

# DP IB Maths: AI HL



Your notes

## 3.2 Geometry of 3D Shapes

### Contents

- \* 3.2.1 3D Coordinate Geometry
- \* 3.2.2 Volume & Surface Area



Your notes

## 3.2.1 3D Coordinate Geometry

### 3D Coordinate Geometry

#### How does the 3D coordinate system work?

- In three-dimensional space we can label where any object is using the x-y-z coordinate system
- In the 3D cartesian system, the x- and y- axes usually represent lateral space (length and width) and the z-axis represents vertical height

#### What can we do with 3D coordinates?

- If we have two points with coordinates  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  then we should be able to find:
  - The **midpoint** of the two points
  - The **distance** between the two points
- If the coordinates are labelled A and B then the line segment between them is written with the notation [AB]

#### How do I find the midpoint of two points in 3D?

- The midpoint is the **average (middle) point**
  - It can be found by finding the middle of the x-coordinates and the middle of the y-coordinates
- The coordinates of the midpoint will be

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

- This is given in the formula booklet, you do not need to remember it

#### How do I find the distance between two points in 3D?

- The distance between two points with coordinates  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  can be found using the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

- This is given in the formula booklet, you do not need to remember it



Your notes

### Worked example

The points A and B have coordinates  $(-2, 1, 5)$  and  $(4, -3, 2)$  respectively.

- i) Calculate the distance of the line segment AB.

Formula for the distance of a line segment:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

↙ in formula booklet

$$A: (-2, 1, 5) \quad B: (4, -3, 2)$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ x_1 & y_1 & z_1 \end{matrix} \quad \begin{matrix} \uparrow & \uparrow & \uparrow \\ x_2 & y_2 & z_2 \end{matrix}$

Substitute:

$$\begin{aligned}
 d &= \sqrt{(-2 - 4)^2 + (1 - (-3))^2 + (5 - 2)^2} \\
 &= \sqrt{(-6)^2 + 4^2 + 3^2} \\
 &= \sqrt{36 + 16 + 9} \\
 &= \sqrt{61}
 \end{aligned}$$

$$d = 7.81 \text{ units (3 sf)}$$

- ii) Find the midpoint of [AB].



Your notes

Formula for the midpoint of a line segment:

$$MP = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

in formula booklet

$$A: (-2, 1, 5) \quad B: (4, -3, 2)$$

$x_1$   $y_1$   $z_1$        $x_2$   $y_2$   $z_2$

Substitute:

$$\begin{aligned} MP &= \left( \frac{-2+4}{2}, \frac{1+(-3)}{2}, \frac{5+2}{2} \right) \\ &= \left( \frac{2}{2}, -\frac{2}{2}, \frac{7}{2} \right) \end{aligned}$$

$$MP = (1, -1, 3.5)$$



Your notes

## 3.2.2 Volume & Surface Area

### Volume of 3D Shapes

#### What is volume?

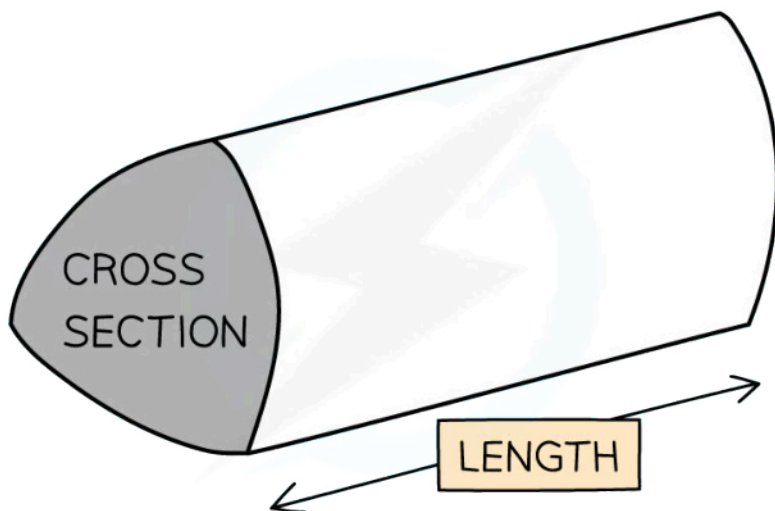
- The volume of a 3D shape is a measure of how much 3D space it takes up
  - A 3D shape is also called a **solid**
- You need to be able to calculate the volume of a number of common shapes

