HL IB Physics

Your notes

Gravitational Fields

Contents

Your notes

Newton's Law of Gravitation

Newton's Law of Gravitation

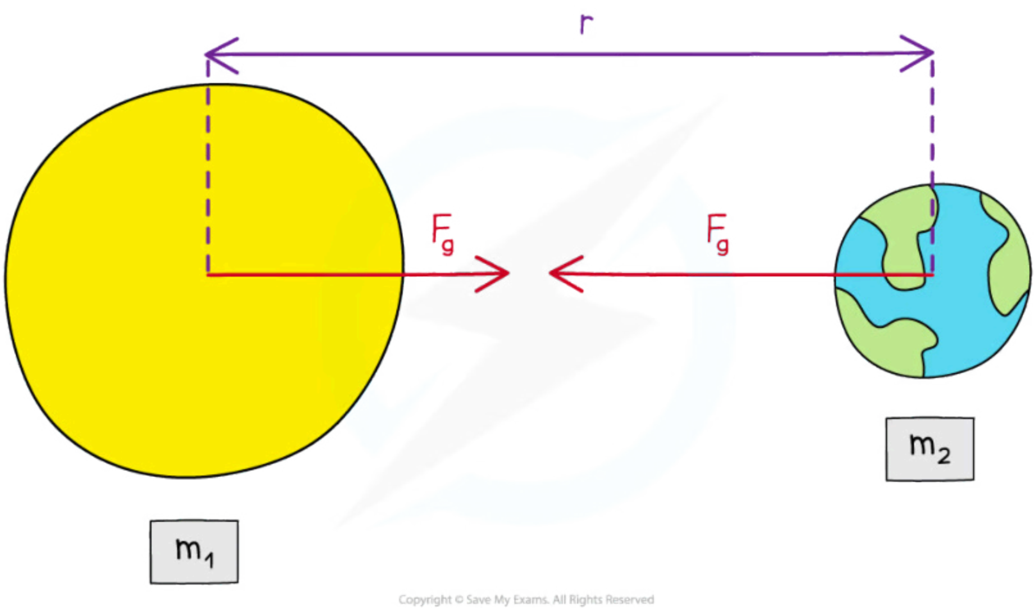
- The gravitational force between two bodies outside a uniform field, e.g. between the Earth and the Sun, is defined by Newton's Law of Gravitation
- Newton's Law of Gravitation states that:

  The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation

- All planets and stars are assumed to be point masses
- In equation form, this can be written as:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Where:
  - F = gravitational force between two masses (N)
  - G = Newton's Gravitational Constant
  - m_1 and m_2 = mass of body 1 and mass of body 2 (kg)
  - r = distance between the centre of the two masses (m)



The gravitational force between two masses outside a uniform field is defined by Newton's Law of Gravitation

- Although planets are not point masses, their separation is much larger than their radius
  - Therefore, Newton's law of gravitation applies to planets orbiting the Sun

- The $F \propto \frac{1}{r^2}$ relation is called the inverse square law
- This means that when a mass is twice as far away from another, its force due to gravity reduces by $(½)^2$ = ¼

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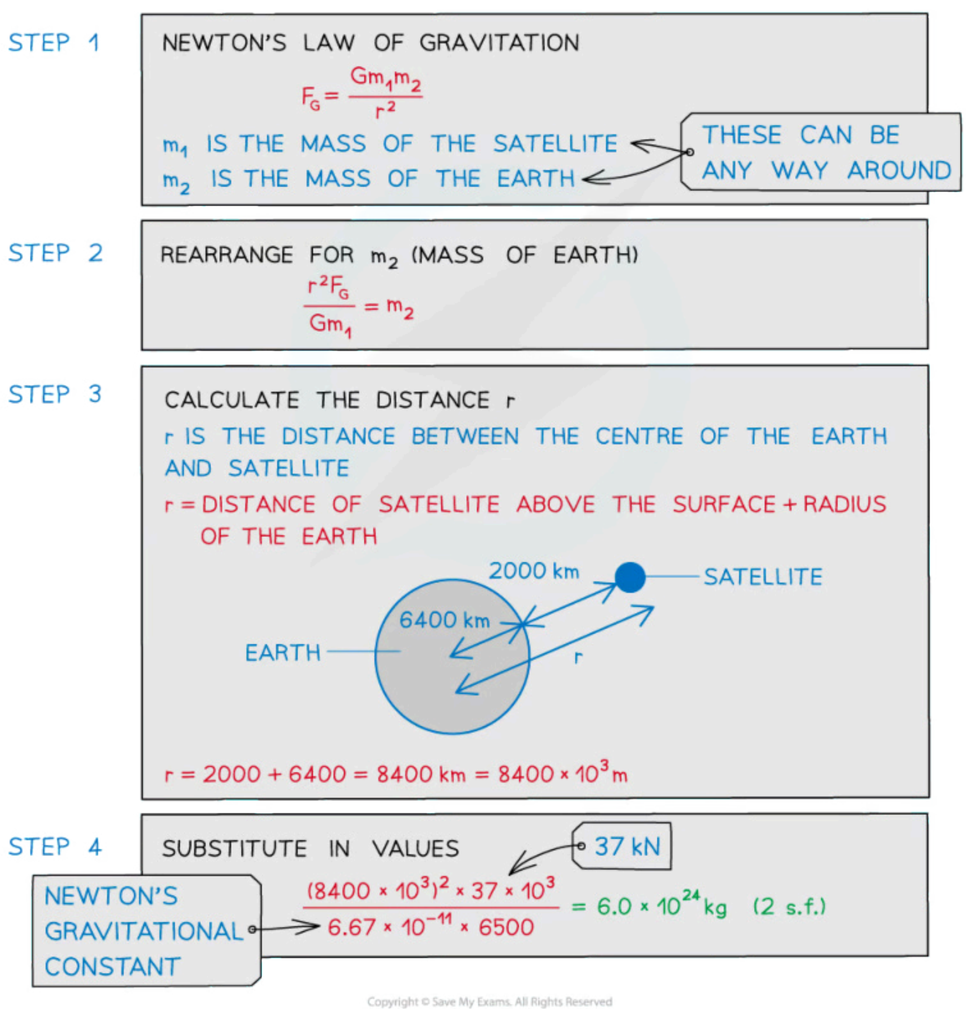
Worked example

A satellite of mass 6500 kg orbits at 2000 km above the Earth's surface. The gravitational force between the Earth and the satellite is 37 kN.

Calculate the mass of the Earth.

Radius of the Earth = 6400 km

Answer:



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Examiner Tip

A common mistake in exams is to forget to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured from the **centre** of the mass, which is from the **centre** of the planet.

Make sure to **square** the separation *r* in the equation!

Your notes

Gravitational Field Strength

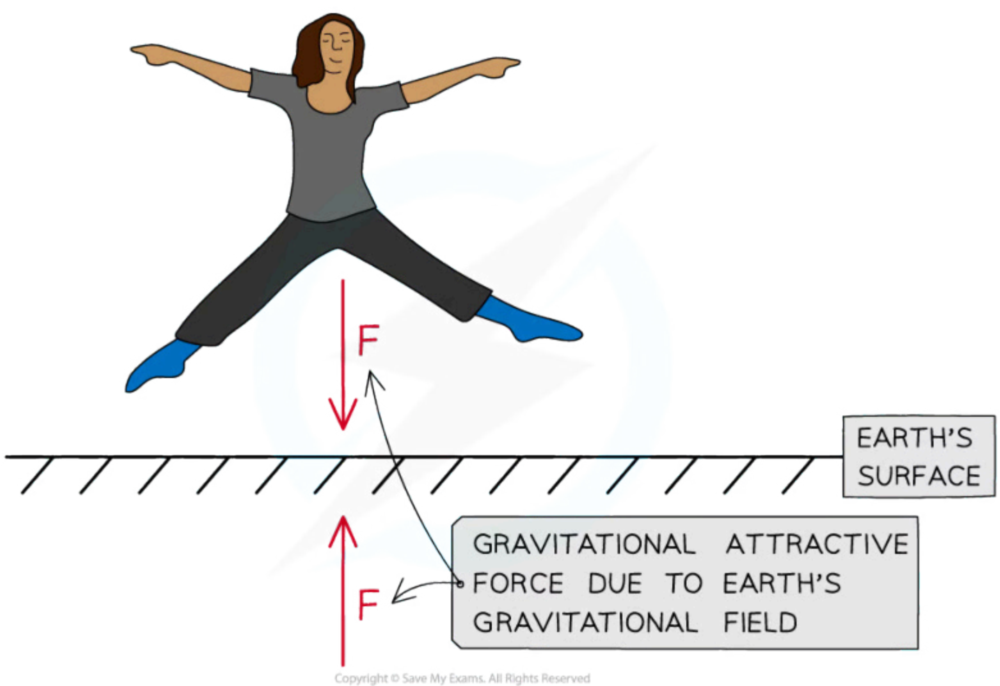
Your notes

Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
  - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- A gravitational field is defined as:

  A region of space where a test mass experiences a force due to the gravitational attraction of another mass

- The direction of the gravitational field is always towards the centre of the mass causing the field
  - Gravitational forces are **always** attractive
- Gravity has an infinite range, meaning it affects **all** objects in the universe
  - There is a **greater** gravitational force around objects with a **large mass** (such as planets)
  - There is a **smaller** gravitational force around objects with a **small mass** (almost negligible for atoms)



The Earth's gravitational field produces an attractive force. The force of gravity is always attractive

- The gravitational field strength at a point is defined as:

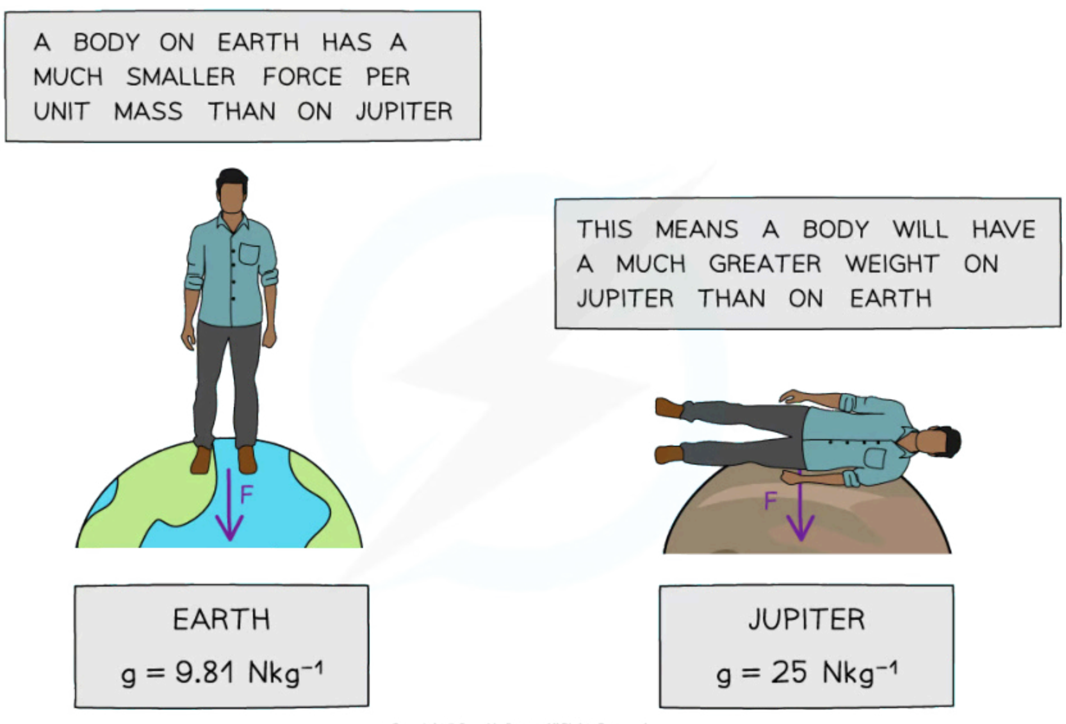
  The force per unit mass experienced by a test mass at that point

