces on an object in equilibrium form a closed vector triangle

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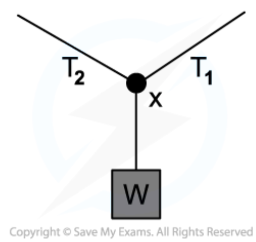
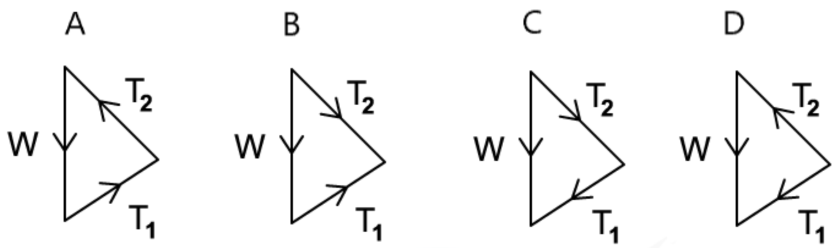
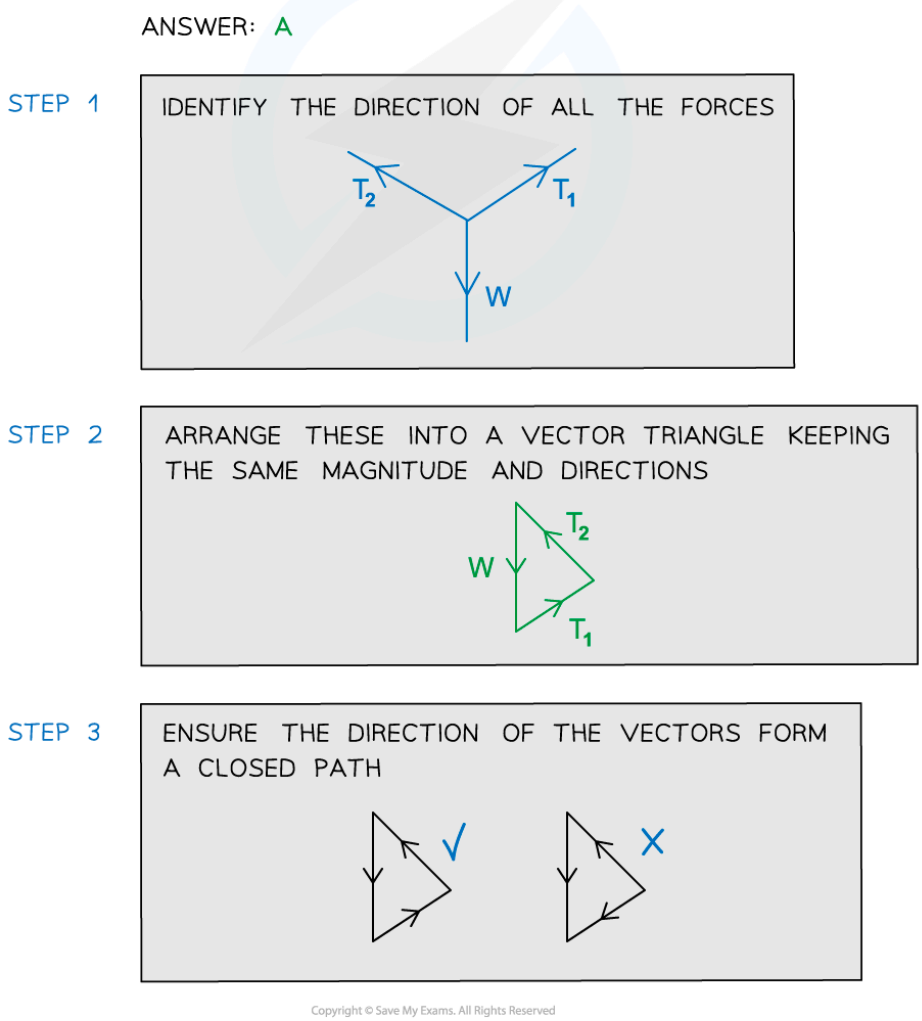
Worked example

A weight hangs in equilibrium from a cable at point **X**. The tensions in the cables are T_1 and T_2 as shown.



Which diagram correctly represents the forces acting at point **X**?