#### How do I find the volume of cuboids, prisms and cylinders?

- A prism is a 3-D shape that has two identical **base** shapes connected by parallel **edges**
  - A prism has the same base shape all the way through
  - A **prism** takes its name from its base
- To find the **volume** of any prism use the formula:

$$\text{Volume of a prism} = Ah$$

- Where **A** is the area of the cross section and **h** is the base height
  - **h** could also be the length of the prism, depending on how it is oriented
- This is in the formula booklet in the **prior learning** section at the beginning
- The base could be any shape so as long as you know its area and length you can calculate the volume of any prism



Copyright © Save My Exams. All Rights Reserved





Your notes

- Note two special cases:
  - To find the volume of a cuboid use the formula:

**Volume of a cuboid = length × width × height**

$$V = lwh$$

- The volume of a **cylinder** can be found in the same way as a prism using the formula:

**Volume of a cylinder =  $\pi r^2 h$**

- where  $r$  is the radius,  $h$  is the height (or length, depending on the orientation)
- Note that a cylinder is technically not a prism as its base is not a polygon, however the method for finding its volume is the same
- Both of these are **in the formula booklet** in the **prior learning** section

### How do I find the volume of pyramids and cones?

- In a **right-pyramid** the apex (the joining point of the triangular faces) is vertically above the centre of the base
  - The base can be any shape but is usually a square, rectangle or triangle
- To calculate the volume of a **right-pyramid** use the formula

$$V = \frac{1}{3} Ah$$

- Where  $A$  is the area of the base,  $h$  is the height
- Note that the height must be **vertical to the base**
- A **right cone** is a circular-based pyramid with the vertical height joining the apex to the centre of the circular base
- To calculate the volume of a **right-cone** use the formula

$$V = \frac{1}{3} \pi r^2 h$$

- Where  $r$  is the radius,  $h$  is the height
- These formulae are both given in the formula booklet

### How do I find the volume of a sphere?

- To calculate the volume of a **sphere** use the formula

$$V = \frac{4}{3} \pi r^3$$

- Where  $r$  is the radius
  - the line segment from the centre of the sphere to the surface

- This formula is given in the formula booklet

### Examiner Tip

- Remember to make use of the formula booklet in the exam as all the volume formulae you need will be here
  - Formulae for basic 3D objects (cuboid, cylinder and prism) are in the **prior learning** section
  - Formulae for other 3D objects (pyramid, cone and sphere) are in the **Topic 3: Geometry** section



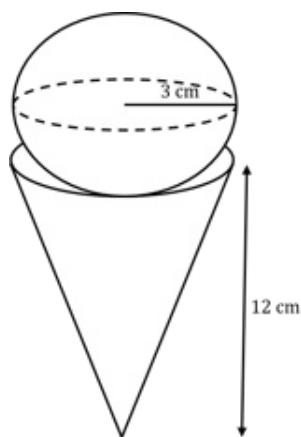
Your notes



Your notes

### Worked example

A dessert can be modelled as a right-cone of radius 3 cm and height 12 cm and a scoop of ice-cream in the shape of a sphere of radius 3 cm. Find the total volume of the ice-cream and cone.



Volume of a sphere:  $V = \frac{4}{3} \pi r^3$  (In formula booklet)

Substitute:  $r = 3 \Rightarrow V = \frac{4}{3} \pi \times 3^3$   
 $= 36\pi$

Volume of a right cone:  $V = \frac{1}{3} \pi r^2 h$  (In formula booklet)

Substitute:  $r = 3, h = 12 \Rightarrow V = \frac{1}{3} \pi (3)^2 (12)$   
 $= 36\pi$

Total Volume =  $72\pi \text{ cm}^3$

**Total Volume =  $226 \text{ cm}^3$  (3sf)**





Your notes

## Surface Area of 3D Shapes

### What is surface area?

- The surface area of a 3D shape is the sum of the areas of all the **faces** that make up a shape
  - A **face** is one of the flat or curved surfaces that make up a 3D shape
  - It often helps to consider a 3D shape in the form of its 2D net

