

**IB HL**

**C** 6 hours **2** 35 questions

Structured Questions

# **Gravitational Fields**

Newton's Law of Gravitation / Gravitational Field Strength / Gravitational Field Lines / Gravitational Potential (HL) / Gravitational Potential Energy in a Non-Uniform Field (HL) / Gravitational Potential Energy Equation (HL) / Gravitational Potential Gradient (HL) / Gravitational Equipotential Surfaces (HL) / Kepler's Laws of Planetary Motion / Escape Speed (HL) / Orbital Motion, Speed & Energy (HL) /…



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# **Easy Questions**

**1 (a)** State Newton's Law of Gravitation.

**(2 marks)**

**(b)** Newton's Law of Gravitation can also be written in equation form:

$$
F = G \frac{Mm}{r^2}
$$

Match the terms in the equation with the correct definition and unit:







**(4 marks)**



**(c)** Newton's Law of Gravitation applies to point masses. Although planets are not point masses, the law also applies to planets orbiting the sun.

State why Newton's Law of Gravitation can apply to planets.

**(1 mark)**

(d) The mass of the Earth is 6.0  $\times$  10<sup>24</sup> kg. A satellite of mass 5000 kg is orbiting at a height of 8500 km above the centre of the Earth.

Calculate the gravitational force between the Earth and the satellite.

**(4 marks)**



**2 (a)** The circular motion of a moon in orbit around a planet can be described by:

$$
V = \sqrt{\frac{GM}{r}}
$$

Define each of the terms in the equation above and give the unit:



**(4 marks)**

**(b)** The moon Europa orbits the planet Jupiter at a distance of 670 900 km. The mass of Jupiter is 1.898  $\times$  10<sup>27</sup> kg.

Calculate the linear velocity of Europa.



**(c)** The mass of Europa is  $4.8 \times 10^{22}$  kg.

Calculate the gravitational force between Jupiter and Europa.

# **(2 marks)**

**(d)** A second, hypothetical planet orbits Jupiter at a radius twice that of Europa, with the same mass. The gravitational force between two bodies is based on a 1  $\overline{r^2}$  rule.

Determine the force between Jupiter and the second planet as a fraction of the the force between Europa and Jupiter.

**(2 marks)**



# **3 (a)** Complete the definition of Kepler's third law using words or phrases from the selection below:

For planets or satellites in a ........................... about the same central body, the ........................... of the time period is ........................... to the ........................... of the radius of the orbit.



**(b)** Kepler's third law can also be represented by the equation:

$$
T^2 = \frac{4\,\pi^2 r^3}{GM}
$$

Define each of the terms in the equation above and give the unit:





# **(4 marks)**

**(3 marks)**

**(c)** Venus has an orbital period, *T* of 0.61 years and its orbital radius, *r* is 0.72 AU from the Sun.

Using these numbers, show that Kepler's Third Law,  $T^2$   $\propto$   $r^3$  is true for Venus. No unit conversions are necessary.

**(d)** Kepler's Third Law  $T^2 \propto r^3$  can be represented graphically on log paper.

On the axes below, sketch a graph of  $T^2 \propto r^3$  for our solar system, marking on the position of the Earth.







**4 (a)** A satellite orbits the Earth in a clockwise direction.



Show on the diagram:

(i) (ii) The centripetal force acting on the satellite when it is in orbit, *F*. [2] The linear velocity of the satellite when it is in orbit, *v*. [2]

- **(4 marks)**
- **(b)** State the name of the force which provides the centripetal force required to keep the satellite orbiting in a circular path.

**(1 mark)**



**(c)** The satellite has a mass of 7000 kg is in geostationary orbit and is constantly fixed above the same point on the Earth's surface. The radius of the geostationary orbit is 42 000 km. The Earth has a mass of 6.0 × 10 $^{24}$  kg.

Calculate the force required to keep the satellite in this orbit.