ANSWER: A
STEP 1
IDENTIFY THE DIRECTION OF ALL THE FORCES
T2
T1
W
STEP 2
ARRANGE THESE INTO A VECTOR TRIANGLE KEEPING THE SAME MAGNITUDE AND DIRECTIONS
W
T2
T1
STEP 3
ENSURE THE DIRECTION OF THE VECTORS FORM A CLOSED PATH
√
X
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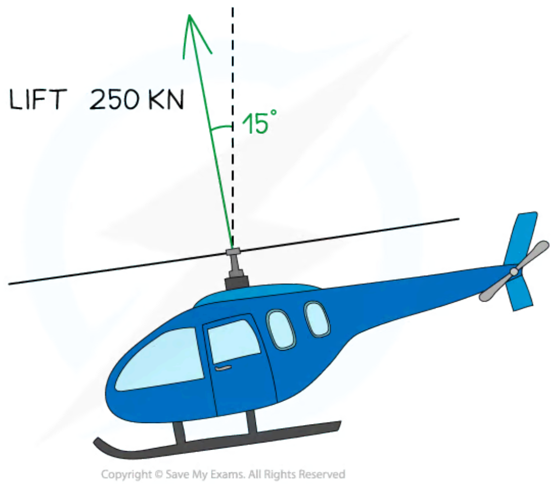
Worked example

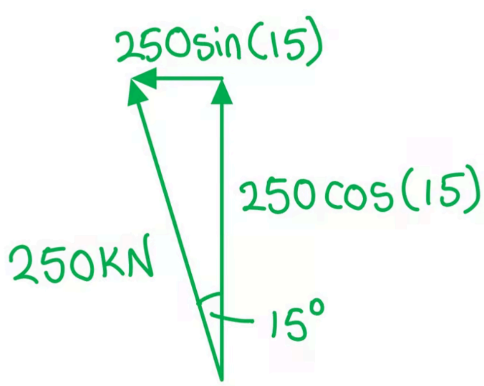
A helicopter provides a lift of 250 kN when the blades are tilted at 15° from the vertical.



Calculate the horizontal and vertical components of the lift force.

Answer:

Step 1: Draw a vector triangle of the resolved forces



Step 2: Calculate the vertical component of the lift force

Vertical component of force = 250 × cos(15) = 242 kN

Step 3: Calculate the horizontal component of the lift force

Horizontal component of force = 250 × sin(15) = 64.7 kN

Your notes

Examiner Tip

When labelling force vectors, it is important to use conventional and appropriate naming or symbols such as:

- ***w*** or weight force or ***mg***
- ***N*** *or* ***R*** for normal reaction force (depending on your local context either of these could be acceptable)

Using unexpected notation can lead to losing marks so try to be consistent with expected conventions.

