SL IB Physics

Your notes

Work, Energy & Power

Contents

Principle of Conservation of Energy

Principle of Conservation of Energy

Your notes

- The Principle of conservation of energy states that:

  Energy cannot be created or destroyed, it can only be transferred from one form to another

- This means the total amount of energy in a closed system **remains constant**, although how much of **each form** there is **may change**
- In physics, a **system** is defined as:

  An object or group of objects

- Defining the system in physics is a way of **narrowing** the parameters to **focus** only on what is relevant to the situation being observed
- When a system is in **equilibrium**, nothing changes and so nothing happens
- When there is a **change** in a system, things happen, and when things happen, **energy is transferred**

Types of Energy

FORM	WHAT IS IT?
KINETIC	THE ENERGY OF A MOVING OBJECT.
GRAVITATIONAL POTENTIAL	THE ENERGY SOMETHING GAINS WHEN YOU LIFT IT UP, AND WHICH IT LOSES WHEN IT FALLS.
ELASTIC	THE ENERGY OF A STRETCHED SPRING OR ELASTIC BAND.(SOMETIMES CALLED STRAIN ENERGY)
CHEMICAL	THE ENERGY CONTAINED IN A CHEMICAL SUBSTANCE.
NUCLEAR	THE ENERGY CONTAINED WITHIN THE NUCLEUS OF AN ATOM.
INTERNAL	THE ENERGY SOMETHING HAS DUE TO ITS TEMPERATURE (OR STATE). (SOMETIMES REFERRED TO AS THERMAL OR HEAT ENERGY)

Your notes

- Kinetic energy, gravitational potential energy, and elastic potential energy are collectively known as **mechanical** energy types

Energy Dissipation

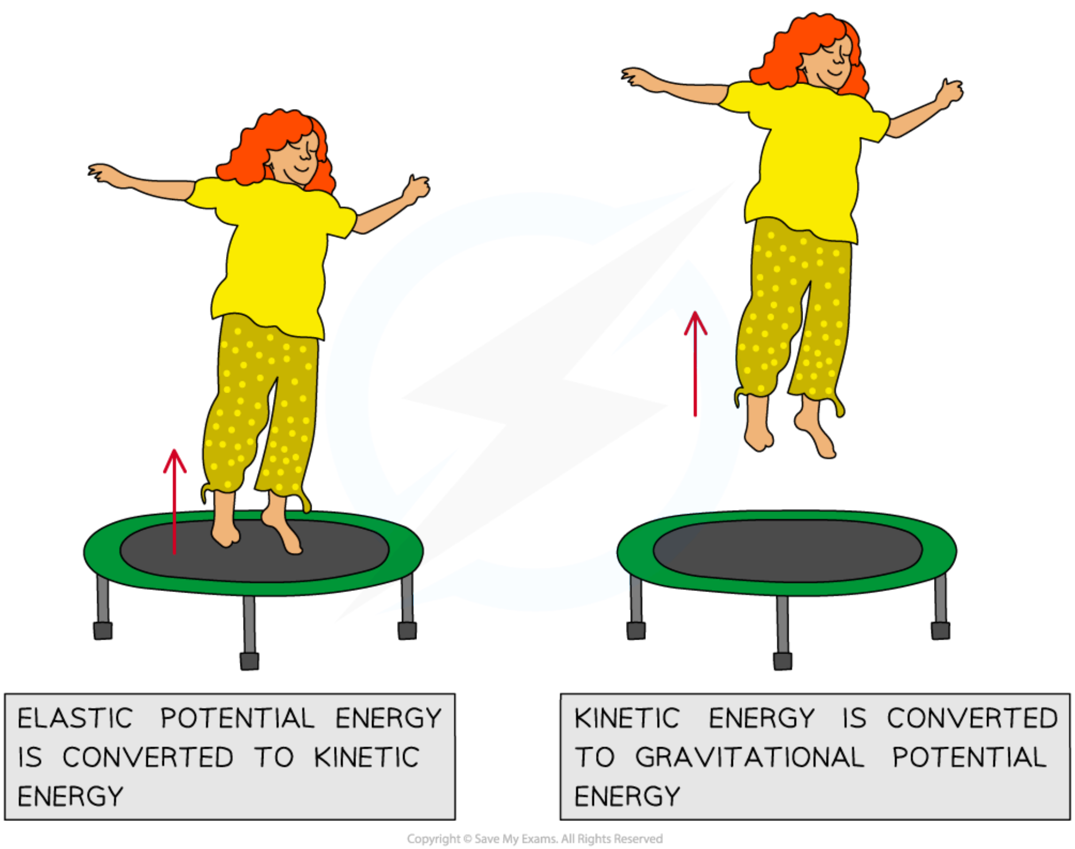
- No energy transfer is 100% efficient
- When energy is transformed from one form to another, some of the energy is **dissipated** to the surroundings
- Dissipated energy usually ends up as **thermal energy** transferred to the **surroundings** where it cannot be easily used for another purpose
- Therefore, dissipated energy is usually regarded as **wasted energy**