- This can be written in equation form as:

Your notes

$$g = \frac{F}{m}$$

- Where:
  - g = gravitational field strength ($N\ kg^{-1}$)
  - F = force due to gravity, or weight (N)
  - m = mass of test mass in the field (kg)
- This equation shows that:
  - On planets with a large value of g, the gravitational force per unit mass is **greater** than on planets with a smaller value of g
- An object's mass remains the **same** at all points in space
  - However, on planets such as Jupiter, the **weight** of an object will be greater than on a less massive planet, such as Earth
  - This means the gravitational force would be so high that humans, for example, would not be able to fully stand up



A person's weight on Jupiter would be so large that a human would be unable to fully stand up

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** r (or diameter) of the planet
  - The **mass** M (or density) of the planet
- This can be shown by equating the equation $F = mg$ with Newton's law of gravitation:

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$$F = \frac{GMm}{r^2}$$

- Substituting the force *F* with the gravitational force *mg* leads to:

$$mg = \frac{GMm}{r^2}$$

- Cancelling the mass of the test mass *m* leads to the equation:

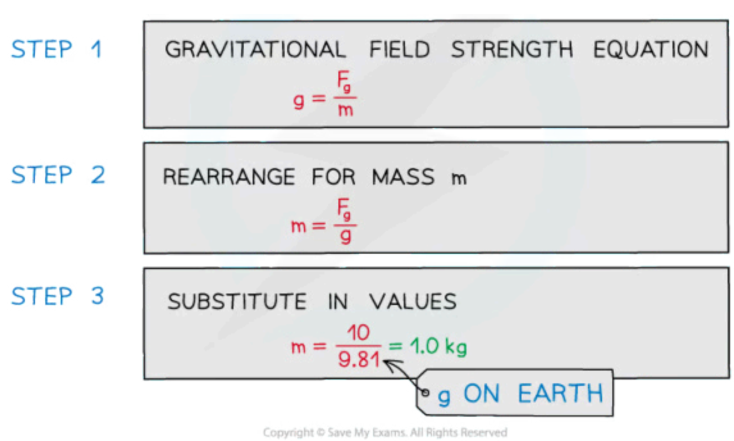
$$g = \frac{GM}{r^2}$$

- Where:
  - *G* = Newton's Gravitational Constant
  - *M = mass of the body causing the field* (kg)
  - *r* = distance from the mass where you are calculating the field strength (m)
- This equation shows that:
  - The gravitational field strength *g* depends only on the mass of the body *M* causing the field
  - Hence, objects with any mass *m* in that field will experience the **same gravitational field strength**
  - The gravitational field strength *g* is **inversely proportional** to the **square** of the radial distance, r^2

Worked example

Calculate the mass of an object with weight 10 N on Earth.

Answer:



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Worked example

The mean density of the Moon is $\frac{3}{5}$ times the mean density of the Earth. The gravitational field strength on the Moon is $\frac{1}{6}$ the gravitational field strength on Earth.

Determine the ratio of the Moon's radius r_M to the Earth's radius r_E.

Answer:

Step 1: Write down the known quantities

- g_M = gravitational field strength on the Moon, ρ_M = mean density of the Moon
- g_E = gravitational field strength on the Earth, ρ_E = mean density of the Earth

$$\rho_M = \frac{3}{5}\rho_E$$

$$g_M = \frac{1}{6}g_E$$

Step 2: Write down the equations for the gravitational field strength, volume and density

Gravitational field strength: $g = \frac{GM}{r^2}$

Volume of a sphere: $V = \frac{4}{3}\pi r^3 \Rightarrow V \propto r^3$

Density: $\rho = \frac{M}{V} \Rightarrow M = \rho V = \frac{4}{3}\pi\rho r^3 \Rightarrow M \propto \rho r^3$

Step 3: Substitute the relationship between M and r into the equation for g

$$g \propto \rho\frac{(r^3)}{r^2} \Rightarrow g \propto \rho r$$

Step 4: Find the ratio of the gravitational field strength

$$g_M \propto \rho_M r_M$$

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$$g_E \propto \rho_E r_E$$

$$g_M = \frac{1}{6} g_E \quad \Rightarrow \quad \rho_M r_M = \frac{1}{6} \rho_E r_E$$

Step 5: Substitute the ratio of the densities into the equation

$$\left(\frac{3}{5}\rho_E\right) r_M = \frac{1}{6} \rho_E r_E$$

$$\frac{3}{5} r_M = \frac{1}{6} r_E$$

Step 6: Calculate the ratio of the radii

$$\frac{r_M}{r_E} = \frac{1}{6} \div \frac{3}{5} = \frac{5}{18} = 0.28$$

Examiner Tip

There is a big difference between *g* and *G* (sometimes referred to as 'little g' and 'big G' respectively), *g* is the gravitational field strength and G is Newton's gravitational constant. Make sure not to use these interchangeably!

Remember the equation $density = \frac{mass}{volume}$, which may come in handy with some calculations.

The equation for the volume of common shapes is in your data booklet.

Your notes

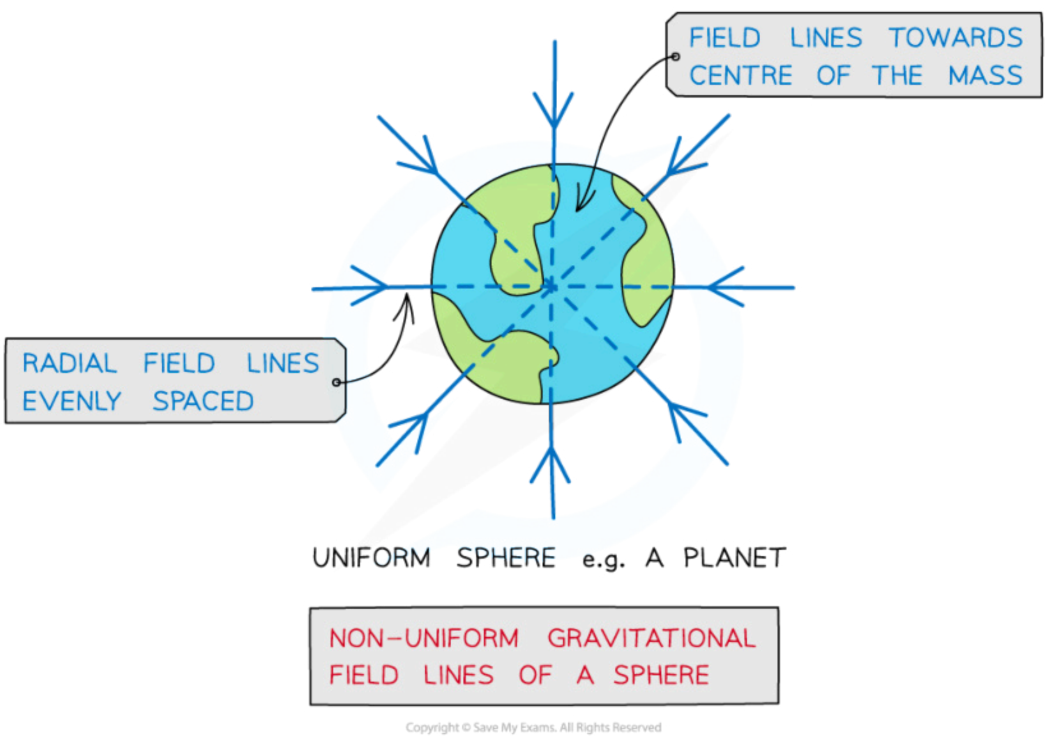
Gravitational Field Lines

Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as a point mass when:

  A body covers a very large distance compared to its size, so, to study its motion, its size or dimensions can be neglected

- An example of this is field lines around planets



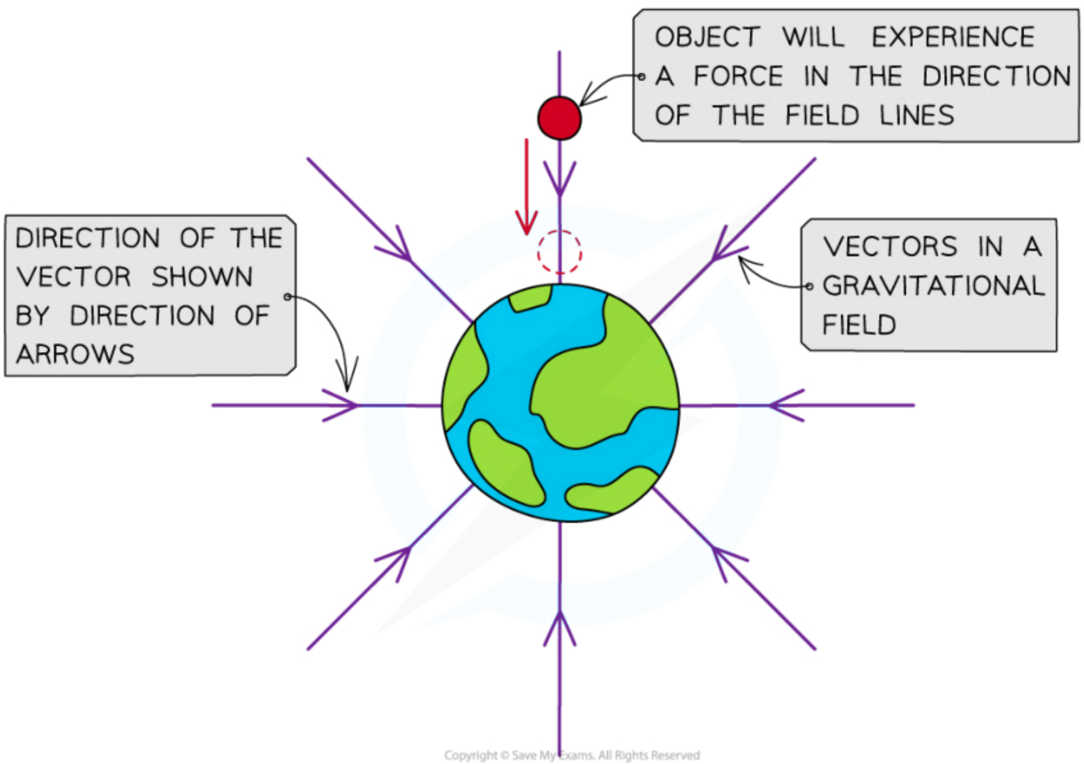
Gravitational field lines around a uniform sphere are identical to those on a point mass

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength *g* is different depending on how far an object is from the centre of mass of the sphere
- Newton's universal law of gravitation is extended to spherical masses of uniform density by assuming that their mass is concentrated at their centre i.e point masses