### How do I find the surface area of cuboids, pyramids and prisms?

- Any prisms and pyramids that have polygons as their bases have only flat faces
  - The surface area is simply found by adding up the areas of these flat faces
  - Drawing a 2D net will help to see which faces the 3D shape is made up of

### How do I find the surface area of cylinders, cones and spheres?

- Cones, cylinders and spheres all have curved faces so it is not always as easy to see their shape
  - The net of a **cylinder** is made up of two identical circles and a rectangle
  - The rectangle is the curved surface area and is harder to identify
  - The length of the rectangle is the same as the circumference of the circle
  - The area of the **curved surface area** is

$$A = 2\pi rh$$

- where  $r$  is the radius,  $h$  is the height
    - This is given in the formula book in the prior learning section
    - The area of the **total surface area of a cylinder** is
- $$A = 2\pi rh + 2\pi r^2$$
- This is **not** given in the formula book, however it is easy to put together as both the area of a circle and the area of the curved surface area are given
  - The net of a **cone** consists of the circular base along with the curved surface area
    - The area of the **curved surface area** is

$$A = \pi rl$$

- Where  $r$  is the radius and  $l$  is the **slant height**
  - This is **given in the formula book**
    - Be careful not to confuse the slant height,  $l$ , with the vertical height,  $h$
    - Note that  $r$ ,  $h$  and  $l$  will create a **right-triangle** with  $l$  as the hypotenuse
  - The area of the **total surface area of a cone** is

$$A = \pi rl + \pi r^2$$

- This is **not** given in the formula book, however it is easy to put together as both the area of a circle and the area of the curved surface area are given
- To find the surface area of a **sphere** use the formula

$$A = 4\pi r^2$$

- where  $r$  is the radius (line segment from the centre to the surface)
- This is given in the formula booklet, you do not have to remember it



Your notes

### Examiner Tip

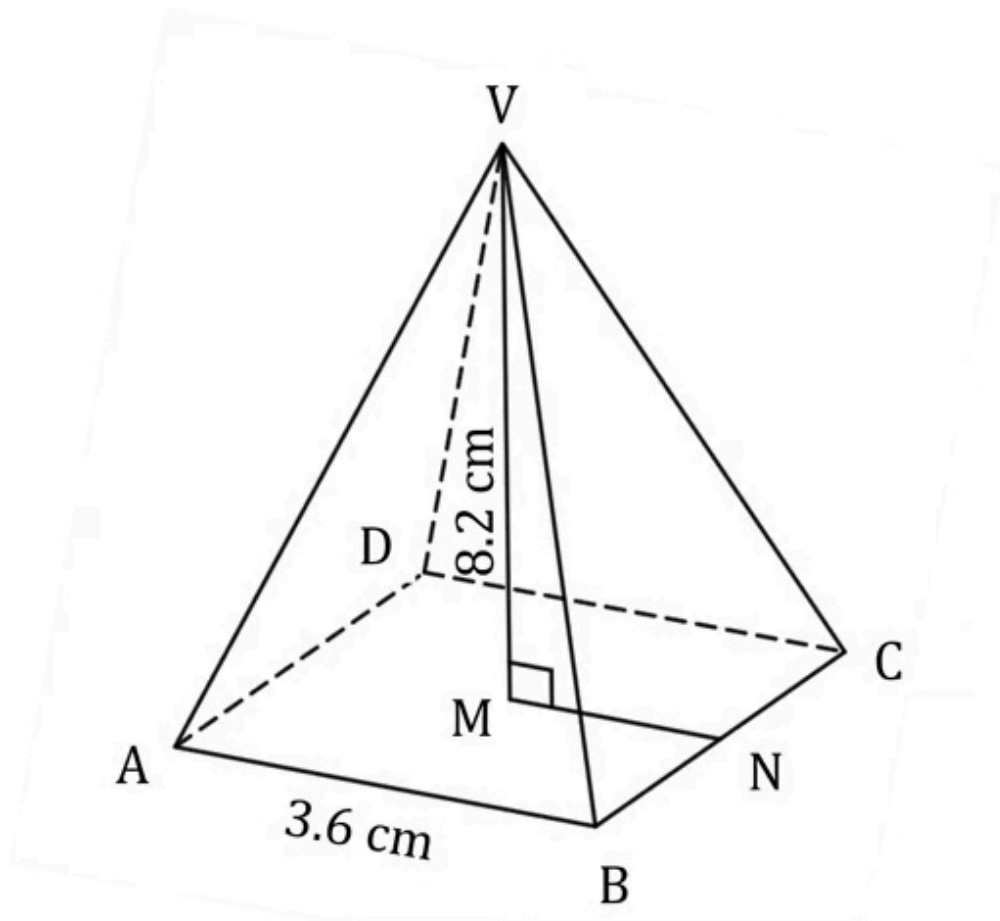
- Remember to make use of the formula booklet in the exam as all the area formulae you need will be here
  - Formulae for basic 2D shapes (parallelogram, triangle, trapezoid, circle, curved surface of a cylinder) are in the **prior learning** section
  - Formulae for other 2D shapes (curved surface area of a cone and surface area of a sphere) are in the **Topic 3: Geometry** section