- A kettle transforms **electrical** energy into **thermal** energy
- The **thermal** energy in the **heating element** is transferred to **thermal** energy in the **water**
  - Some thermal energy is also transferred to the **plastic casing**
  - Some thermal energy is also **dissipated** to the surrounding air
- The energy transfers that are useful for heating the water are considered **useful** energy transfers
- The energy transfers that are not useful for heating the water are considered **wasted** energy transfers

Applications of Energy Conservation

- In mechanical systems, the **energy transferred** is equivalent to the **work done**
  - A falling object (in a vacuum, where no energy is not dissipated into the surroundings) transfers its gravitational potential energy into kinetic energy
  - Horizontal mass on a spring transfers its elastic potential energy into kinetic energy
  - A battery or cell transfers its chemical energy into electrical energy
  - A car transfers chemical energy from the fuel into kinetic energy of the car
  - A person bouncing on a trampoline is transferring energy from elastic potential to kinetic to gravitational potential

Your notes



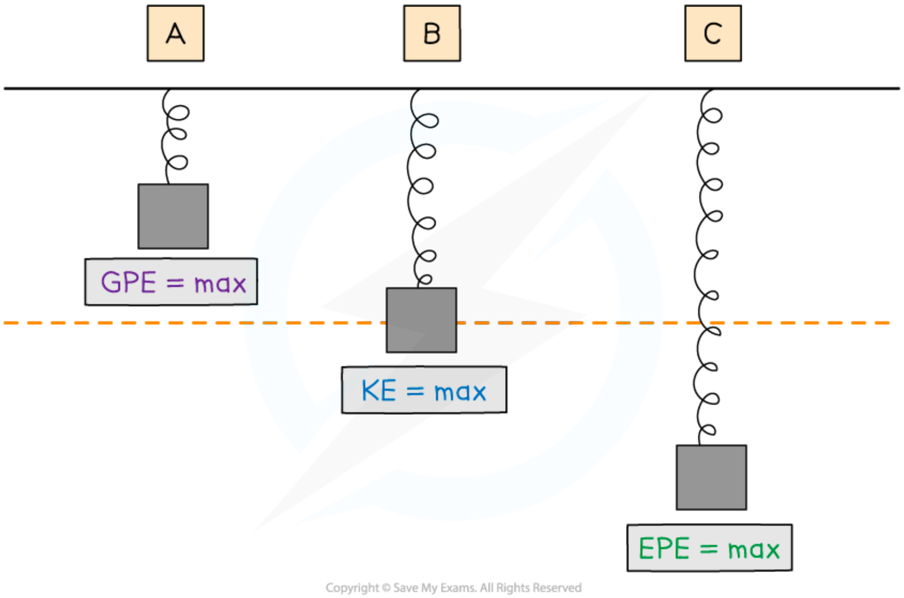
Energy transfers whilst jumping on a trampoline

- There may also be work done against resistive forces such as **friction**
- For example, if an object travels up a rough inclined surface, then

Loss in kinetic energy = Gain in gravitational potential energy + Work done against friction

Spring Energy Conservation

- When a vertical spring oscillates, its energy is converted into other forms
- Although the total energy of the spring will remain constant, it will have changing amounts of:
  - **Elastic** potential energy (EPE)
  - **Kinetic** energy (KE)
  - **Gravitational** potential energy (GPE)



Your notes

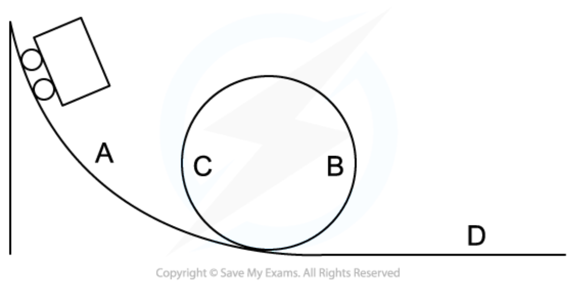
- At position **A:**
  - The spring has some EPE because it is slightly compressed
  - Its KE is **zero** because it is stationary
  - Its **GPE is at a maximum** because the mass is at its highest point
- At position **B:**
  - The spring has some EPE because it is slightly stretched
  - Its **KE is at a maximum** as it passes through the equilibrium position at its maximum speed
  - It has some GPE because the mass is still raised
- At position **C:**
  - The spring has its **maximum** EPE because the spring is at its maximum extension
  - Its KE is **zero** because it is stationary
  - Its **GPE is at a minimum** because the mass is at its lowest point
- For a **horizontal** mass on a spring system, you do not need to consider the gravitational potential energy because this does not change

Your notes

Worked example

The diagram shows a rollercoaster going down a track.