# **(3 marks)**

**(d)** All satellites in geostationary orbit are found at the same distance from the centre of the Earth, and are travelling at the same speed.

The equation linking speed of a satellite *v* and it's orbital radius, *r* is:

$$
v^2 = \frac{GM}{r}
$$

where *G* is the gravitational constant and *M* is the mass of the Earth.

Discuss why the speed is the same for every satellite in geostationary orbit, including the relevance of the satellite's mass.

**(2 marks)**



**5 (a)** Define the following terms:

(i) (ii) Gravitational field strength Gravitational field [2] [2]

**(4 marks)**

**(b)** Gravitational field strength can be written in equation form as:

$$
g = \frac{F}{m}
$$

Define each of the terms in the equation above and give the unit:





**(c)** An astronaut of mass 80 kg stands on the Moon which has a gravitational field strength of 1.6 N  $kg^{-1}$ .

Calculate the weight of the astronaut on the Moon.

**(3 marks)**

(d) The mass of the Earth is 5.972  $\times$  10<sup>24</sup> kg and sea level on the surface of the Earth is 6371 km.

Show that the gravitational field strength,  $g$ , is about 9.86 N kg<sup>-1</sup> at sea level.



**6 (a)** Define the term gravitational field.

**(2 marks)**

**(b)** An equation to describe field strength is:

$$
field \ strength = \frac{X}{Y}
$$

Define X and Y in terms of a gravitational field.

**(2 marks)**

**(c)** Based on your answer to part (b), define the terms in the following equation:

$$
g = \frac{F}{m}
$$

**(1 mark)**

**(d)** The following text is about uniform gravitational fields.

Complete the following sentences by circling the correct words:

A gravitational field is a region of space in which objects with **mass / charge** will experience a force.

The direction of the gravitational field is always directed **away from / towards** the centre of the mass.

Gravitational forces are always **attractive / repulsive** and cannot be **attractive / repulsive**.



**7 (a)** State the definition for the gravitational potential at a point.



**(b)** Explain why gravitational potential is always negative.

#### **(2 marks)**

**(c)** A satellite orbiting the moon, M, is moved from orbit A to orbit B:



The gravitational potential due to the moon of each of these orbits is:

Orbit A:  $-2.10$  MJ  $\text{kg}^{-1}$ Orbit B:  $-1.65$  MJ kg<sup>-1</sup>

Calculate the gravitational potential difference as the satellite moves from orbit A to orbit B.



**(d)** The satellite has a mass of 950 kg.

Calculate the work done in moving the satellite from orbit A to orbit B.

**(2 marks)**



**8 (a)** The gravitational potential,  $V_g^{}$  around a planet can be calculated using the equation:

$$
V_g = -\frac{Gm}{r}
$$

Where *G* is the gravitational constant, *m* is the mass of the planet and *r* is the distance from the centre of the planet.

The mass of the Earth is 5.97  $\times$  10<sup>24</sup> kg.

Calculate the gravitational potential at a point 4.23  $\times$  10<sup>7</sup> m from the centre of the Earth.

**(2 marks)**

**(b)** The gravitational potential on the surface of the Earth is –6.25  $\times$  10<sup>7</sup> J kg<sup>-1</sup>.

Calculate the gravitational potential difference between the surface of the Earth and a point 4.23  $\times$  10<sup>7</sup> m from the centre of the Earth from part (a).

**(3 marks)**

**(c)** Calculate the work done in taking a 5.0 kg mass from the surface of the Earth to a point 4.23  $\times$  10<sup>7</sup> m from the centre of the Earth.

**(2 marks)**



**(d)** (i) (ii) (iii) State the magnitude of the gravitational potential at a point where the Earth's gravitational effect is negligible. [1] Calculate the gravitational potential difference between the Earth's surface (from part b) and the point where the Earth's gravitational effect is negligible [3] Calculate the work done in taking the 5.0 kg mass from the surface of the Earth to the point where the Earth's gravitational effect is negligible. [2]

**(6 marks)**

**9 (a)** A planet is in orbit around a star.

State two reasons why a centripetal force is needed for a planet to maintain a circular orbit.





