

DP IB Maths: AI SL



Your notes

3.1 Geometry Toolkit

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Your notes

3.1.1 Coordinate Geometry

Basic Coordinate Geometry

What are cartesian coordinates?

- **Cartesian** coordinates are basically the x-y coordinate system
 - They allow us to label where things are in a two-dimensional plane
- In the 2D cartesian system, the horizontal axis is labelled x and the vertical axis is labelled y

What can we do with coordinates?

- If we have two points with coordinates (x_1, y_1) and (x_2, y_2) then we should be able to find
 - The **midpoint** of the two points
 - The **distance** between the two points
 - The **gradient** of the line between them

How do I find the midpoint of two points?

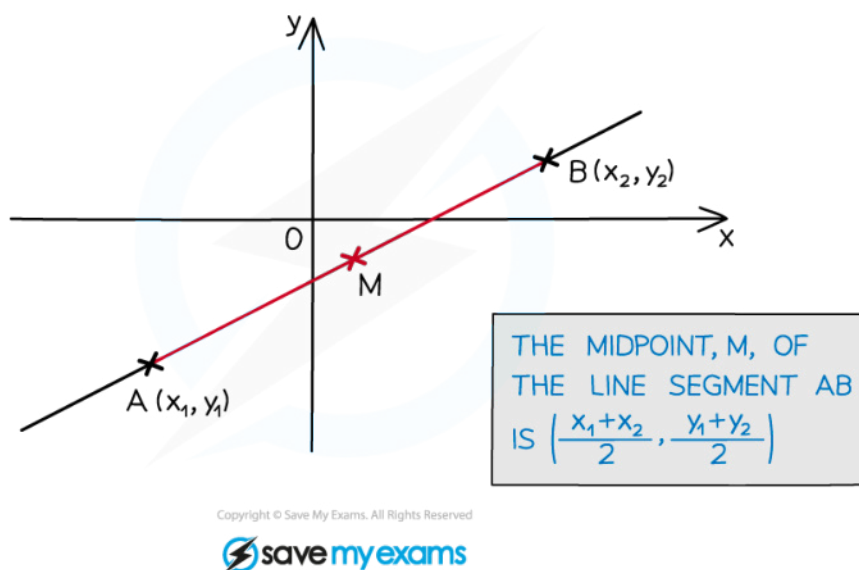
- The midpoint is the **average (middle) point**
 - It can be found by finding the middle of the x-coordinates and the middle of the y-coordinates
- The coordinates of the midpoint will be

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- This is given in the formula booklet under the prior learning section at the beginning



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How do I find the distance between two points?

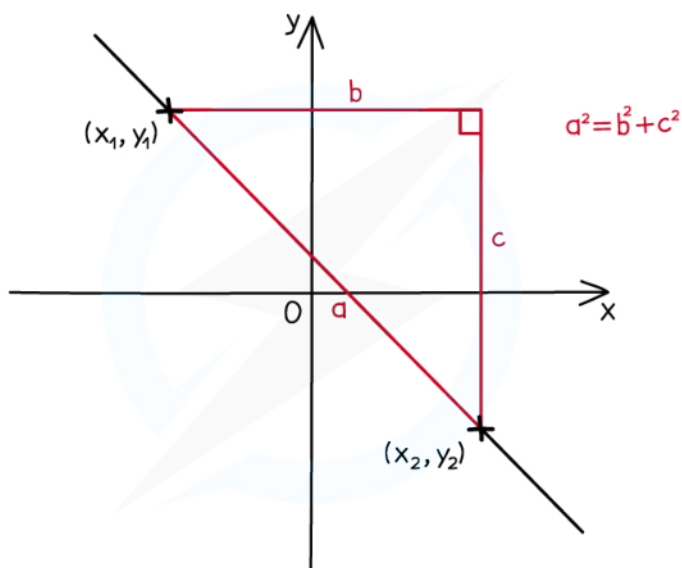
- The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) can be found using the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- This is given in the formula booklet in the *prior learning* section at the beginning
- Pythagoras' Theorem $a^2 = b^2 + c^2$ is used to find the length of a line between two coordinates
- If the coordinates are labelled A and B then the line segment between them is written with the notation [AB]



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How do I find the gradient of the line between two points?

- The gradient of a line between two points with coordinates (x_1, y_1) and (x_2, y_2) can be found using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- This is given in the formula booklet under section 2.1 Gradient formula

- This is usually known as $m = \frac{\text{rise}}{\text{run}}$



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Worked example

Point A has coordinates (3, -4) and point B has coordinates (-5, 2).