The rollercoaster takes the path A → B → C → D.



Which statement is true about the energy changes that occur for the rollercoaster down this track?

A. KE - GPE - GPE - KE

B. KE - GPE - KE - GPE

C. GPE - KE - KE - GPE

D. GPE - KE - GPE - KE

ANSWER: D

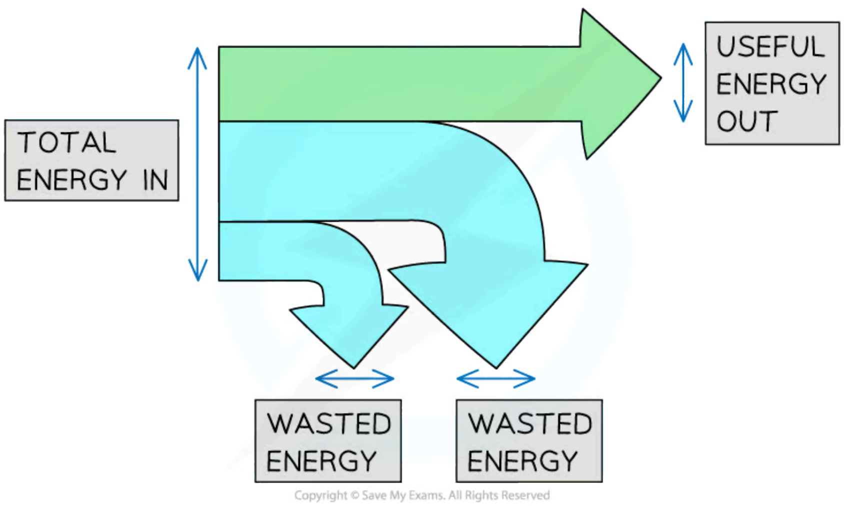
- **At point A:**
  - The rollercoaster is raised above the ground, therefore it has **GPE**
  - As it travels down the track, **GPE** is converted to **KE** and the roller coaster speeds up
- **At point B:**
  - **KE** is converted to **GPE** as the rollercoaster rises up the loop
- **At point C:**
  - This **GPE** is converted back into **KE** as the rollercoaster travels back down the loop
- **At point D:**
  - The flat terrain means the rollercoaster only has **KE**

Sankey Diagrams

Your notes

Sankey Diagrams

- **Sankey diagrams** are used to represent energy transfers
- The arrow in a Sankey diagram represents the transfer of energy:
  - The end of the arrow pointing to the **right** represents the energy that ends up in the **desired** store (the **useful energy output**)
  - The end(s) that point(s) down represents the **wasted energy**

