

DP IB Maths: AA SL



Your notes

1.4 Proof & Reasoning

Contents

* 1.4.1 Proof



Your notes

1.4.1 Proof

Language of Proof

What is proof?

- Proof is a series of logical steps which show a result is **true** for all specified numbers
 - 'Seeing' that a result works for a few numbers is not enough to **show** that it will work for all numbers
 - Proof allows us to show (usually algebraically) that the result will work for **all values**
- You must be familiar with the notation and language of proof
- LHS and RHS are standard abbreviations for left-hand side and right-hand side
- **Integers** are used frequently in the language of proof
 - The set of **integers** is denoted by \mathbb{Z}
 - The set of **positive integers** is denoted by \mathbb{Z}^+

How do we prove a statement is true for all values?

- Most of the time you will need to use algebra to show that the left-hand side (LHS) is the same as the right-hand side (RHS)
 - You **must not** move terms from one side to the other
 - Start with one side (usually the LHS) and manipulate it to show that it is the same as the other
- A **mathematical identity** is a statement that is true for all values of x (or θ in trigonometry)
 - The symbol \equiv is used to identify an identity
 - If you see this symbol then you can use proof methods to show it is true
- You can complete your proof by stating that $\text{RHS} = \text{LHS}$ or writing QED

Examiner Tip

- You will need to show each step of your proof clearly and set out your method in a logical manner in the exam
 - Be careful not to skip steps



Your notes

Worked example

Prove that $(2x - 2)(x - 3) + 2(x - 1) = 2(x - 2)(x - 1)$.

Work with LHS first:

Expand brackets:

$$\text{LHS: } (2x - 2)(x - 3) + 2(x - 1)$$

Note: The diagram shows blue arrows indicating the FOIL method for the first part of the expression.

$$2x^2 - 6x - 2x + 6 + 2x - 2$$

Simplify, take care with negatives:

$$2x^2 - 6x + 4$$

Factorise the 2:

$$2(x^2 - 3x + 2)$$

Factorise remaining quadratic:

$$2(x - 2)(x - 1) = \text{RHS as required.}$$

$$(2x - 2)(x - 3) + 2(x - 1) = 2(x - 2)(x - 1)$$



Your notes

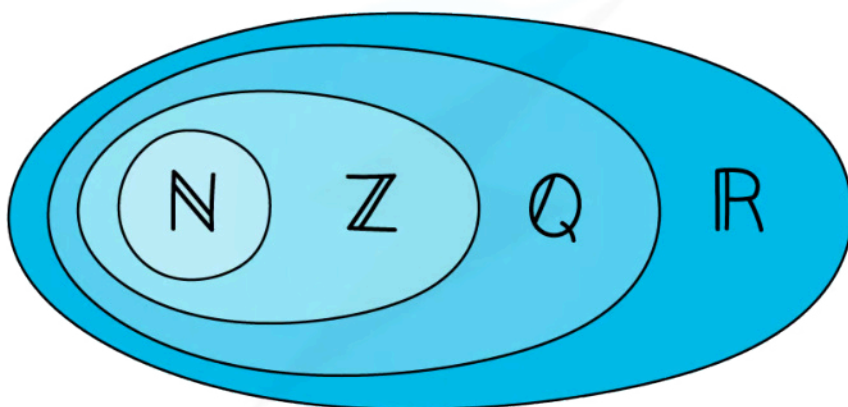
Proof by Deduction

What is proof by deduction?

- A mathematical and logical argument that shows that a result is true

How do we do proof by deduction?

- A proof by deduction question will often involve showing that a result is true for all integers, consecutive integers or even or odd numbers
 - You can begin by letting an integer be n
 - Use conventions for even ($2n$) and odd ($2n - 1$) numbers
- You will need to be familiar with sets of numbers (\mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{R})
 - \mathbb{N} - the set of **natural numbers**
 - \mathbb{Z} - the set of **integers**
 - \mathbb{Q} - the set of **quotients (rational numbers)**
 - \mathbb{R} - the set of **real numbers**



Copyright © Save My Exams. All Rights Reserved



Examiner Tip

- Try the result you are proving with a few different values
 - Use a sequence of them (eg 1, 2, 3)
 - Try different types of numbers (positive, negative, zero)
- This may help you see a pattern and spot what is going on



Your notes

Worked example

Prove that the sum of any two consecutive odd numbers is always even.

Let $2n - 1$ be an odd number
↑
must be even

Let two consecutive odd integers be:

$2n - 1$, $2n + 1$
↖ next odd number

Then their sum is:

$$\begin{aligned} 2n - 1 + 2n + 1 &\equiv 4n \\ &= 2(2n) \end{aligned}$$

Any multiple of 2 must be even.