**(d)** Calculate the escape speed of the spacecraft from the planet-star system.

**(2 marks)**



**10 (a)** The moon Ganymede moves around the planet Jupiter in a circular orbit.

State the force responsible for this motion and outline why it does no work on Ganymede.

**(2 marks)**

**(b)** The graph shows the variation of the gravitational potential between Jupiter and Ganymede with distance from the centre of Jupiter. The distance from Jupiter is expressed as a fraction of the total distance between the centre of Jupiter and the centre of Ganymede.



Identify a word equation for the distance at which the gravitational force between Jupiter and Ganymede is zero.

**(1 mark)**



**(c)** By equating the expressions for the gravitational force due to Jupiter and Ganymede, where gravitational potential is at a maximum, show that:

$$
\frac{M_J}{(0.9)^2} = \frac{M_G}{(0.1)^2}
$$

Where  $M_G$  is the mass of Ganymede and  $M_J$  is the mass of Jupiter.

**(4 marks)**

**(d)** Taking the mass of Jupiter as 1.898  $\times$  10<sup>27</sup> kg, determine the mass of Ganymede.

**(2 marks)**



**11 (a)** Titan is a moon of Saturn and is much smaller than Earth. The radius of Earth is 2.48 times the radius of Titan and it is 40 times more massive.



Determine the ratio of the escape velocities of Earth and Titan

**(2 marks)**

**(b)** Hence calculate the escape velocity from Titan.

#### **(1 mark)**

**(c)** Titan orbits Saturn at a distance between the centres of mass, *R* with an orbital period of revolution *T*.

State which forces must be in equilibrium for this orbit to be maintained.

**(1 mark)**

**(d)** Hence, by using equations for centripetal acceleration and gravitational force, derive an expression in terms of the mass of Saturn for the time period, *T*.



**12 (a)** The forces between two masses are described as having an inverse-square relationship.



**(2 marks)**

**(b)** A planet with mass *M* and radius *R* has escape velocity  $v_{esc}$ .

A prototype rocket is launched with a speed which is only half of the escape speed.

Write an expression, in terms of *M* and *R*, which can be used to calculate the total energy of the rocket at its maximum height.

**(2 marks)**

**(c)** Write an expression in terms of *M* and *R* to calculate the total energy at the launch of the prototype rocket in part (b).

**(4 marks)**

**(d)** The prototype rocket, fired with half the escape velocity on Earth, launches with the energy determined in part (c). Assume that air resistance is negligible.

Determine, in terms of *R*, the maximum height *h* which the rocket achieves upon launch.





# **Medium Questions**

**1 (a)** The distance from the Earth to the Sun is 1.5  $\times$  10<sup>11</sup> m. The mass of the Earth is 6  $\times$  10<sup>24</sup> kg and the mass of the Sun is 3.3  $\times$  10<sup>5</sup> times the mass of the Earth.

Estimate the gravitational force between the Sun and the Earth.

#### **(2 marks)**

**(b)** Mars is 1.5 times further away from the Sun than the Earth and is 10 times lighter than Earth.

Predict the gravitational force between Mars and the Sun.

**(1 mark)**

**(c)** Determine the acceleration of free fall on a planet 20 times as massive as the Earth and with a radius 10 times larger.

**(2 marks)**

**(d)** Calculate the orbital speed of the Earth around the Sun.



**2 (a)** A satellite orbits the Earth with mass *M* above the equator with a period, *T* equal to 48 hours. The mass of the Earth is 5.972  $\times$  10<sup>24</sup> kg.



**(4 marks)**

**(b)** The mean radius of Earth is  $6.37 \times 10^6$  m.

Calculate the height of the satellite above the Earth's surface.

**(3 marks)**

**(c)** The Hubble Space Telescope is in orbit around the Earth at a height of 490 km above the Earth's surface.

Calculate Hubble's speed.