- i) Calculate the distance of the line segment AB.

$$\begin{array}{cc}
 A: (3, -4) & B: (-5, 2) \\
 \uparrow \quad \uparrow & \uparrow \quad \uparrow \\
 x_1 \quad y_1 & x_2 \quad y_2
 \end{array}$$

Formula for distance between two points:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

sub coordinates for A and B into the formula:

$$\begin{aligned}
 d &= \sqrt{(3 - (-5))^2 + (-4 - 2)^2} \\
 &= \sqrt{8^2 + (-6)^2} = \sqrt{100}
 \end{aligned}$$

$$d = 10 \text{ units}$$

- ii) Find the gradient of the line connecting points A and B.



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$$A: (3, -4) \quad B: (-5, 2)$$

x_1 y_1 x_2 y_2

Formula for gradient of a line segment:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Sub coordinates for A and B into the formula:

$$m = \frac{2 - (-4)}{-5 - 3} = \frac{6}{-8} = -\frac{3}{4}$$

$$m = -\frac{3}{4}$$

iii) Find the midpoint of [AB].

$$A: (3, -4) \quad B: (-5, 2)$$

x_1 y_1 x_2 y_2

Formula for the midpoint of two coordinates:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Sub values in:

$$\text{Midpoint} = \left(\frac{3 + (-5)}{2}, \frac{-4 + 2}{2} \right) = (-1, -1)$$

$$\text{Midpoint} = (-1, -1)$$



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Perpendicular Bisectors

What is a perpendicular bisector?

- A perpendicular bisector of a line segment cuts the line segment in half at a right angle
 - Perpendicular lines meet at right angles
 - Bisect means to cut in half
- Two lines are perpendicular if the **product of their gradients is -1**

How do I find the equation of the perpendicular bisector of a line segment?

- To find the equation of a straight line you need to find
 - The gradient of the line
 - A coordinate of a point on the line
- To find the equation of the **perpendicular bisector** of a line segment follow these steps:
 - STEP 1: Find the coordinates of the midpoint of the line segment
 - We know that the perpendicular bisector will cut the line segment in half so we can use the midpoint of the line segment as the known coordinate on the bisector
 - STEP 2: Find the gradient of the line segment
 - STEP 3: Find the gradient of the perpendicular bisector
 - This will be -1 divided by the gradient of the line segment
 - STEP 4: Substitute the gradient of the perpendicular bisector and the coordinates of the midpoint into an equation for a straight line
 - The **point-gradient** form $y - y_1 = m(x - x_1)$ is the easiest
 - STEP 5: Rearrange into the required form
 - Either $y = mx + c$ or $ax + by + d = 0$
 - These equations for a straight line are given in the formula booklet



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Worked example

Point A has coordinates (4, -6) and point B has coordinates (8, 6). Find the equation of the perpendicular bisector to [AB]. Give your answer in the form $ax + by + d = 0$.

Step 1: find the coordinates of the midpoint:

$$\begin{array}{ccc}
 A: (4, -6) & B: (8, 6) & \\
 \uparrow & \uparrow & \\
 x_1 & y_1 & x_2 & y_2 & \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
 \end{array}$$

Sub values in:

$$\text{Midpoint} = \left(\frac{4 + 8}{2}, \frac{-6 + 6}{2} \right) = (6, 0)$$

Step 2: Find the gradient of [AB]:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - -6}{8 - 4} = \frac{12}{4} = 3$$

Step 3: Find the gradient of the perpendicular bisector:

$$m_{\perp} = -\frac{1}{m} = -\frac{1}{3}$$

Step 4: Substitute gradient and coordinate into an equation for a straight line.

$$\begin{array}{l}
 \text{insert coordinates of the midpoint.} \\
 (y - y_1) = m(x - x_1) \\
 (y - 0) = -\frac{1}{3}(x - 6)
 \end{array}$$

Step 5: Rearrange into the form $ax + by + d = 0$

$$\begin{array}{l}
 (y - 0) = -\frac{1}{3}(x - 6) \quad (x - 3) \\
 -3y = x - 6 \quad (+3y)
 \end{array}$$

$$x + 3y - 6 = 0$$



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3.1.2 Arcs & Sectors

Length of an Arc

What is an arc?

- An arc is a part of the **circumference** of a circle
 - It is easiest to think of it as the crust of a single slice of pizza
- The length of an arc depends of the size of the angle at the centre of the circle
- If the angle at the centre is **less than 180°** then the arc is known as a **minor arc**
 - This could be considered as the crust of a single slice of pizza
- If the angle at the centre is **more than 180°** then the arc is known as a **major arc**
 - This could be considered as the crust of the remaining pizza after a slice has been taken away

How do I find the length of an arc?

- The length of an arc is simply a fraction of the circumference of a circle
 - The fraction can be found by dividing the angle at the centre by 360°
- The formula for the length, l , of an arc is

$$l = \frac{\theta}{360} \times 2\pi r$$

- Where θ is the angle measured in degrees
- r is the radius
- This is **in the formula booklet**, you do not need to remember it

Examiner Tip

- Make sure that you read the question carefully to determine if you need to calculate the arc length of a sector, the perimeter or something else that incorporates the arc length!