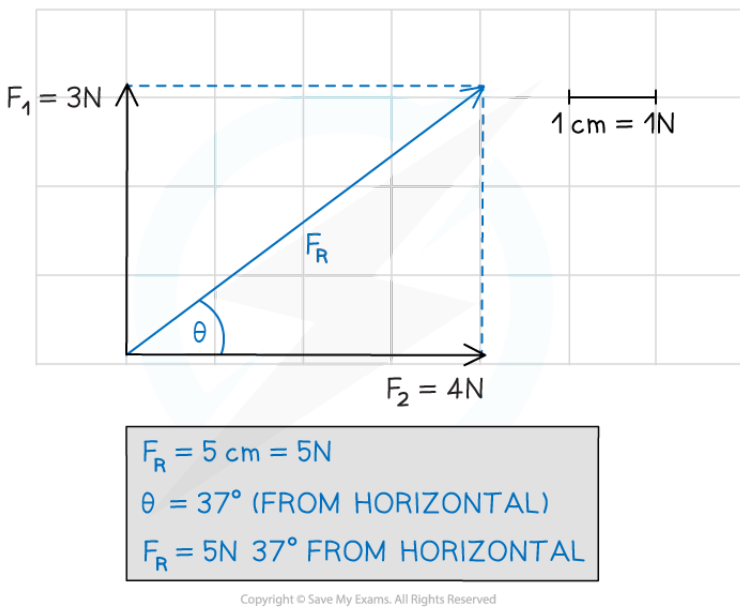
Your notes

Scale Diagrams

Your notes

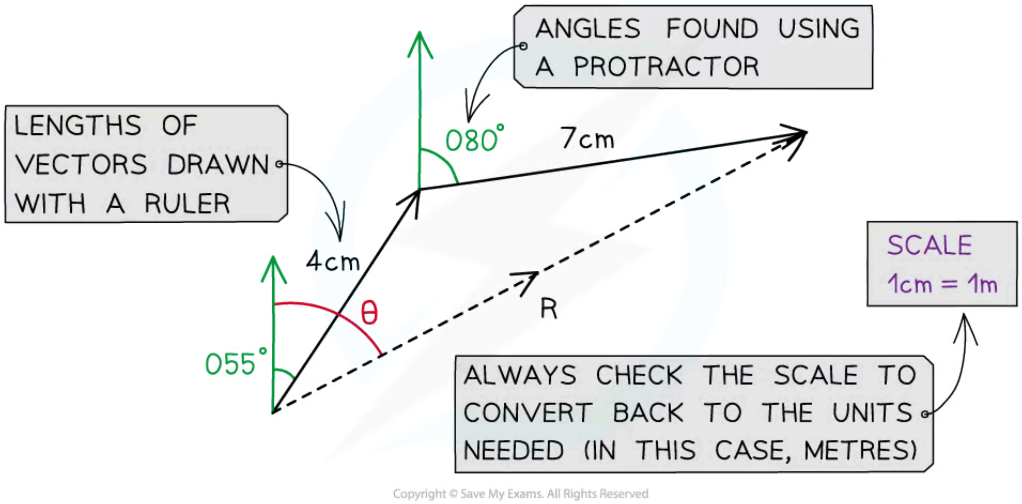
Scale Diagrams

- There are two methods that can be used to combine or resolve vectors
  - **Calculation** – if the vectors are perpendicular
  - **Scale drawing** – if the vectors are not perpendicular
- Calculating vectors using a scale drawing involves drawings the lengths and angles of the vectors accurately using a sharp **pencil**, **ruler** and **protractor**



Vectors can be determined using scale diagrams

- When two vectors are **not** at right angles, the resultant vector can be calculated using a scale drawing
  - **Step 1:** Link the vectors head-to-tail if they aren't already
  - **Step 2:** Draw the resultant vector using the triangle or parallelogram method
  - **Step 3:** Measure the length of the resultant vector using a ruler
  - **Step 4:** Measure the angle of the resultant vector (from North if it is a bearing) using a protractor



A scale drawing of two vector additions. The magnitude of resultant vector R is found using a rule and its direction is found using a protractor

- Note that with scale drawings, a scale may be given for the diagram such as 1 cm = 1 km since only limited lengths can be measured using a ruler
- The final answer is always converted back to the units needed in the diagram
  - Eg. For a scale of 1 cm = 2 km, a resultant vector with a length of 5 cm measured on your ruler is actually 10 km in the scenario

Your notes

Worked example

A hiker walks a distance of 6 km due east and 10 km due north.

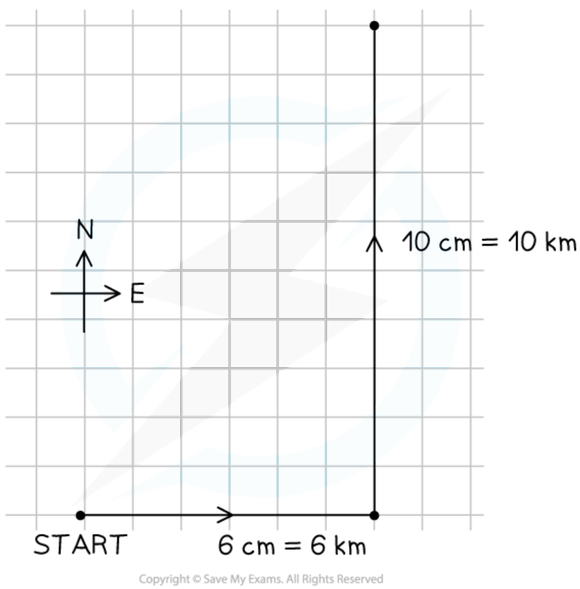
By making a scale drawing of their route, find the magnitude of their displacement and its direction from the horizontal.