**(3 marks)**

**(d)** Calculate the magnitude of the gravitational field on the Hubble Space Telescope at this height above the Earth's surface.

**(2 marks)**







**(3 marks)**

**(b)** Show that the orbital speed of Europa is 14 km  $s^{-1}$ .

**(3 marks)**

**(c)** Deduce the mass of Jupiter.

**(3 marks)**

**(d)** Ganymede is the largest of Jupiter's Moons. It has an orbital period of 7.15 days and an orbital speed of 10.880 km  $s^{-1}$ .

Calculate the orbital radius of Ganymede, in km.



**4 (a)** Define Newton's universal law of gravitation.

#### **(2 marks)**

**(b)** The diagram shows a satellite orbiting the Earth. The satellite is part of the network of global–positioning satellites (GPS) that transmit radio signals used to locate the position of receivers that are located on the Earth.



When the satellite is directly overhead the microwave signal reaches the receiver 62 ms after leaving the satellite.

Calculate the height of the satellite above the surface of the Earth.

**(2 marks)**

**(c)** Explain why the satellite is accelerating towards the centre of the Earth even though its orbital speed is constant.



**(d)** The radius of Earth is  $6.4 \times 10^6$  m.

Calculate the gravitational field strength of the Earth at the position of the satellite.

Mass of Earth =  $6.0 \times 10^{24}$  kg

**(2 marks)**



**5 (a)** A satellite is in a circular orbit around a planet of mass *M*.



Sketch arrows to represent the velocity and acceleration of the satellite.

**(b)** Show that the angular speed, *ω* is related to the orbital radius *r* by

$$
r = \sqrt[3]{\frac{GM}{\omega^2}}
$$

**(2 marks)**

**(c)** Because of friction with the upper atmosphere, the satellite slowly moves into another circular orbit with a smaller radius before.

Suggest the effect of this on the satellites angular speed.

**(d)** Titus and Enceladus are two of Saturn's moons. Data about these moons are given in the table.



Determine the mass of Saturn.



(i) State what is meant by the gravitational field strength at a point.

**6 (a)**

(ii) Show that the potential *V* at a distance *r* above the surface of a planet with radius *R* is given by  $g(R + r)$ .

[2]

[1]

# **(3 marks)**

**(b)** Sketch a graph on the axes provided below to show the relationship between the gravitational potential *V* with distance *r* above the surface of the planet.



**(2 marks)**



**(c)** An asteroid, with an initial negligible speed, is gathering pace and is now on a collisioncourse with the planet.

Estimate the speed of the asteroid when it reaches the top of the planet's atmosphere, which stretches for 15 km above the planet's surface.

Use the following data:

- Radius of the planet =  $3.0 \times 10^6$  m
- Gravitational field strength at the top of the atmosphere = 2.5 N  $kg^{-1}$

**(d)** As the asteroid enters the planet's atmosphere, it begins as a small point of light which grows much brighter and faster as it moves towards the surface of the planet.

Discuss these observations. Your answer should make reference to *g* as well as the effect of the planet's atmosphere.

**(4 marks)**



**7 (a)** The diagram shows the gravitational field produced by planet P.



Outline how this diagram shows that the gravitational field strength of planet P decreases with distance from the surface.

**(2 marks)**

**(b)** The diagram shows part of the surface of planet P. The gravitational potential at the surface of planet P is –6 V and the gravitational potential at point X is –2 V.



On the grid, sketch and label the equipotential surface corresponding to a gravitational potential of –4 V.

### **(3 marks)**

**(c)** A meteorite, very far from planet P, begins to fall to the surface with a negligibly small initial speed. The mass of planet P is 3.0  $\times$  10<sup>21</sup> kg and its radius is 2.3  $\times$  10<sup>6</sup> m.

Calculate the speed at which the meteorite will hit the surface, assuming planet P has no atmosphere.