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Worked example

A circular pizza has had a slice cut from it, the angle of the slice that was cut was 38° . The radius of the pizza is 12 cm. Find

- i) the length of the outside crust of the slice of pizza (the minor arc),

A diagram will help:



Formula for the length of an arc:

$$l = \frac{\theta}{360} \times 2\pi r$$

Substitute:

$$\begin{aligned} l &= \frac{38}{360} \times 2\pi (12) \\ &= \frac{38\pi}{15} = 7.9587\dots \text{ cm} \end{aligned}$$

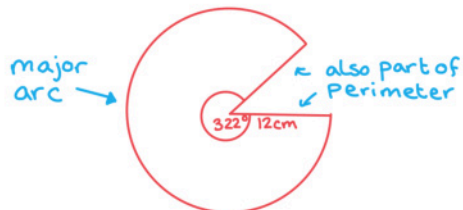
Length of crust = 7.96 cm (3sf)

- ii) the perimeter of the remaining pizza.



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A diagram will help:



Formula for the Length of an arc:

$$l = \frac{\theta}{360} \times 2\pi r$$

Substitute:

$$\begin{aligned} l &= \frac{322}{360} \times 2\pi (12) \\ &= \frac{322\pi}{15} \leftarrow \text{Length of major arc} \end{aligned}$$

Find perimeter:

$$\begin{aligned} P &= \text{major arc} + \text{radius} + \text{radius} \\ &= \frac{322\pi}{15} + 12 + 12 = 91.4395... \text{ cm} \end{aligned}$$

$$\text{Perimeter} = 91.4 \text{ cm (3sf)}$$



Your notes

Area of a Sector

What is a sector?

- A sector is a part of a circle enclosed by two radii (radiuses) and an arc
 - It is easier to think of this as the shape of a single slice of pizza
- The area of a sector depends of the size of the angle at the centre of the sector
- If the angle at the centre is **less than 180°** then the sector is known as a **minor sector**
 - This could be considered as the shape of a single slice of pizza
- If the angle at the centre is **more than 180°** then the sector is known as a **major sector**
 - This could be considered as the shape of the remaining pizza after a slice has been taken away

How do I find the area of a sector?

- The area of a sector is simply a fraction of the area of the whole circle
 - The fraction can be found by dividing the angle at the centre by 360°
- The formula for the area, A , of a sector is

$$A = \frac{\theta}{360} \times \pi r^2$$

- Where θ is the angle measured in degrees
- r is the radius
- This is **in the formula booklet**, you do not need to remember it



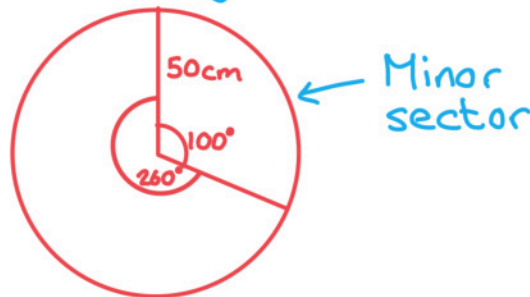
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 **Worked example**

Jamie has divided a circle of radius 50 cm into two sectors; a minor sector of angle 100° and a major sector of angle 260° . He is going to paint the minor sector blue and the major sector yellow. Find

- i) the area Jamie will paint blue,

Start with a diagram:



Formula for the area of a sector:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\begin{aligned} \text{Substitute: } A &= \frac{100}{360} \times \pi \times 50^2 \\ &= \frac{6250}{9} \pi \\ &= 2181.66... \text{ cm}^2 \end{aligned}$$

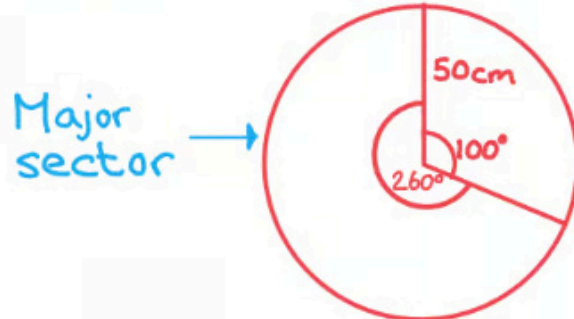
Blue area = 2180 cm^2 (3sf)

- ii) the area Jamie will paint yellow.



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Start with a diagram:



Formula for the area of a sector:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

Substitute: $A = \frac{260}{360} \times \pi \times 50^2$

$$= \frac{16250}{9} \pi$$

$$= 5672.32... \text{ cm}^2$$

Yellow area = 5670 cm² (3sf)