Your notes

Representing Gravitational Fields

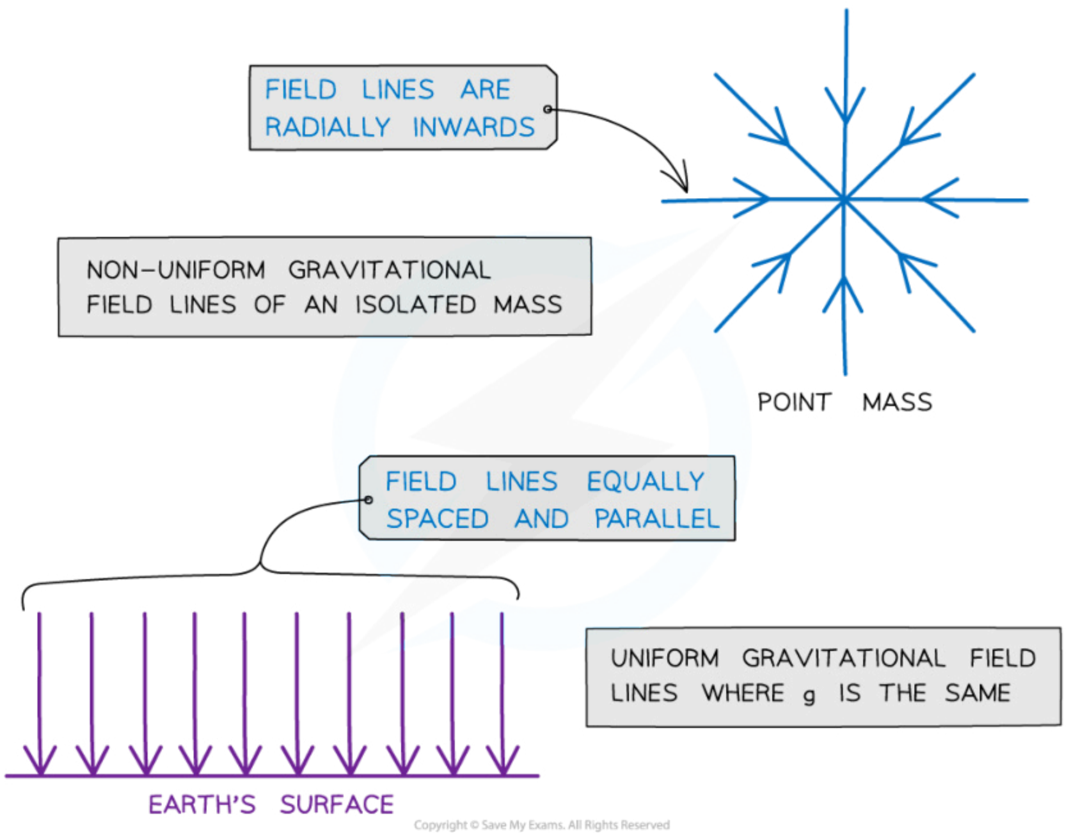
- Gravitational fields represent the **action** of gravitational forces between masses, the direction of these forces can be shown using vectors
  - The direction of the **vector** shows the direction of the **gravitational force** that would be exerted on a **mass** if it was placed at that position in the field
  - These vectors are known as **field lines** (or 'lines of force')
- The direction of a gravitational field is represented by gravitational field lines
  - Therefore, gravitational field lines also show the direction of **acceleration** of a mass placed in the field
- Gravitational field lines are always directed toward the centre of mass of a body
  - This is because gravitational forces are **attractive only** (they are never repulsive)
  - Therefore, masses **always** attract each other via the gravitational force
- The gravitational field around a point mass will be **radial** in shape and the field lines will always point towards the centre of mass



The direction of the gravitational field is shown by the vector field lines

- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface

Your notes



Gravitational field lines for a point mass and a uniform gravitational field

- Radial fields are considered **non-uniform fields**
  - The gravitational field strength *g* is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
  - The gravitational field strength *g* is the same throughout

Examiner Tip

Always label the arrows on the field lines! Gravitational forces are attractive only. Remember:

- For a radial field: it is towards the centre of the sphere or point charge
- For a uniform field: towards the surface of the object e.g. Earth

Your notes

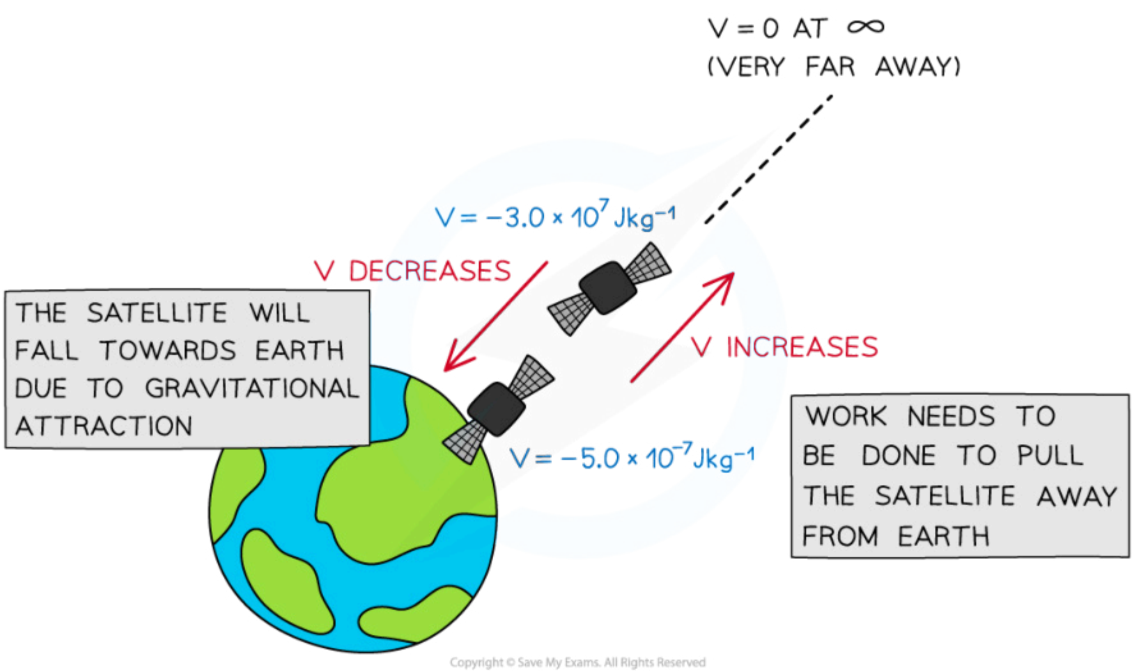
Gravitational Potential (HL)

Gravitational Potential

- The gravitational potential V at a point can, therefore, be defined as:

The work done per unit mass in bringing a test mass from infinity to a defined point

- Gravitational potential is measured in $J\ kg^{-1}$
- It is always has a **negative** value because:
  - It is defined as having a value of **zero** at **infinity**
  - Since the gravitational force is **attractive**, work must be done **on** a mass to reach infinity
- On the surface of a mass (such as a planet), gravitational potential has a negative value
  - The value becomes less negative, i.e. it increases, with distance from that mass
- Work has to be done **against** the gravitational pull of the planet to take a unit mass away from the planet
- The gravitational potential at a point depends on:
  - The **mass** of the object
  - The **distance** from the centre of mass of the object to the point



Gravitational potential decreases as the satellite moves closer to the Earth

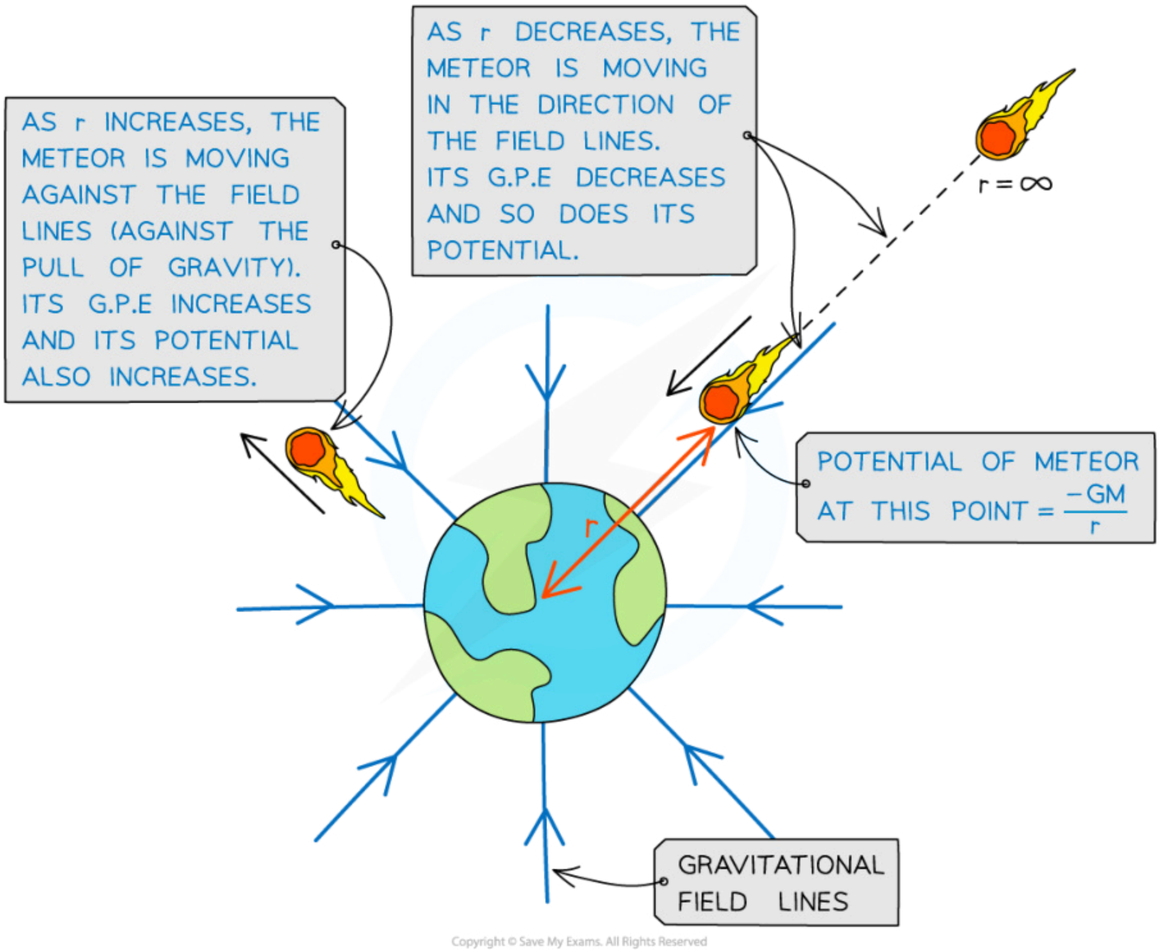
Calculating Gravitational Potential

- The equation for gravitational potential V is defined by the mass M and distance r:

$$V_g = -\frac{GM}{r}$$

Your notes

- Where:
  - V_g = gravitational potential (J kg^{-1})
  - G = Newton's gravitational constant
  - M = mass of the body producing the gravitational field (kg)
  - r = distance from the centre of the mass to the point mass (m)
- The gravitational potential always is negative near an isolated mass, such as a planet, because:
  - The potential when *r* is at infinity (∞) is defined as zero
  - Work must be done to move a mass away from a planet (V becomes less negative)
- It is also a scalar quantity, unlike the gravitational field strength which is a vector quantity
- Gravitational forces are always **attractive**, this means as *r* decreases, positive work is done by the mass when moving from infinity to that point
  - When a mass is closer to a planet, its gravitational potential becomes smaller (more negative)
  - As a mass moves away from a planet, its gravitational potential becomes larger (less negative) until it reaches 0 at infinity
- This means when the distance *r* becomes very large, the gravitational force tends rapidly towards zero the further away the point is from a planet



Gravitational potential increases and decreases depending on whether the object is travelling towards or against the field lines from infinity

Your notes

Worked example

A planet has a diameter of 7600 km and a mass of 3.5 × 10^{23} kg. A meteor of mass 6000 kg accelerates towards the planet from infinity.