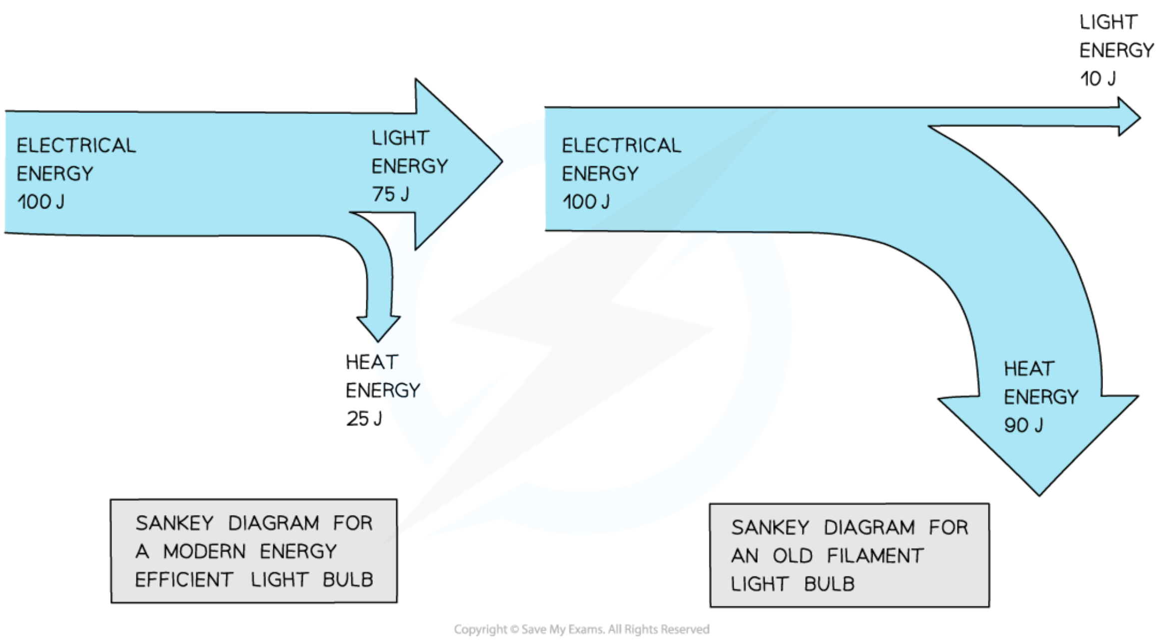


Total energy in, wasted energy and useful energy out shown on a Sankey diagram

- The width of each arrow is proportional to the amount of energy going to each store
- As a result of the conversation of energy:

Total energy in = Useful energy out + Wasted energy

- A Sankey diagram for a modern, efficient light bulb will look very different from that for an old filament light bulb
- A more efficient light bulb has **less** wasted energy
  - This is shown by the smaller arrow downward, representing energy transferred by heating

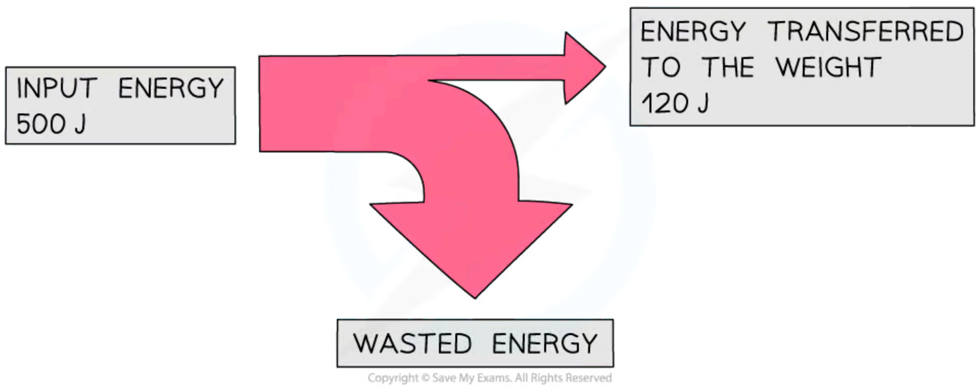


Sankey diagram for modern vs. old filament light bulb

Your notes

Worked example

An electric motor is used to lift a weight. The diagram represents the energy transfers in the motor.



Determine the amount of wasted energy.

Answer:

Step 1: State the conservation of energy

- Energy cannot be created or destroyed, it can only be moved from one store to another
- This means that:

Total energy in = Useful energy out + Wasted energy

Step 2: Rearrange the equation for the wasted energy

Wasted energy = Total energy in – Useful energy out

Step 3: Substitute the values from the diagram

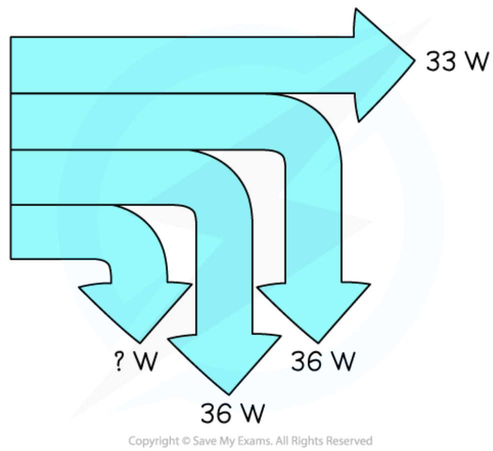
500 – 120 = **380 J**

Worked example

A small electric car is driven by a 120 watt motor.

The useful power output of the motor is measured to be 33 W. 36 W of power is wasted on friction losses and a further 36 W is wasted on traction losses.

Further power is lost by the electric car during operation. This situation is shown in the diagram below.



Determine the remaining power loss for the electric car when it is operating.