Answer:

Step 1: Choose a sensible scale

- The distances are 6 and 10 km, so a scale of 1 cm = 1 km will fit easily on the page, but be large enough for an accurate scale drawing

Step 2: Draw the two components using a ruler and make the measurements accurate to 1 mm



Step 3: Add the resultant vector, remembering the start and finish points of the journey

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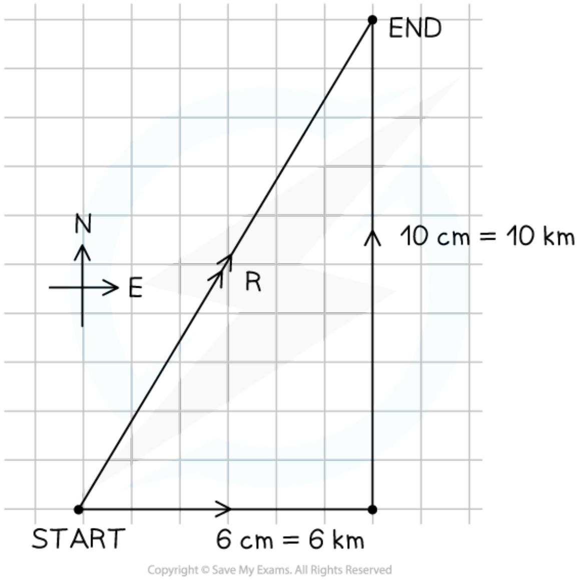
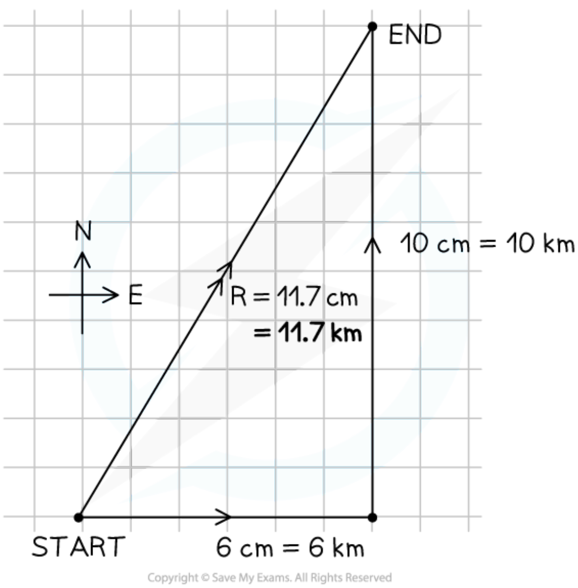


Step 4: Carefully measure the length of the resultant and convert using the scale



Step 5: Measure the angle between the vector and the horizontal line

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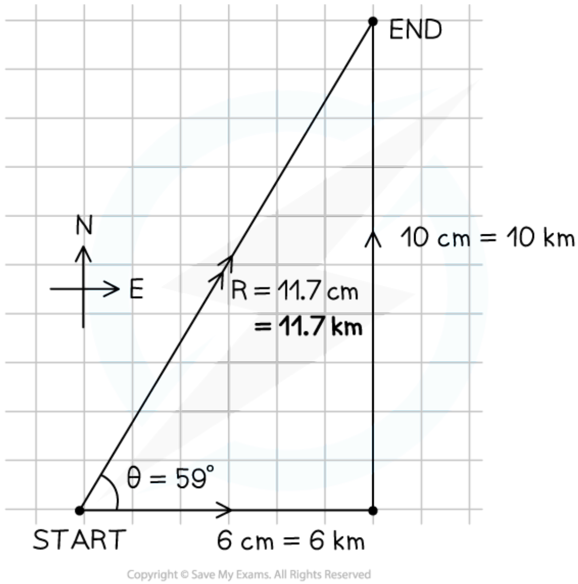

Step 6: Write the complete answer, giving both magnitude and direction

- Magnitude: R = 11.7 km
- Direction: θ = 59°

Examiner Tip

It should be noted that some of the examples used on this page demonstrate the use of scale diagrams where the vectors are placed at right angles - it would be quicker to determine the resultant force of these via calculation as simple trigonometry can be used

Scale diagram questions will typically involve vector triangles that do not contain a right angle