**(3 marks)**

**(d)** Without detailed calculation, state and explain the effect on the meteorite's impact velocity if planet P's mass was twice as large.

**(2 marks)**







**(c)** While the Earth orbits the Sun, terrestrial shuttles often enter orbit around Earth. One such shuttle is launched with a kinetic energy  $E_{\mathsf{K}}$  given by the expression below:

$$
E_K = \frac{5GM_Em}{8R_E}
$$

where *G* is the gravitational constant,  $M_{\mathsf{E}}$  is the mass of Earth, and  $m$  is the mass of the shuttle. Deduce that the shuttle cannot escape the gravitational field of the Earth.

**(2 marks)**

**(d)** Show that, if the shuttle enters an orbit of radius *R* about the Earth, then its total energy is given by -- $GM_{\overline E}$ m  $\overline{2R}^-$  stating an appropriate assumption required.





**9 (a)** Evaluate this statement of Newton's law of gravitation: "The gravitational force between two masses is proportional to the masses and inversely proportional to the square of the distance between them."

**(2 marks)**

**(b)** A satellite of mass *m* orbits a planet of mass *M*. If the orbital radius is *R* and the orbital period is *T*, show that the ratio  $R^3$  $\overline{I^2}$  is constant.

**(3 marks)**

**(c)** Calculate the change in gravitational potential energy of the satellite, of mass 39 kg, as it moves from an orbit of height 1100 km above the Earth's surface to one of height 2100 km.

Use the following data:

- Mass of Earth =  $6.0 \times 10^{24}$  kg
- Average radius of Earth = 6.4  $\times$  10 $^{6}$  m

**(3 marks)**

**(d)** Explain whether the gravitational potential energy has increased, decreased or stayed the same when the orbit changes as in part (c).

**(2 marks)**



**10 (a)** Binary star systems involve two stars that orbit a common centre of gravity. One such system is shown.



Each star has a mass *M* and orbital radius *r*, such that their separation is 2*r*.

Deduce that the time period *T* of each star's orbit is related to the orbital radius *r* by the following equation:

$$
T^2 = \frac{16\pi^2 r^3}{GM}
$$

**(3 marks)**

**(b)** Show that the kinetic energy of each star in the binary system is given by:

$$
E_K = \frac{GM^2}{8r}
$$

**(c)** Hence, show that the total energy of the binary star system is given by the equation:

$$
E = -\frac{GM^2}{4r}
$$

**(2 marks)**

**(d)** The binary system radiates energy in the form of gravitational waves.

Deduce that the stars move closer to each other as the binary system emits gravitational waves.



# **Hard Questions**

- **1 (a)** The gravitational field strength on the moon's surface is 1.63 N kg<sup>-1</sup>. It has a diameter of 3480 km.
	- (i) (ii) Calculate the mass of the moon [2] State the assumption necessary for part (i) [1]

**(3 marks)**

**(b)** The ISS orbits the Earth at an average distance of 408 km from the surface of the Earth.





The following data are available:

- Average distance between the centre of the Earth and the centre of the Moon = 3.80  $\times$  10 $^{8}$  m
- Mass of the Earth = 5.97  $\times$  10<sup>24</sup> kg
- Radius of the Earth =  $6.37 \times 10^6$  m

Calculate the maximum gravitational field strength experienced by the ISS. You may assume that both the Moon and the ISS can be positioned at any point on their orbital path.

**(4 marks)**

**(c)** Show that the gravitational field strength *g* is proportional to the radius of a planet *r* and its density *ρ.*

**(3 marks)**

**(d)** Two planets X and Y are being compared by a group of astronomers. They have different masses.

Planet X has a density *ρ* and the gravitational field strength on its surface is *g*. The density of planet Y is three times that of planet X and the gravitational field strength on its surface is 9 times that of planet X.

Use the equation you derived in part (c) to show that the mass of planet Y is roughly 80 times larger than the mass of planet X.