Answer:

Step 1: State the conservation of energy

- Energy cannot be created or destroyed, it can only be moved from one store to another
- This also applies to power
- This means that:

$$\text{Total power in} = \text{Useful power out} + \text{Friction losses} + \text{Traction losses} + \text{Wasted power}$$

Step 2: Rearrange the equation for the wasted power

$$\text{Wasted power} = \text{Total power in} - (\text{Useful power out} + \text{Friction losses} + \text{Traction losses})$$

Step 3: Substitute the values from the diagram

$$120 - (33 + 36 + 36) = \mathbf{15\ W}$$

Your notes

Your notes

Examiner Tip

- Drawing good Sankey diagrams takes practice.
- Start by planning your diagram:
  - How wide are you going to make the input arrow?
  - How wide will the 'useful energy out' arrow need to be?
  - How wide must the 'wasted energy' arrow be?
- Next, start drawing the diagram one step at a time:
  - Draw the left hand side of the arrow, along with the line going across the top
  - Next add the 'useful energy out' arrow, making sure it is the correct width
  - Now carefully mark the start and end of the wasted arrow – make sure your marks are the correct distance apart!
  - Finally join the markings together, finishing the 'wasted energy' arrow

Your notes

Work Done

Work Done

- The **work done** by a force is equivalent to a **transfer of energy**
  - The units of work done are newton metres
  - 1 N m = 1 J
- The work done by a **resultant force** on a system is equal to the change in energy in that system

- Mechanical work is defined as

  The transfer of energy when an external force causes an object to move over a certain distance

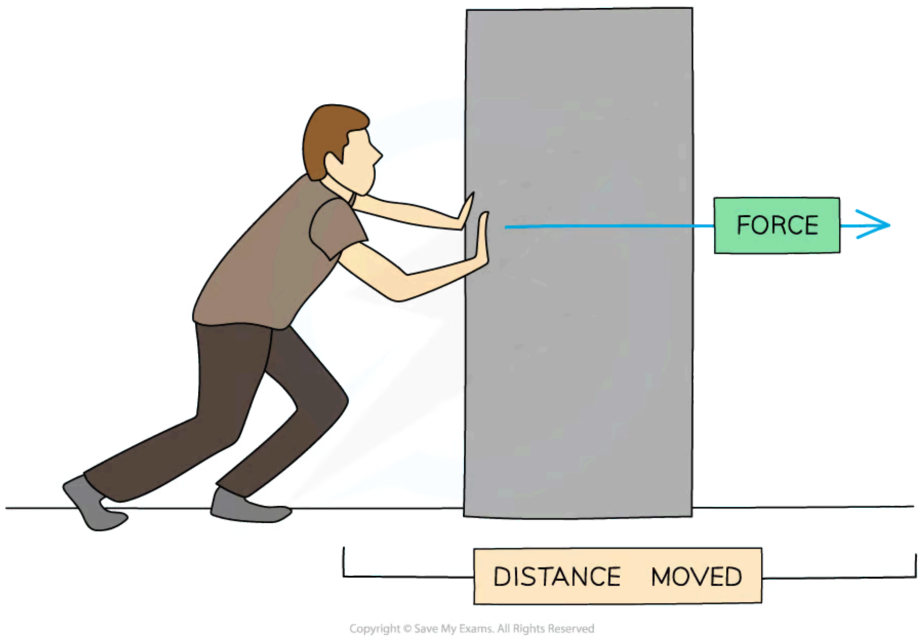
- If a constant force is applied in the line of an object's displacement (i.e. parallel to it), the work done can be calculated using the equation:

$$W = Fs$$

- Where:
  - W = work done (J)
  - F = constant force applied (N)
  - s = displacement (m)

- In the diagram below, the man's pushing force on the block is doing work as it is transferring energy to the block

Your notes



Work is done when a force is used to move an object over a distance

- When pushing a block, **work is done against friction** and energy is transferred from the man to the block
- The kinetic energy is transferred to other forms of energy such as heat and sound