Calculate the gravitational potential of the rock at a distance of 400 km above the planet's surface.

Answer:

- The gravitational potential at a point is

$$V_g = -\frac{GM}{r}$$

- Where *r* is the distance from the centre of the planet to the point i.e. the radius of the planet + the height above the planet's surface

$$r = \frac{7600}{2} + 400 = 4200 \text{ km}$$

- And *M* is the mass of the larger mass, i.e. the planet (not the meteor)

$$V_g = -\frac{(6.67\times10^{-11})\times(3.5\times10^{23})}{4200\times10^{3}} = -5.6\times10^{6}\ \text{J kg}^{-1}$$

Examiner Tip

Notice the red herring in the worked example. You do not need the mass *m* of the meteor, as *M* in the equation for gravitational potential is only the mass of the **object creating the gravitational field**. *m* will come into play with gravitational potential **energy**.

Your notes

Gravitational Potential Energy in a Non-Uniform Field (HL)

Gravitational Potential Energy in a Non-Uniform Field

- In a radial field, gravitational potential energy (GPE) describes the energy an object possesses due to its **position** in a gravitational field
- The gravitational potential energy of a system is defined as:
  The work done to assemble the system from infinite separation of the components of the system
- Similarly, the gravitational potential energy of a point mass is defined as:
  The work done in bringing a mass from infinity to a point

Near the Earth's Surface

- The gravitational potential energy near the Earth's surface is equal to

$$E_p = mg\Delta h$$

- The GPE on the surface of the Earth is taken to be zero
  - This means work is done to **lift** the object
- This equation can **only** be used for objects that are **near the Earth's surface**
  - This is because, near Earth's surface, the gravitational field is approximated to be **uniform**
  - Far away from the Earth's surface, the gravitational field is **radial** because the Earth is a **sphere**

Examiner Tip

You should be able to interpret areas under curves by thinking about what the **product** of the quantities on the axes would represent. Since, in this case, **force × distance = work done**, then it follows that the **area** under the curve represents the **change** in **energy** between two points. Specifically, this would be a change in **gravitational potential energy**.

The equation GPE = $mg\Delta h$ is very **rarely** used in this topic. This is only relevant for objects **on a planet's surface**.

The only difference between GPE and g is GPE = mg where m is the mass of the object in the gravitational field of mass M.

This equation is **not** given on your data booklet, but you must understand its significance

Gravitational Potential Energy Equation (HL)

Your notes

Work Done on a Mass

- When a mass is moved against the force of gravity, work is required
  - This is because gravity is **attractive**, therefore, energy is needed to work against this attractive force
- The work done in moving a mass *m* is given by:

$$\Delta W = m\Delta V_g$$

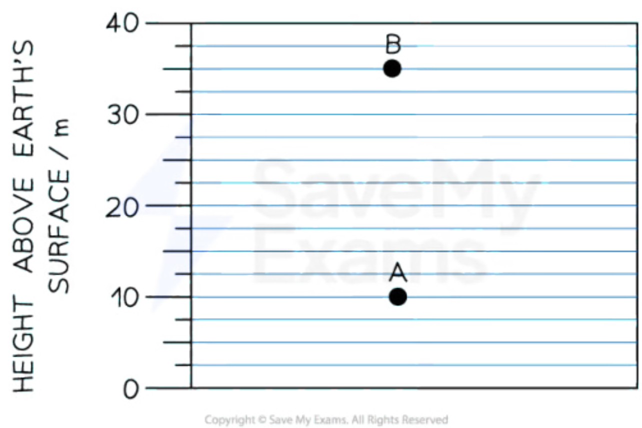
- Where:
  - ΔW = change in work done (J)
  - m = mass (kg)
  - ΔV_g = change in gravitational potential (J kg^{-1})

Your notes

Worked example

A particle of mass 50 g is moved vertically from point A to point B, as shown in the diagram.

Take the gravitational field strength to be 10 N kg^{-1}.



Determine

(a) the potential difference between A and B

(b) the work done in moving the mass from A to B

Answer:

(a)

- The work done in moving a mass in a gravitational field is:

$$W = m\Delta V \text{ and } W = mg\Delta h \text{ (close to the Earth's surface)}$$

$$m\Delta V = mg\Delta h \quad \Rightarrow \quad \Delta V = g\Delta h$$

- Where the change in height is Δh = 35 – 10 = 25 m
- Therefore, the potential difference between A and B is:

$$\Delta V = 10 \times 25 = 250 \text{ J kg}^{-1}$$

(b)

- The work done in moving the mass from A to B is:

$$W = m\Delta V$$

$$W = (50 \times 10^{-3}) \times 250 = 12.5 \text{ J}$$

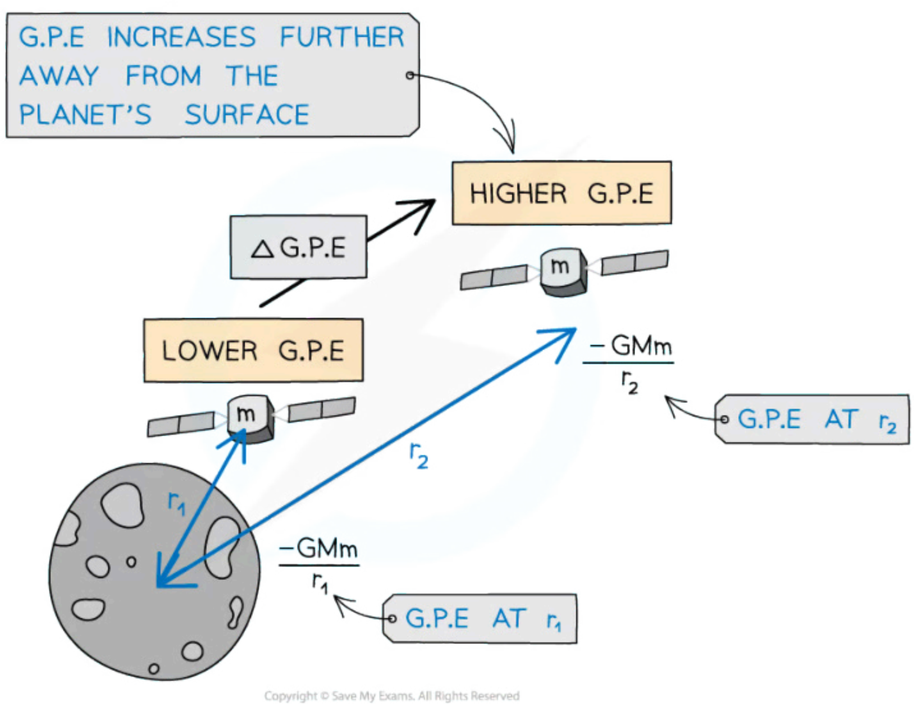
Your notes

Gravitational Potential Energy Equation

- In a radial field, gravitational potential energy (GPE) describes the energy an object possesses due to its **position** in a gravitational field
- The gravitational potential energy of a system is defined as:
  The work done to assemble the system from infinite separation of the components of the system
- Similarly, the gravitational potential energy of a point mass is defined as:
  The work done in bringing a mass from infinity to a point
- The equation for GPE of two point masses *m* and *M* at a distance *r* is:

$$E_p = -\frac{Gm_1 m_2}{r}$$

- Where:
  - G = universal gravitational constant (N m^2 kg^{-2})
  - m_1 = larger mass producing the field (kg)
  - m_2 = mass moving within the field of M (kg)
  - r = distance between the centre of m and M (m)



Gravitational potential energy increases as a satellite leaves the surface of the Moon (of mass M)

- Recall that Newton's Law of Gravitation relates the magnitude of the force *F* between two masses *M* and *m*:

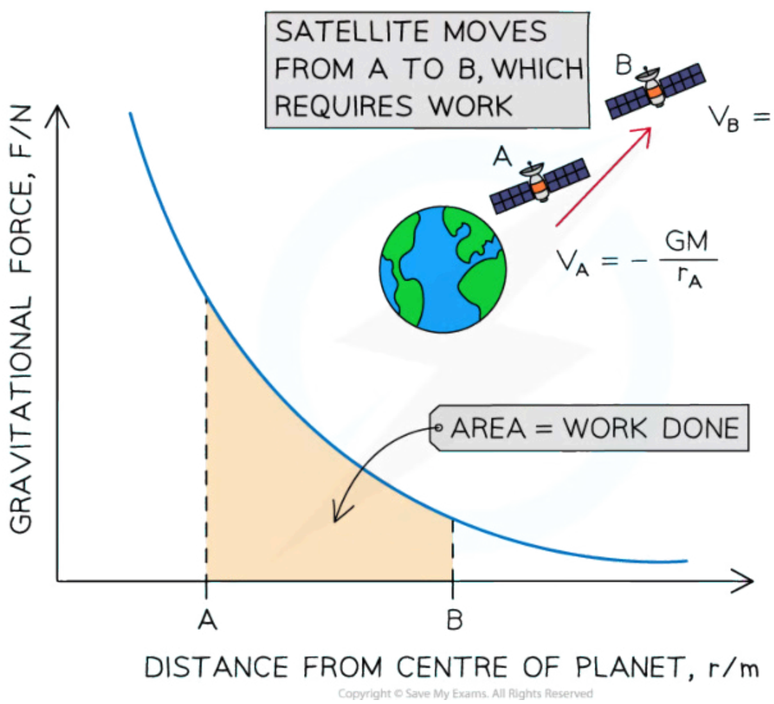
Your notes

$$F = \frac{Gm_1 m_2}{r^2}$$

- Therefore, a **force-distance** graph would be a curve, because *F* is **inversely proportional** to r^2, or:

$$F \propto \frac{1}{r^2}$$

- The product of **force** and **distance** is equal to work done (or energy transferred)
- Therefore, the **area** under the **force**-**distance** graph for gravitational fields is equal to the **work done**
  - In the case of a mass *m* moving further away from a mass *M*, the potential **increases**
  - Since gravity is attractive, this requires **work to be done** on the mass *m*
  - The area between two points under the **force**-**distance** curve, therefore, gives the change in **gravitational potential energy** of mass *m*



Work is done on the satellite of mass m to move it from A to B, because gravity is attractive. The area under the curve represents the magnitude of energy transferred

Change in Gravitational Potential Energy

- Two points at different distances from a mass will have **different** gravitational potentials
  - This is because the gravitational potential **increases** with distance from a mass
- Therefore, there will be a **gravitational potential difference** ΔV between the two points

$$\Delta V = V_f - V_i$$

Your notes

- Where:
  - V_i = initial gravitational potential (J kg^{-1})
  - V_f = final gravitational potential (J kg^{-1})
- The change in work done against a gravitational field is equal to the change in **gravitational potential energy** (GPE)
  - When V = 0, then the GPE = 0
- It is usually more useful to find the **change** in the GPE of a system
  - For example, a satellite lifted into space from the Earth's surface
- The change in GPE when a mass moves towards, or away from, another mass is given by:

$$\Delta E_p = -\frac{Gm_1 m_2}{r_2} - \left(-\frac{Gm_1 m_2}{r_1}\right)$$

$$\Delta E_p = Gm_1 m_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Where:
  - m_1 = mass that is producing the gravitational field (e.g. a planet) (kg)
  - m_2 = mass that is moving in the gravitational field (e.g. a satellite) (kg)
  - r_1 = first distance of m from the centre of M (m)
  - r_2 = second distance of m from the centre of M (m)
- The change in potential ΔV_g is the same, without the mass of the object m_2:

$$\Delta V_g = -\frac{Gm_1}{r_2} - \left(-\frac{Gm_1}{r_1}\right)$$

$$\Delta V_g = Gm_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Work is done when an object in a planet's gravitational field moves **against** the gravitational field lines i.e. **away** from the planet

Your notes

Worked example

A spacecraft of mass 300 kg leaves the surface of Mars up to an altitude of 700 km.