**(4 marks)**



**2 (a)** The gravitational field strength on the surface of a particular moon is 2.5 N kg<sup>-1</sup>. The moon orbits a planet of similar density, but the diameter of the planet is 50 times greater than the moon.

Calculate the gravitational field strength at the surface of the planet.

**(3 marks)**

**(b)** Two planets P and Q are in concentric circular orbits about a star S.



The radius of P's orbit is *R* and the radius of Q's orbit is 2*R*. The gravitational force between P and Q is *F* when angle SPQ is 90° as shown.

Deduce an equation for the gravitational force between P and Q, in terms of *F*, when they are nearest to each other.



**(c)** Planet P is twice the mass of planet Q.

Sketch the gravitational field lines between the two planets on the image below.

Label the approximate position of the neutral point.



**(2 marks)**



**3 (a)** The distance between the Sun and Mercury varies from 4.60  $\times$  10<sup>10</sup> m to 6.98  $\times$  10<sup>10</sup> m. The gravitational attraction between them is *F* when they are closest together.

Show that the minimum gravitational force between the Sun and Mercury is about 43% of *F.*

**(3 marks)**

**(b)** Mercury has a mass of 3.30  $\times$  10<sup>23</sup> kg and a mean diameter of 4880 km. A rock is projected from its surface vertically upwards with a velocity of 6.0 m  $\mathsf{s}^{-1}.$ 

Calculate how long it will take for the rock to return to Mercury's surface.

**(3 marks)**

**(c)** Venus is approximately 5.00  $\times$  10<sup>10</sup> m from Mercury and has a mass of 4.87  $\times$  10<sup>24</sup> kg. A satellite of mass 1.50 × 10<sup>4</sup> kg is momentarily at point P, which is 1.75 × 10<sup>10</sup> from Mercury, which itself has a mass of 3.30  $\times$  10<sup>23</sup> kg.



Calculate the magnitude of the resultant gravitational force exerted on the satellite when it is momentarily at point P.

**(6 marks)**



**4 (a)** A student has two unequal, uniform lead spheres.

Lead has a density of 11.3  $\times$  10<sup>3</sup> kg m<sup>-3</sup>. The larger sphere has a radius of 200 mm and a mass of 170 kg. The smaller sphere has a radius of 55 mm.

The surfaces of two lead spheres are in contact with each other, and a third, iron sphere of mass 20 kg and radius 70 mm is positioned such that the centre of mass of all three spheres lie on the same straight line.



Calculate the distance between the surface of the iron sphere and the surface of the larger lead sphere which would result in no gravitational force being exerted on the larger sphere.

**(3 marks)**

**(b)** Calculate the resultant gravitational field strength on the surface of the iron sphere.



**(c)** The smaller lead sphere is removed. The separation distance between the surface of the iron sphere and the large lead sphere is *r*.

Sketch a graph on the axes provided showing the variation of gravitational field strength *g* between the surface of the iron sphere and the surface of the lead sphere.





**5 (a)** A kilogram mass rests on the surface of the Earth. A spherical region S, whose centre of mass is underneath the Earth's surface at a distance of 3.5 km, has a radius of 2 km. The density of rock in this region is 2500 kg  $\text{m}^{-3}$ .



Determine the size of the force exerted on the kilogram mass by the matter enclosed in S, justifying any approximations.

# **(3 marks)**

- **(b)** If the region S consisted of oil of density 900 kg  $m^{-3}$  instead of rock, the force recorded on the kilogram mass would reduce by approximately 2.9  $\times$  10<sup>-4</sup> N.
	- (i) Suggest how gravity meters may be used in oil prospecting.
	- (ii) Determine the uncertainty within which the acceleration of free fall needs to be measured if the meters are to detect such a quantity of oil.

[2]

[1]



**(c)** A spherical hollow is made in a lead sphere of radius *R,* such that its surface touches the outside surface of the lead sphere on one side and passes through its centre on the opposite side. The mass of the sphere before it was made hollow is *M*.