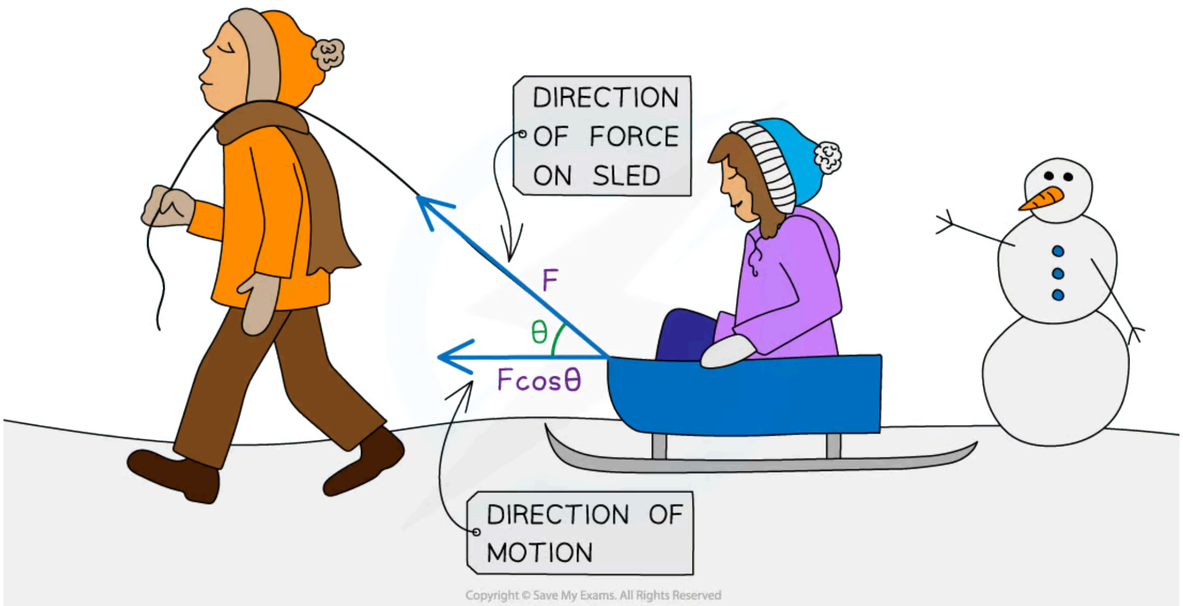
- When plotting a graph of average force applied against displacement, the **area** under the graph is equal to the **work done**

- Sometimes the direction of motion of an object is **not parallel** to the direction of the force
- If the force is at an **angle θ** to the object's displacement, the work done is calculated by:

$$W = Fs\cos\theta$$

- Where ***θ*** is the angle, in degrees, between the direction of the force and the motion of the object
  - When ***θ*** is 0 (the force is in the direction of motion) then $\cos\theta = 1$ and $W = Fs$

- For **horizontal** motion, cos θ is used
- For **vertical** motion, sin θ is used
  - Always consider the horizontal and vertical components of the force
  - The component needed is the one that is **parallel to the displacement**



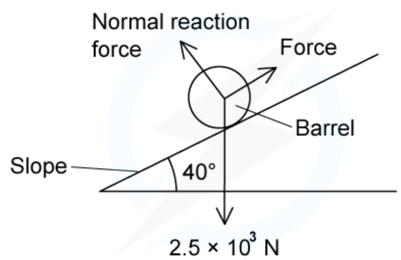
When the force is at an angle, only the component of the force in the direction of motion is considered for the work done

Your notes

Your notes

Worked example

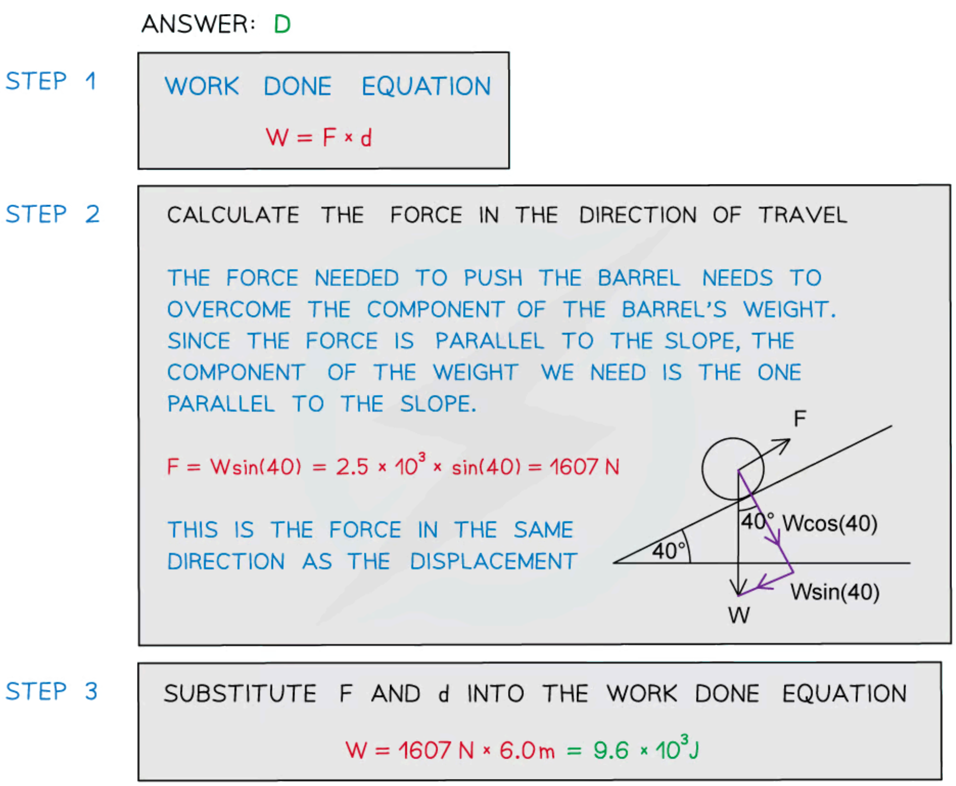
The diagram shows a barrel of weight 2.5×10^3 N on a frictionless slope inclined at 40° to the horizontal.