Calculate the work done by the spacecraft.

- Radius of Mars = 3400 km
- Mass of Mars, $m_1 = 6.40 \times 10^{23}$ kg

Answer:

- The change in GPE is equal to

$$\Delta E_p = Gm_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

- Where
  - r_1 = radius of Mars = 3400 km
  - r_2 = radius + altitude = 3400 + 700 = 4100 km

$$\Delta E_p = (6.67 \times 10^{-11}) \times (6.40 \times 10^{23}) \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3} \right)$$

Work done by satellite: $\Delta E_p = 643.1 \times 10^6 = 640 \text{ MJ}$ (2 s.f.)

Your notes

Worked example

A satellite of mass 1450 kg moves from an orbit of 980 km above the Earth's surface to a lower orbit of 480 km.

Calculate the change in gravitational potential energy of the satellite.

- Mass of the Earth = 5.97×10^{24} kg
- Radius of the Earth = 6.38×10^{6} m

Answer:

Step 1: Write down the known quantities

- Initial height above Earth's surface, h_1 = 980 km
- Final height above Earth's surface, h_2 = 480 km
- Mass of the satellite, m_1 = 1450 kg
- Mass of the Earth, $m_2 = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $R = 6.38 \times 10^{6}$ m

Step 2: Write down the equation for change in gravitational potential energy

$$\Delta E_p = Gm_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Step 3: Convert distances into standard units and include Earth radius

- Distance from centre of Earth to higher orbit:

$$r_1 = h_1 + R$$

$$r_1 = (980 \times 10^3) + (6.38 \times 10^6) = 7.36 \times 10^6 \text{ m}$$

- Distance from centre of Earth to lower orbit:

$$r_2 = h_2 + R$$

$$r_2 = (480 \times 10^3) + (6.38 \times 10^6) = 6.86 \times 10^6 \text{ m}$$

Step 4: Substitute values into the equation

$$\Delta E_p = (6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times 1450 \times \left(\frac{1}{7.36 \times 10^6} - \frac{1}{6.86 \times 10^6} \right)$$

Change in gravitational potential energy: $\Delta E_p = 5.72 \times 10^{9}$ J

Examiner Tip

Make sure to not confuse the ΔE_p equation with $\Delta E = mg\Delta h$, they look similar but refer to quite **different situations.**

The more familiar equation is only relevant for an object lifted in a **uniform gravitational field**, meaning very close to the Earth's surface, where we can model the field as uniform.

The new equation for E_p does not include g. The gravitational field strength, which is different on different planets, does not remain constant as the distance from the surface increases. Gravitational field strength **falls away** according to the inverse square law.

The change in gravitational potential energy **is** the work done.

Your notes

Gravitational Potential Gradient (HL)

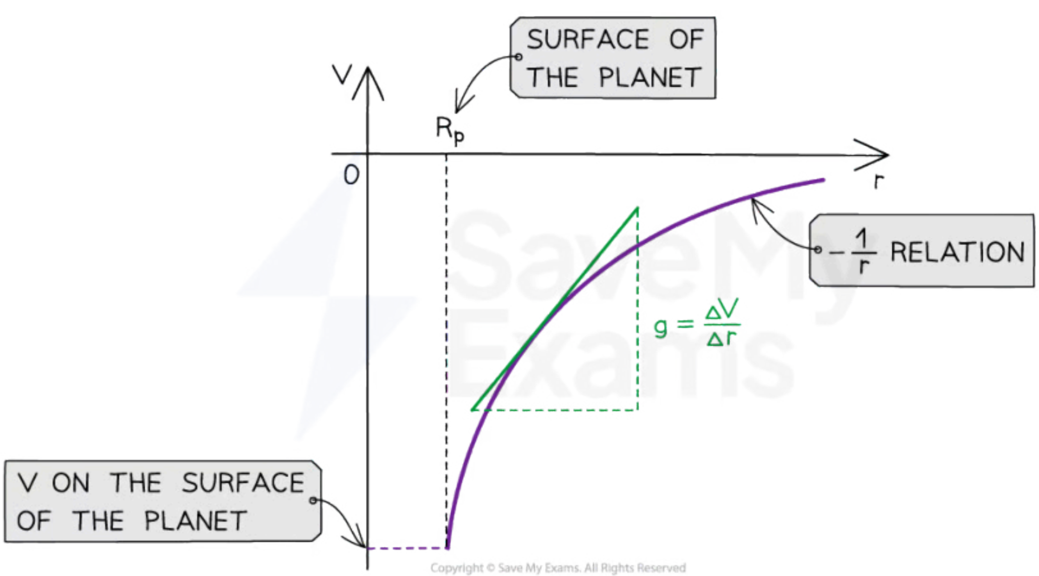
Your notes

Gravitational Potential Gradient

- A gravitational field can be defined in terms of the variation of gravitational potential at different points in the field:
  The gravitational field at a particular point is equal to the negative gradient of a potential-distance graph at that point
- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the gravitational potential in a gravitational field and are always drawn **perpendicular** to the field lines
- The potential gradient in a gravitational field is defined as:
  The rate of change of gravitational potential with respect to displacement in the direction of the field
- Gravitational field strength, *g* and the gravitational potential, *V* can be graphically represented against the distance from the centre of a planet, *r*

$$g = -\frac{\Delta V_g}{\Delta r}$$

- Where:
  - g = gravitational field strength ($N\ kg^{-1}$)
  - ΔV_g = change in gravitational potential ($J\ kg^{-1}$)
  - Δr = distance from the centre of a point mass (m)
- The graph of V_g against r for a planet is:



The gravitational potential and distance graphs follow a -1/r relation

Your notes

- **The key features of this graph are:**
  - The values for V_g are all negative (because the graph is drawn below the horizontal *r* axis)
  - As *r* increases, V_g against *r* follows a $-\frac{1}{r}$ relation
  - The **gradient** of the graph at any particular point is the value of *g* at that point,

$$g = -V_g \times -\frac{1}{r} = \frac{V_g}{r}$$

  - The graph has a shallow increase as *r* increases
- To calculate *g*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the gravitational potential equation:

$$V_g = -\frac{GM}{r}$$

where G and *M* are constant

Worked example

Determine the change in gravitational potential when travelling from 3 Earth radii (from Earth's centre) to the surface of the Earth.

Take the mass of the Earth to be 5.97×10^{24} kg and the radius of the Earth to be 6.38×10^6 m.

Answer:

Step 1: List the known quantities

- Mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $r_E = 6.38 \times 10^6$ m
- Initial distance, $r_1 = 3r_E = 3 \times (6.38 \times 10^6)$ m $= 1.914 \times 10^7$ m
- Final distance, $r_2 = r_E = 6.38 \times 10^6$ m
- Gravitational constant, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

Step 2: Write down the equation for potential difference

$$\Delta V_g = -GM_E\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Step 3: Substitute the values into the equation

$$\Delta V_g = -(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times \left(\frac{1}{6.38 \times 10^6} - \frac{1}{1.914 \times 10^7}\right)$$

$\Delta V_g = -4.16 \times 10^7$ J kg^{-1}

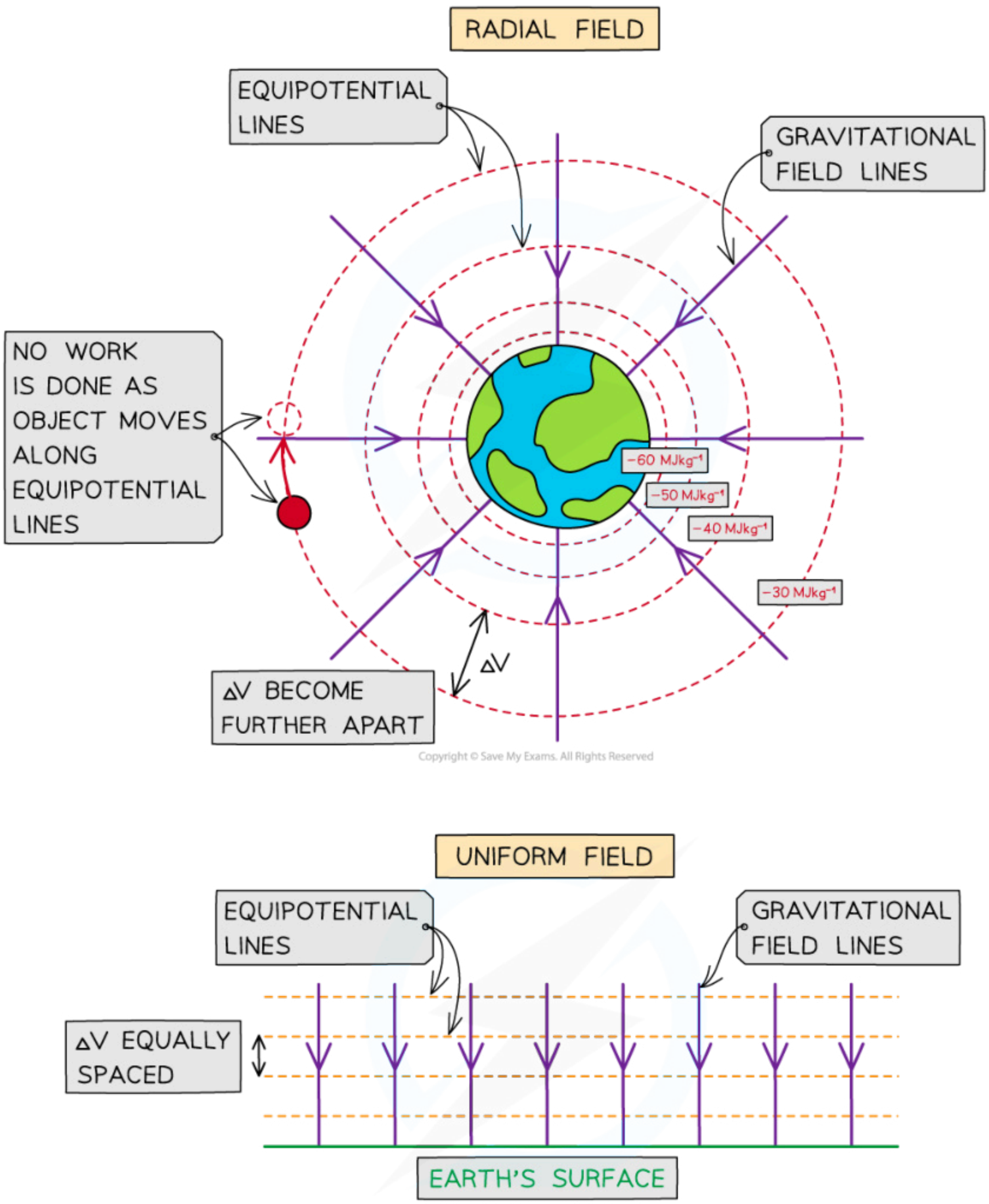
Your notes

Gravitational Equipotential Surfaces (HL)

Gravitational Equipotential Surfaces

- Equipotential lines (when working in 2D) and surfaces (when working in 3D) join together points that have the same gravitational potential
- These are always:
  - **Perpendicular** to the gravitational field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)
- In a radial field (e.g. a planet), the equipotential lines:
  - Are concentric circles around the planet
  - Become further apart further away from the planet
  - Remember: **radial** field is made up of lines which follow the **radius** of a circle
- In a uniform field (e.g. near the Earth's surface), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
  - Remember: **uniform** field is made up of lines which are a **uniform** distance apart
- Potential gradient is defined by the **equipotential lines**
- **No work is done** when moving along an equipotential line or surface, only **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$

Your notes



Gravitational equipotential lines in a non-uniform and uniform gravitational field

Examiner Tip

Remember equipotential lines do **not** have arrows, since they have no particular direction and are not vectors.