Your notes

 **Worked example**

In the diagram below  $ABCD$  is the square base of a right pyramid with vertex  $V$ . The centre of the base is  $M$ . The sides of the square base are  $3.6$  cm and the vertical height is  $8.2$  cm.

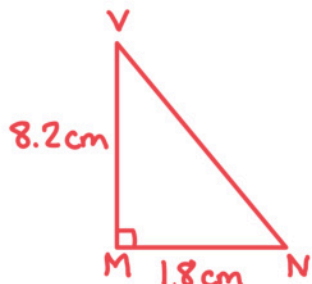


- i) Use the Pythagorean Theorem to find the distance  $VN$ .



Your notes

Sketch the triangle MNV:



M is the midpoint  
so  $MN = 3.6 \div 2$

By the Pythagorean Theorem:

$$\begin{aligned} VN^2 &= \sqrt{VM^2 + MN^2} \\ &= \sqrt{8.2^2 + 1.8^2} \\ &= \sqrt{70.48} \end{aligned}$$

$$VN = 8.40 \text{ cm (3sf)}$$

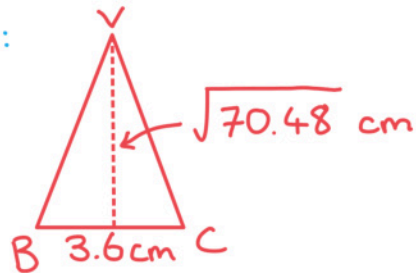
- ii) Calculate the area of the triangle ABV.



Your notes

$$\text{Area } \triangle ABV = \text{area } \triangle BCV$$

Sketch  $\triangle BCV$ :



$$\text{Area of a triangle} = \frac{1}{2}bh$$

$$\text{Substitute } b = 3.6, h = \sqrt{70.48}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}(3.6)(\sqrt{70.48}) \\ &= 15.111\dots \text{ cm}^2 \end{aligned}$$

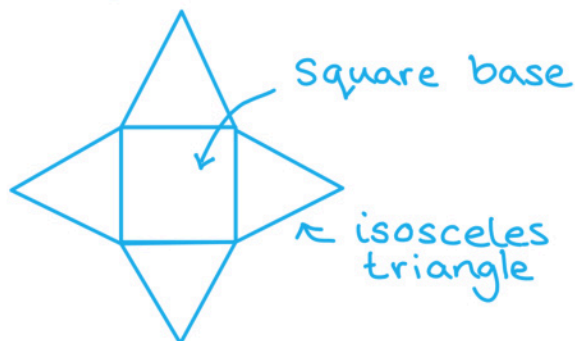
$$\text{Area } \triangle ABV = 15.1 \text{ cm}^2$$

- iii) Find the surface area of the right pyramid.



Your notes

Considering the net may help:



$$\text{Surface area} = \text{area square} + 4(\text{area triangle})$$

$$\begin{aligned} SA &= 3.6^2 + 4(15.111\dots) \\ &= 73.405\dots \text{ cm}^2 \end{aligned}$$

$$SA = 73.4 \text{ cm}^2 \text{ (3sf)}$$