A force is applied to the barrel to move it up the slope at a constant speed. The force is parallel to the slope.

What is the work done in moving the barrel a distance of 6.0 m up the slope?

A. 7.2×10^3 J **B.** 2.5×10^4 J **C.** 1.1×10^4 J **D.** 9.6×10^3 J

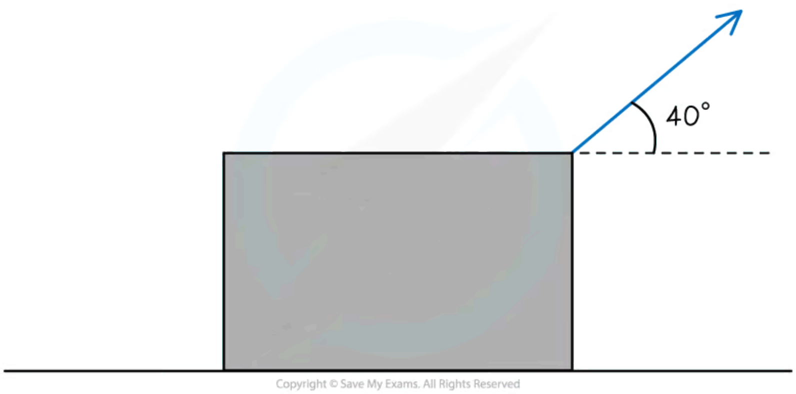


Your notes

Worked example

An 80 kg person pulls a 15 kg box along using a rope which is at 40° from the horizontal as shown below. The person is pulling with a force of 40 N and moves the box 20 m horizontally from its starting position against a constant friction force of 5.0 N.

Calculate the work that has been done on the box in the direction of its motion.

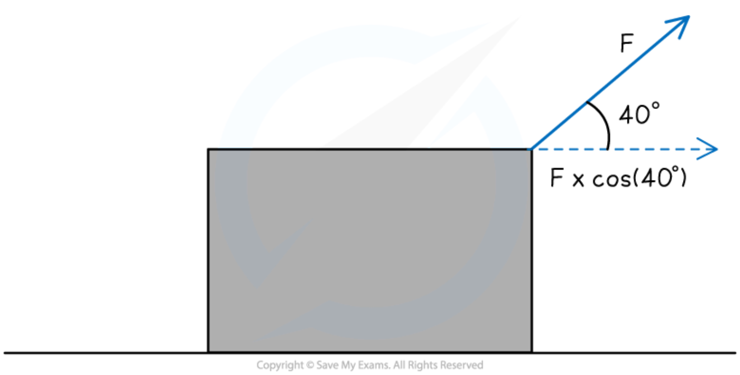


Answer

Step 1: List the known quantities

- The angle between the rope and the horizontal, θ = 40°
- The pulling force (along rope) = 40 N
- Horizontal distance moved by box, s = 20 m
- Frictional force = 5.0 N