Make sure to draw any straight lines with a ruler or a straight edge.

Your notes

Your notes

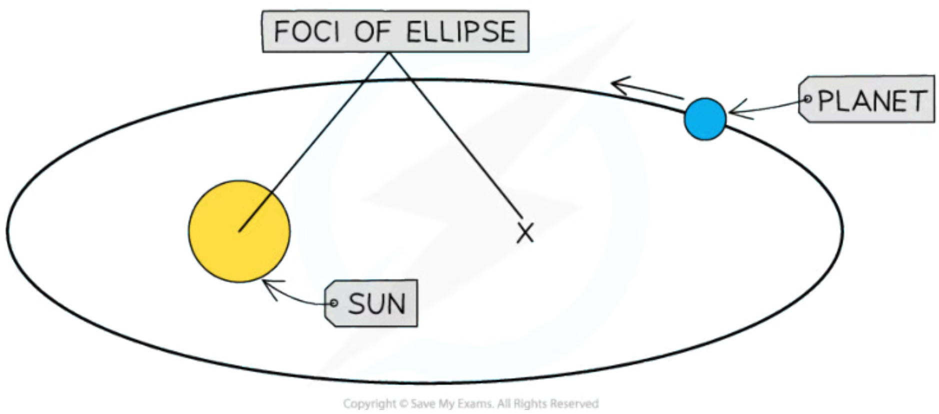
Kepler's Laws of Planetary Motion

Kepler's Laws of Planetary Motion

Kepler's First Law

- Kepler's First Law describes the **shape** of planetary orbits
- It states:

The orbit of a planet is an ellipse, with the Sun at one of the two foci



The orbit of all planets are elliptical, and with the Sun at one focus

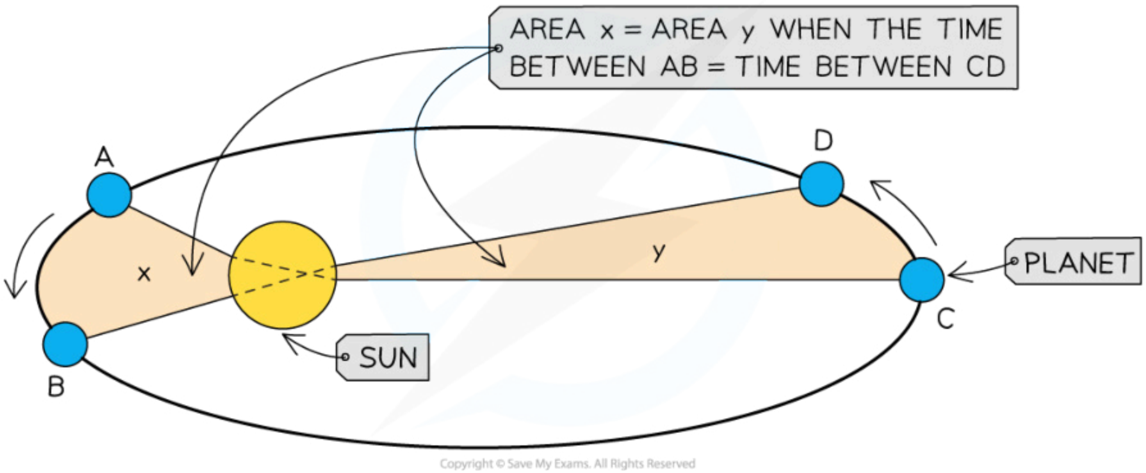
- An ellipse is just a 'squashed' circle
  - Some planets, like Pluto, have highly elliptical orbits around the Sun
  - Other planets, like Earth, have near circular orbits around the Sun

Kepler's Second Law

- Kepler's Second Law describes the **motion** of all planets around the Sun
- It states:

A line segment joining the Sun to a planet sweeps out equal areas in equal time intervals

Your notes



- The consequence of Kepler's Second Law is that planets move **faster** nearer the Sun and **slower** further away from it

Kepler's Third Law

- Kepler's Third Law states

For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit

- This law describes the relationship between the **time** of an orbit and its **radius**

$$T^2 \propto r^3$$

- Where:
  - T = orbital time period (s)
  - r = mean orbital radius (m)

Time Period & Orbital Radius Relation

- Since a planet or a satellite is travelling in circular motion when in order, its orbital time period T to travel the circumference of the orbit $2\pi r$, the linear speed v is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed v from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

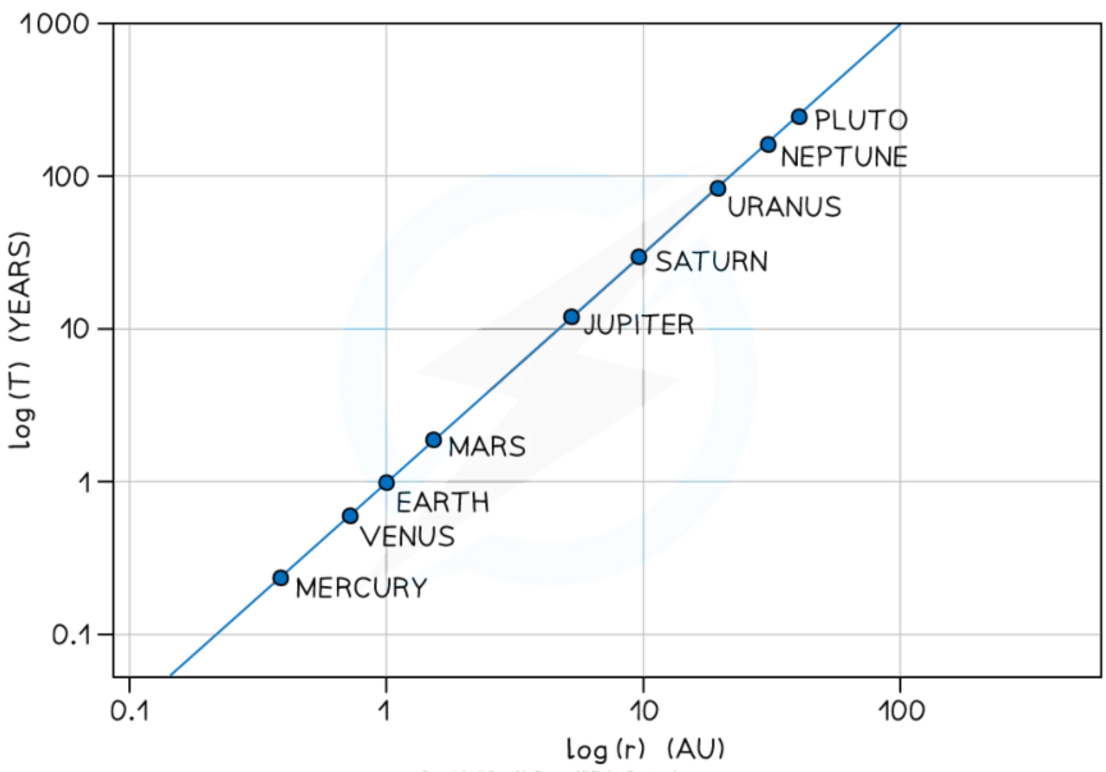
- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period T and orbital radius r:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

- Where:
  - T = time period of the orbit (s)
  - r = orbital radius (m)
  - G = Gravitational Constant
  - M = mass of the object being orbited (kg)
- The relationship between T and r can be shown using a logarithmic plot

$$T^2 \propto r^3 \quad \Rightarrow \quad 2 \log T \propto 3 \log r$$

- The graph of log T in years against log r in AU (astronomical units) for the planets in our solar system is a straight-line graph:



The logarithmic graph of log T against log r gives a straight line

- The graph does not go through the origin since it has a negative y-intercept
  - Only the graph of log T and log r will produce a straight-line graph, a graph of T vs r would not

Your notes

Worked example

Planets A and B orbit the same star.

Planet A is located an average distance *r* from the star. Planet B is located an average distance 6*r* from the star

What is $\frac{orbital\ period\ of\ planet\ A}{orbital\ period\ of\ planet\ B}$?

A. $\frac{1}{\sqrt[3]{6}}$ **B.** $\frac{1}{\sqrt{6}}$ **C.** $\frac{1}{\sqrt[3]{6^2}}$ **D.** $\frac{1}{\sqrt{6^3}}$

Answer: D

- Kepler's third law states $T^2 \propto r^3$
- The orbital period of planet A: $T_A \propto \sqrt{r^3}$
- The orbital period of planet B: $T_B \propto \sqrt{(6r)^3}$
- Therefore the ratio is equal to:

$$\frac{T_A}{T_B} = \frac{\sqrt{r^3}}{\sqrt{(6r)^3}} = \frac{1}{\sqrt{6^3}}$$

Examiner Tip

You are expected to be able to describe Kepler's Laws of Motion, so make sure you are familiar with how they are worded.

Your notes

Escape Speed (HL)

Escape Speed

- To escape a gravitational field, a mass must travel at, or above, the minimum **escape speed**
  - This is dependent on the mass and radius of the object creating the gravitational field, such as a planet, a moon or a black hole
- Escape speed is defined as:

  The minimum speed that will allow an object to escape a gravitational field with no further energy input

- It is the same for all masses in the same gravitational field
  - For example, the escape speed of a rocket is the same as a tennis ball on Earth
- The escape speed of an object is the speed at which all its kinetic energy has been transferred to gravitational potential energy
- This is calculated by equating the equations:

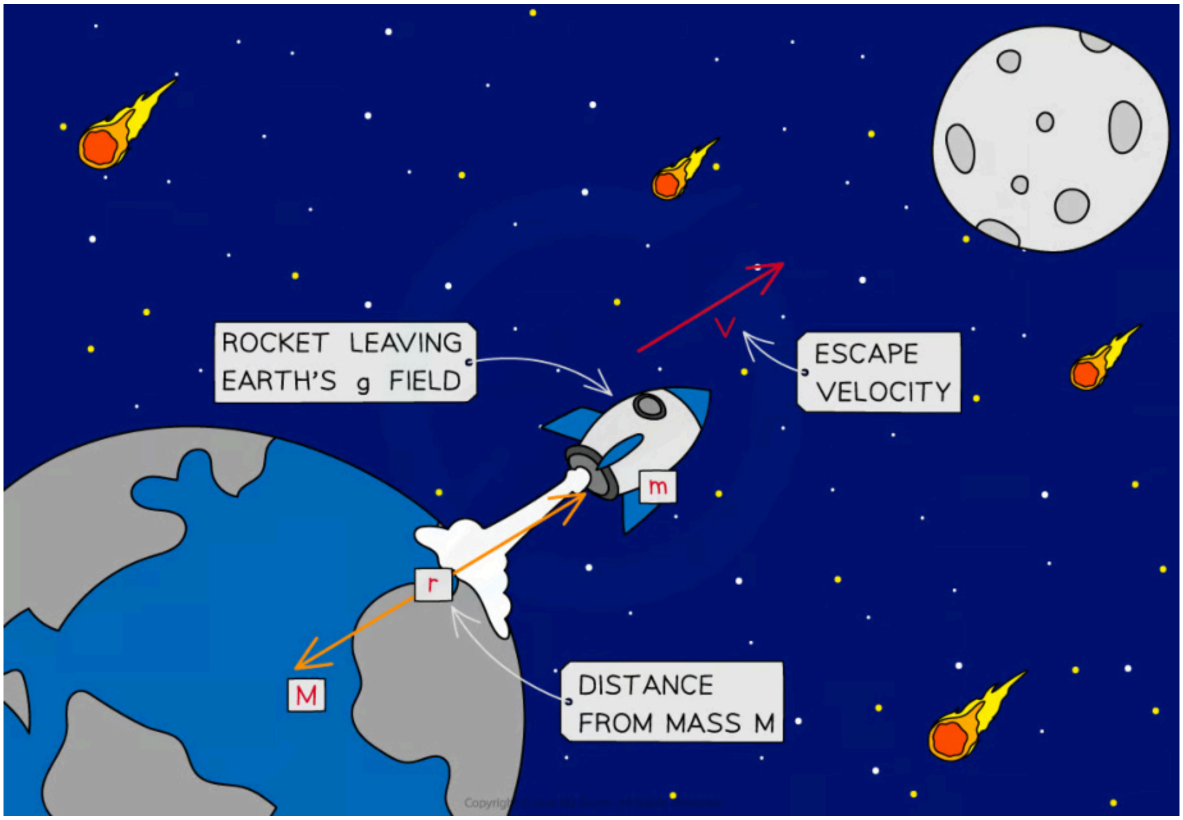
$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{r}$$