Show that the magnitude of the force *F* exerted by the spherical hollow on a small mass *m*, placed at a distance *d* from its centre, is given by:

$$
F = \frac{GMm}{d^2} \left( 1 - \frac{1}{8} \left( \frac{2d}{2d - R} \right)^2 \right)
$$
\n[4]

**(4 marks)**



#### **6 (a)** Scientists want to put a satellite in orbit around planet Venus.

Justify how Newton's law of gravitation can be applied to a satellite orbiting Venus, when neither the satellite, nor the planet are point masses.

### **(2 marks)**

**(b)** The satellite's orbital time, *T*, and its orbital radius, *R*, are linked by the equation:

$$
T^2 = kR^3
$$

Venus has a mass of 4.9  $\times$  10<sup>24</sup> kg.

Determine the value of the constant *k*, and give the units in SI base units.

**(6 marks)**

**(c)** One day on Venus is equal to 116 Earth days and 18 Earth hours.

Determine the orbital speed of the satellite in m  $s^{-1}$ .

**(2 marks)**



**7 (a)** An object has a weight of 100 N at a distance of 200 km above the centre of a small planet.

Sketch a labelled graph to show the relationship between the gravitational force, *F*, between two masses and the distance, *r*, between them. Mark at least three points on the graph using the information provided in the question.

# **(3 marks)**

**(b)** The distance along the Earth's surface from the North Pole to the Equator is 1  $\times$  10<sup>7</sup> m.

Calculate the mass of the Earth.

**(2 marks)**

(c) A rocket is sent from the Earth to the moon. The moon has a radius of  $1.74 \times 10^6$  m and the gravitational field strength on its surface is 1.62 N  $\text{kg}^{-1}$ . The radius of the Earth is 6370 km.





The distance between the centre of the Earth and the centre of the moon is 385 000 km.

Calculate the distance above the Earth's surface where there is no resultant gravitational field strength acting on the rocket.

**(4 marks)**



**(d)** The rocket will require a different amount of fuel to get to the moon than it will to return to the Earth.

Explain which journey will require the most fuel.

**(2 marks)**



**8 (a)** A space shuttle of mass  $2 \times 10^6$  kg is travelling from the Earth to the moon. It accelerates uniformly from launch at 5.25 m  $s^{-2}$ .It has enough propellant to provide thrust for the first 124 seconds.

The mass of the Earth is 5.97  $\times$  10<sup>24</sup> kg and the mean radius is 6.37  $\times$  10<sup>6</sup> m.

Calculate the work done by the rocket during the first 124 seconds after launch. State any assumptions you have made.

**(3 marks)**

**(b)** The Moon has a radius approximately 27% that of the Earth, and a mass of 1.2% that of the Earth.

Calculate the gravitational potential at the surface of the moon in terms of the gravitational potential on the surface of the Earth.

**(2 marks)**

**(c)** The gravitational field strength lines between the Earth and the moon can be drawn on a diagram.





Point P is the neutral point between the Earth and the moon where there is no resultant gravitational field.

Sketch the equipotential lines between the Earth and the moon on the diagram.



**9 (a)** Imagine that it was possible to construct a tunnel through the centre of the earth, connecting a point on the surface to the diametrically opposite point. Assume that the earth is perfectly spherical with an evenly distributed mass and that the mass and volume of the tunnel, air resistance and friction are negligible.

Describe the variation in gravitational field strength and how speed of travel would vary if a person were to jump into the hole.

# **(2 marks)**

**(b)** Within a hollow sphere of uniform density, the gravitational field strength is zero.

Using this information, derive an expression for the gravitational field strength at any point in the tunnel in terms of:

- The distance from the centre of the earth = *r*
- The radius of the earth =  $R_{\rm e}$
- The gravitational field strength on the surface of the earth = *g* surf

# **(3 marks)**

**(c)** This case is analogous to a mass bouncing on a spring where  $F = ma = -kx$ , and where in this situation  $k$   $\!=$   $\cdot$ mg surf  $\overline{R}$ . e

The time period for a mass on a spring, *T*, is equal to  $2\pi\sqrt{\frac{m}{L}}$  $\overline{k}$  . For a satellite in orbit at a distance *r*, from the centre of a large body with mass *M*, the orbital speed can be obtained as  $v =$ GM  $\frac{1}{r}$ .