- Where:
  - m = mass of the object in the gravitational field (kg)
  - v_{esc} = escape velocity of the object (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object to be escaped from (i.e. a planet) (kg)
  - r = distance from the centre of mass M (m)
- Since mass m is the same on both sides of the equation, it can cancel on both sides of the equation:

$$\frac{1}{2}v_{esc}^2 = \frac{GM}{r}$$

- Multiplying both sides by 2 and taking the square root gives the equation for escape velocity v_{esc}:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$



For an object to leave the Earth's gravitational field, it will have to travel at a speed greater than the Earth's escape velocity, v

- Rockets launched from the Earth's surface do **not** need to achieve escape velocity to reach their orbit around the Earth
- This is because:
  - They are continuously given energy through fuel and thrust to help them move
  - Less energy is needed to achieve orbit than to escape from Earth's gravitational field
- The escape velocity is **not** the velocity needed to escape the planet but to escape the planet's **gravitational field** altogether
  - This could be quite a large distance away from the planet

Your notes

Worked example

Calculate the escape speed at the surface of the Moon.

- Density of the Moon = 3340 kg m^{-3}
- Mass of the Moon = 7.35 x 10^{22} kg

Answer:

Step 1: List the known quantities

- Gravitational constant, G = 6.67 x 10^{-11} N m^2 kg^{-2}
- Density of the Moon, ρ = 3340 kg m^{-3}
- Mass of the Moon, M = 7.35 x 10^{22} kg

Step 2: Rearrange the density equation for radius *r*

Density: $\rho = \frac{M}{V}$ and volume of a sphere: $V = \frac{4}{3}\pi r^3$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

$$r = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

Step 3: Calculate the radius by substituting in the values

$$r = \sqrt[3]{\frac{3 \times (7.35 \times 10^{22})}{4\pi \times 3340}} = 1.7384 \times 10^6 \text{ m}$$

Step 4: Substitute r into the escape speed equation

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{1.7384 \times 10^6}}$$

Escape speed of the Moon: v_{esc} = 2.37 km s^{-1}

Examiner Tip

When writing the definition of **escape velocity**, avoid terms such as 'gravity' or the 'gravitational pull / attraction' of the planet. It is best to refer to its **gravitational field.** This equation is given on the data sheet, but make sure you know how it is derived.

Your notes

Your notes

Orbital Motion, Speed & Energy (HL)

Orbital Motion, Speed & Energy

- Since most planets and satellites have near-circular orbits, the gravitational force F_G between two bodies (e.g. planet & star, planet & satellite) provides the centripetal force needed to stay in an orbit
  - Both the gravitational force and centripetal force are **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*

$$F_G = F_{circ}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

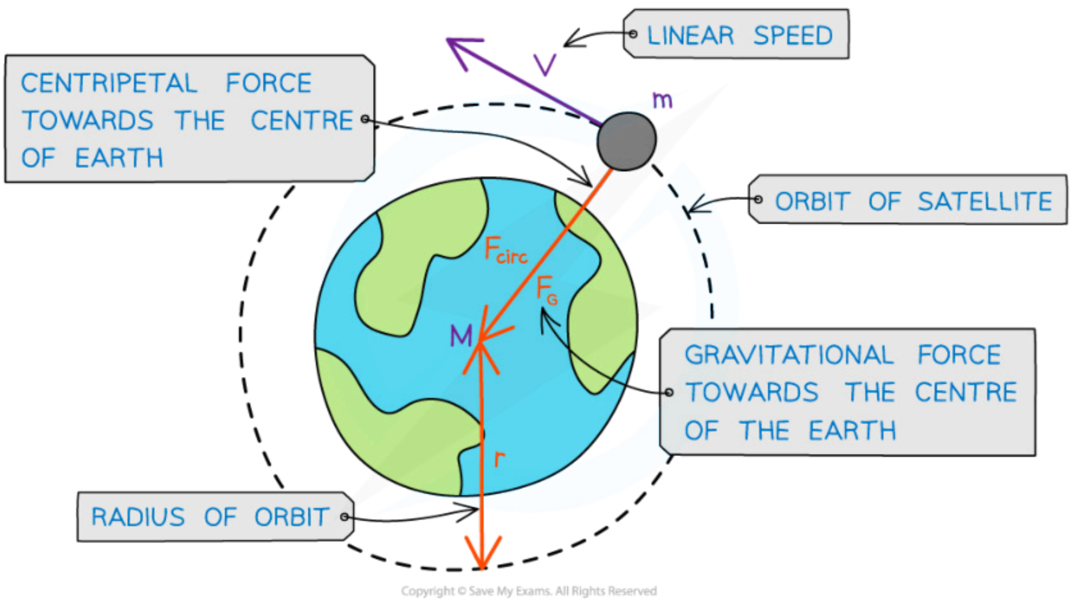
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{GM}{r} \quad \Rightarrow \quad v_{orbital} = \sqrt{\frac{GM}{r}}$$

- Where:
  - $v_{orbital}$ = orbital speed of the smaller mass (m s^{-1})
  - *G* = Newton's Gravitational Constant
  - *M* = mass of the larger mass being orbited (kg)
  - *r* = orbital radius (m)
- This means that all satellites, **whatever their mass**, will travel at the same speed *v* in a particular orbit radius *r*
  - Since the direction of a planet orbiting in circular motion is constantly changing, the **centripetal acceleration** acts towards the planet

Your notes

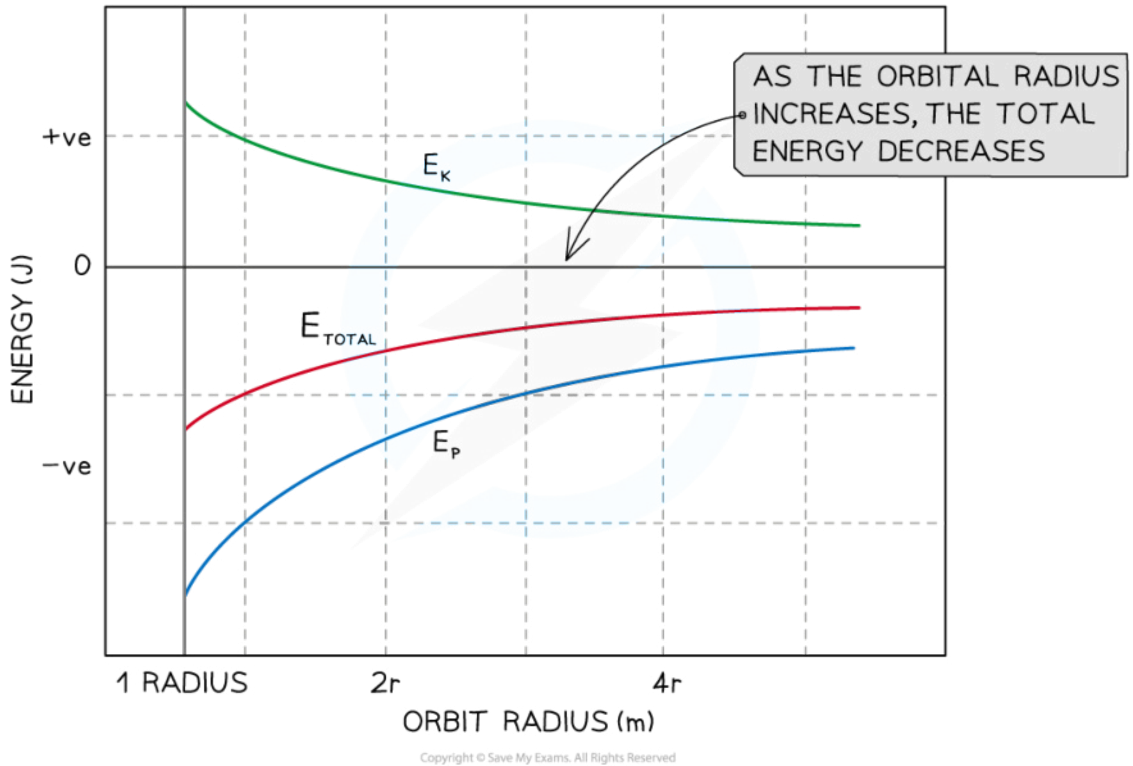


A satellite in orbit around the Earth travels in circular motion

Energy of an Orbiting Satellite

- An orbiting satellite follows a circular path around a planet
- Just like an object moving in circular motion, it has both kinetic energy (E_k) **and** gravitational potential energy (E_p) and its **total** energy is always **constant**
- An orbiting satellite's total energy is calculated by:

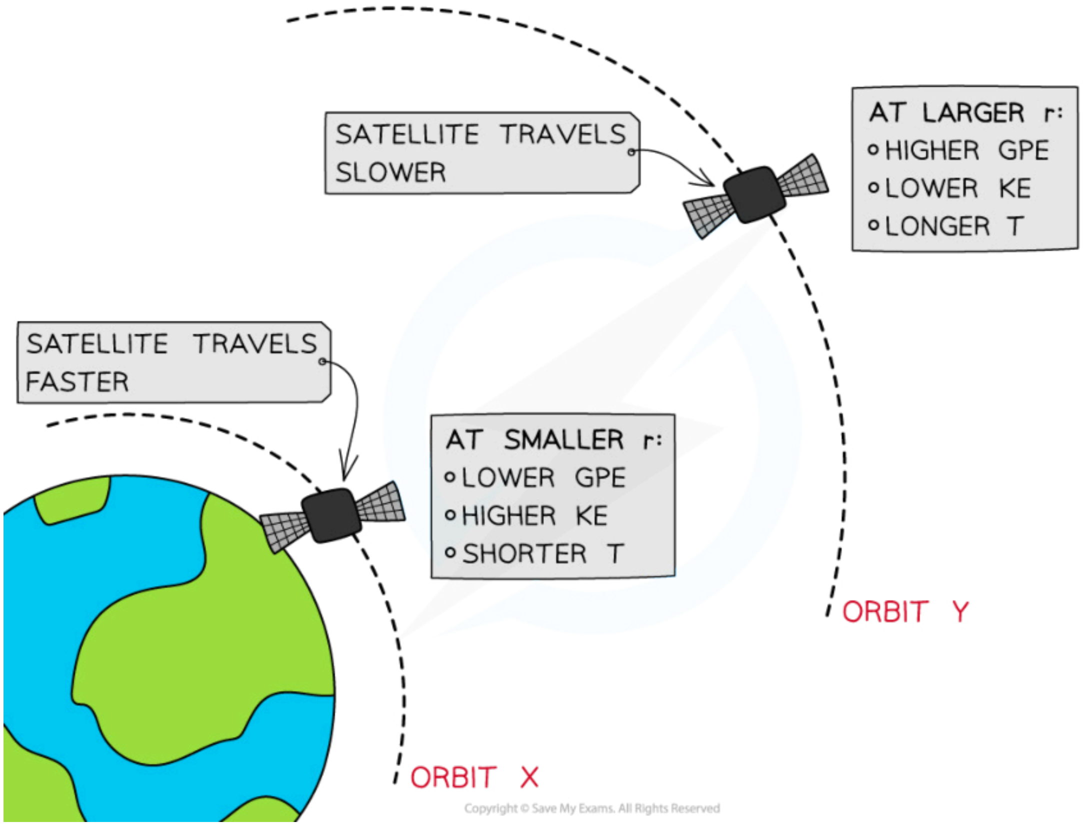
Total energy = Kinetic energy + Gravitational potential energy



A graph showing the kinetic, potential and total energy for a mass at varying orbital distances from a massive body

- This means that the satellite's E_k and E_p are also both constant in a particular orbit
  - If the orbital radius of a satellite **decreases** its E_k **increases** and its E_p **decreases**
  - If the orbital radius of a satellite **increases** its E_k **decreases** and its E_p **increases**

Your notes



At orbit Y, the satellite has greater GPE and less KE than at at orbit X

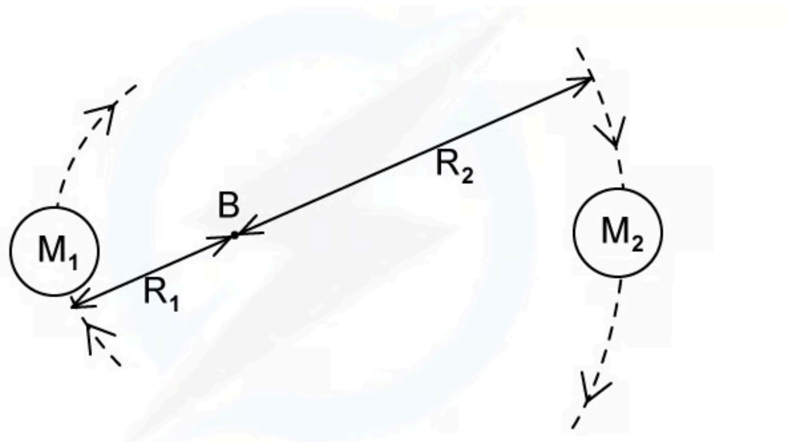
- A satellite is placed in two orbits, X and Y, around Earth
- At orbit X, where the radius of orbit *r* is smaller, the satellite has a:
  - Larger gravitational force on it
  - Higher speed
  - Higher E_k
  - Lower E_p
  - Shorter orbital time period, T
- At orbit Y, where the radius of orbit *r* is larger, the satellite has a:
  - Smaller gravitational force on it
  - Smaller speed
  - Lower E_k
  - Higher E_p
  - Longer orbital time period, T

Your notes

Worked example

A binary star system constant of two stars orbiting about a fixed point **B**.

The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed v and an angular speed ω about **B**.



In terms of G, M_2, R_1 and R_2, write an expression for

(a) the angular speed ω of mass M_1

(b) the time period T of each star

Answer:

(a) Angular speed:

- The centripetal force on mass M_1 is:

$$F = \frac{M_1 v_1^2}{R_1} = \frac{M_1(\omega R_1)^2}{R_1} = M_1 R_1 \omega^2$$

- The gravitational force between the two masses is:

$$F = \frac{GM_1 M_2}{(R_1 + R_2)^2}$$

- Equating these expressions gives:

$$M_1 R_1 \omega^2 = \frac{GM_1 M_2}{(R_1 + R_2)^2}$$

- Rearrange for angular velocity

Your notes

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}}$$

(b) Orbital period:

- The relation between angular speed and orbital period is

$$\omega = \frac{2\pi}{T} \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

- Using the expression for angular velocity from part (a)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1 + R_2)^2}{GM_2}}$$

Your notes

Worked example

Two identical satellites, X and Y, orbit a planet at radii *R* and 3*R* respectively.

Which one of the following statements is **incorrect**?

A. Satellite X has more kinetic energy and less potential energy than satellite Y

B. Satellite X has a shorter orbital period and travels faster than satellite Y

C. Satellite Y has less kinetic energy and more potential energy than satellite X

D. Satellite Y has a longer orbital period and travels faster than satellite X

Answer: D

- Satellite Y is at a larger orbital radius, therefore it will have a **longer** orbital period, since $T^2 \propto R^3$
- Being at a larger orbital radius means the gravitational force will be weaker for Y than for X
- So, satellite Y will travel much **slower** than X as centripetal force: $F \propto v^2$
- Travelling at a slower speed means satellite Y will have **less** kinetic energy, as $E_K \propto v^2$, and, therefore, **more** potential energy than X

	Satellite X	Satellite Y
orbital radius	smaller	larger
orbital period	shorter	longer
orbital speed	faster	slower
kinetic energy	greater	lower
potential energy	lower	greater

- Therefore, all statements are correct except in **D** where it says 'Satellite Y travels faster than satellite X'

Examiner Tip

If you can't remember which way around the kinetic and potential energy increases and decreases, think about the velocity of a satellite at different orbits.

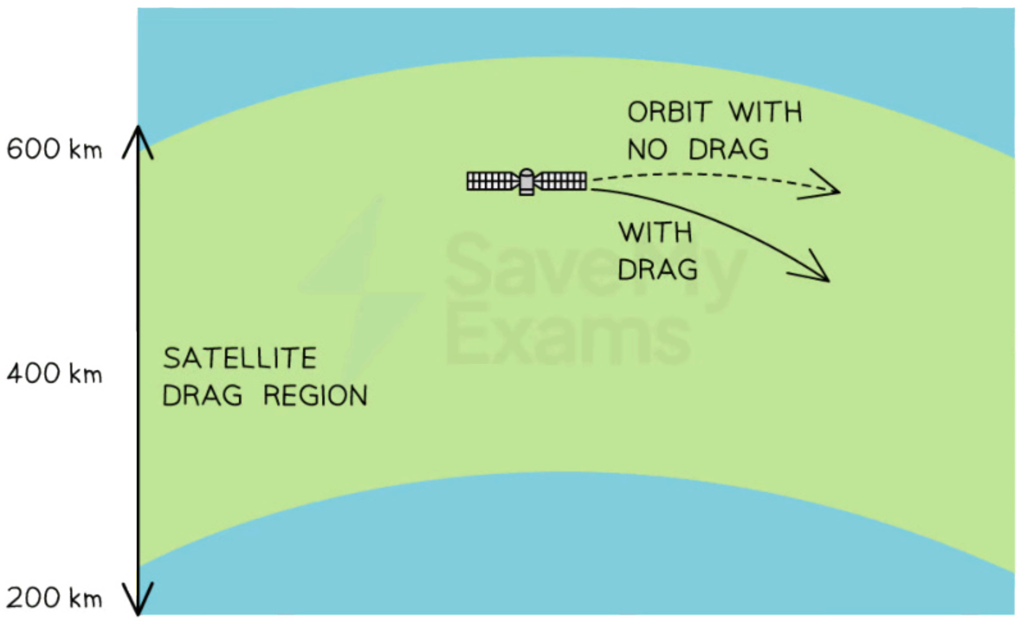
When it is orbiting close to a planet, it experiences a larger gravitational pull and therefore orbits faster. Since the kinetic energy is proportional to v^2, it, therefore, has higher kinetic energy closer to the planet. To keep the total energy constant, the potential energy must decrease too.

Effects of Drag on Orbital Motion (HL)

Your notes

Effects of Drag on Orbital Motion

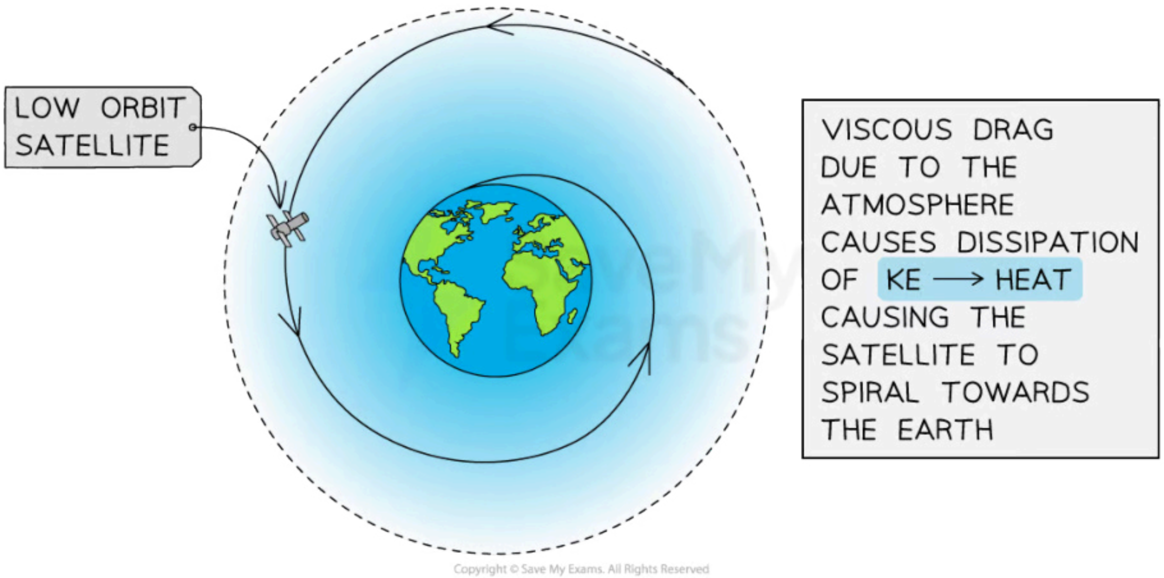
- Satellites in low orbits (<600 km) may be slightly affected by viscous drag, or air resistance
- The effects of drag on the motion of the satellite are usually very small, but over time, it can have a significant effect on the height and speed of the satellite's orbit



Viscous drag can affect the height and speed of a low-orbit satellite as a result of energy dissipation

- The density of the air in the very upper layers of the atmosphere is very low, but not zero
- As a result, satellites travelling through these thin layers of air will experience a small dissipation of kinetic energy into thermal energy
  - This heating is due to the friction between the air particles and the surface of the satellite

Your notes



As a low-orbit satellite loses energy, it spirals towards the Earth as its orbital radius decreases

- As some of the kinetic energy is dissipated into the surroundings, the satellite's total energy is reduced
  - When a satellite loses energy, its **orbital radius decreases**
  - However, as the satellite's orbit becomes lower, some of its **potential** energy is transferred to **kinetic** energy
- Overall, its speed increases and the effects of air resistance become even greater in its lower orbit resulting in greater dissipation of kinetic energy into thermal energy
- If the overall **decrease** in potential energy is **larger** than the overall **increase** in kinetic energy, the **total energy will decrease**

$$\Delta E_{total} < 0 \text{ if } \Delta E_p > \Delta E_k$$