Using this information, show that the time for a traveller to reach the other end of the tunnel is the same as the time taken for a satellite in orbit just above the surface of the Earth to travel through half its orbit.

**(1 mark)**



**10 (a)** In an experiment a coin of 0.5 cm in diameter held at a distance of 55.3 cm from the eye appeared to be exactly the same size as the Moon. The coin was measured using a micrometer screw gauge and the distance to the eye using a metre rule.

The distance to the Moon is 384 400 km and the gravitational field strength on the surface of the Moon is 1.63 N  $\text{kg}^{-1}$ .

By referring to the data

- (i) Calculate the mass of the Moon.
- (ii) Determine the percentage error in the answer.

**(6 marks)**

[3]

[3]

**(b)** The gravitational field strength on the surface of a particular planet is 1.6 N kg<sup>-1</sup>. The planet orbits a star of similar density, but the diameter of the star is 100 times greater than the planet.

Calculate the gravitational field strength at the surface of the star.



**(5 marks)**



- **11 (a)** Work is done on the Soyuz spacecraft which is used to transport astronauts to the International Space Station (ISS). The ISS orbits the Earth at a height of 400 km above the Earth's surface. The Soyuz spacecraft has a mass of 7150 kg.
	- Mass of the Earth = 5.97  $\times$  10<sup>24</sup> kg
	- Mean radius of the Earth =  $6.37 \times 10^6$  m

For one trip from Earth to the ISS:

- (i) Calculate the work done in taking the Soyuz spacecraft from the Earth's surface to the ISS.
- (ii) State an assumption that was made in the calculation.

**(4 marks)**

[3]

 $[1]$ 

- **(b)** Consider the work done on the Soyuz rocket and the ISS.
	- (i) In reality, it takes less work than calculated in part (c) to move the Soyuz rocket from the surface of the Earth to the ISS. Discuss a possible reason for the difference in value.

[2]

(ii) Explain why there is no work done by the ISS when it maintains a constant orbit around the Earth.

[2]



**(4 marks)**



**12 (a)** A binary planet system consists of two stars, A and B.



A has a mass of mass 4.0 × 10 $^{30}$  kg and B has a mass of 8.0 × 10 $^{30}$  kg. The centres of the stars are separated by a distance of 2 × 10 $^8$  km.

Calculate the gravitational potential at the midpoint between the stars.

# **(2 marks)**

**(b)** The amount of energy required to send a space probe of mass 1800 kg from the surface of star A to the midpoint between stars A and B is 4.2  $\times$  10<sup>11</sup> J.

Calculate the gravitational field strength on the surface of star A.

**(4 marks)**

**(c)** The two stars are drifting apart.

Calculate how far star A will have drifted at the point where its gravitational potential energy has decreased by 10 %.



**13 (a)** The orbits of the Earth and Jupiter are very nearly circular, with radii of 150  $\times$  10<sup>9</sup> m and 778  $\times$  10 $^9$  m respectively. It takes Jupiter 11.8 years to complete a full orbit of the Sun.

Show that the values in this question are consistent with Kepler's third law.

# **(2 marks)**

**(b)** Data from the orbits of different planets around our Sun is plotted in a graph of log( $T^2$ ) against log(R<sup>3</sup>) as shown in the graph below, where *T* is the orbital period and *R* is the radius of the planet's orbit.

The values of *T* and *R* have been squared and cubed respectively due to Kepler's Third Law stating that:

$$
T^2 = \frac{4\pi^2 r^3}{GM}
$$





Calculate the percentage error for the mass of the Sun obtained from the graph.

**(4 marks)**