Step 2: Resolve the pulling force in the rope into its horizontal component

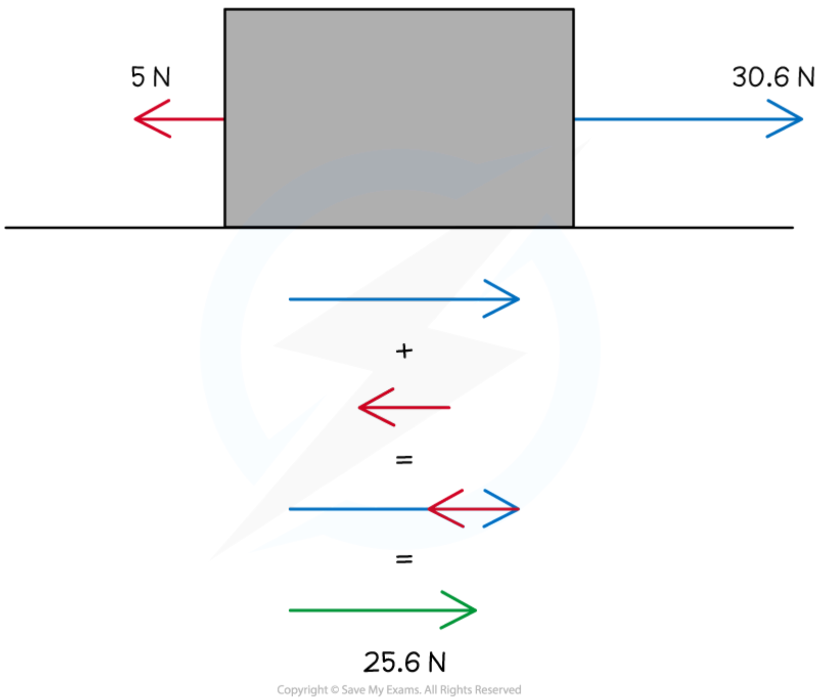


- The horizontal component of the pulling force is the only part of the pulling force aligned with the direction of work
- Hence, that is the component that is needed to continue solving this problem
- The horizontal component can be resolved from:

$$\cos(40) \times 40 = 30.6 \text{ N} \text{ to the right}$$

Step 3: Find the resultant force for the motion

- The resultant force can be found from the interaction between the horizontal component of the pulling force and the friction force:



$$30.6 + (-5) = 25.6 \text{ N} \text{ to the right}$$

Step 4: Calculate the work done

- Use the equation for work done given the resultant force and distance moved in the horizontal plane

$$W = Fs\cos\theta$$

- The **cos** θ has already been accounted for so that the resultant force could be found when combined with friction
- Therefore:

$$W = Fs = 25.6 \times 20 = 512 \text{ J}$$

W = **512 J**

Your notes

Examiner Tip

Sometimes exam questions will include more values than you need to use in the solution - this is purposefully done to confuse you. For example, in the second worked example above, the question supplies the mass of the person and the box, however, these quantities are not needed for the calculation.

Always consider the horizontal and vertical components of the force. The component needed is the one that is **parallel to the displacement**. The equation with $\cos\theta$ is given on your data sheet, but you only need to use this if the force is applied at an angle to the displacement.

Your notes

Your notes

Kinetic Energy

Kinetic Energy

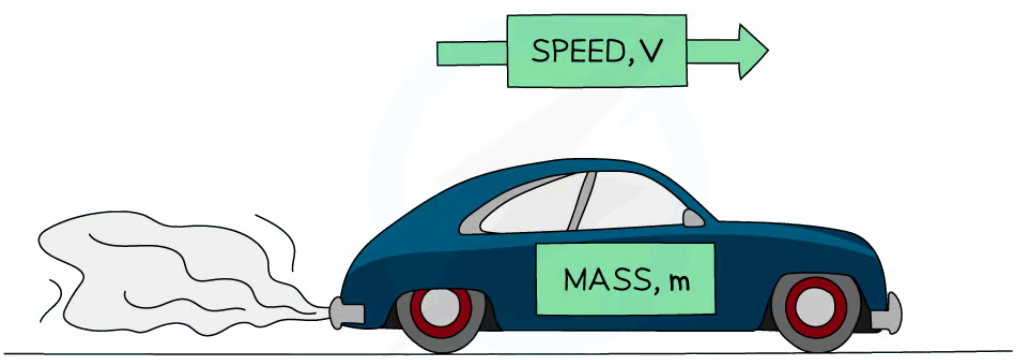
- Kinetic energy (E_k) is the energy an object has due to its **translational motion** (i.e. because it's moving)
  - The **faster** an object is moving, the **greater** its kinetic energy

- When an object is falling, it is **gaining** kinetic energy since it is **accelerating** under gravity
- This energy is transferred from the gravitational potential energy it is losing
- An object will **maintain** this kinetic energy unless its **speed** or **mass** changes

- Kinetic energy can be calculated using the following equation:

$$E_k = \frac{1}{2}mv^2$$

- Where:
  - E_k = kinetic energy (J)
  - m = mass (kg)
  - v = velocity (m s^{-1})



Kinetic energy: The energy an object has when it is moving

- Another quantity that also depends on mass m and velocity v is **momentum**
- Therefore, kinetic energy can be written in terms of momentum p, using the equation

$$E_k = \frac{p^2}{2m}$$

- Where:

- p = momentum (kg m s^{-1})

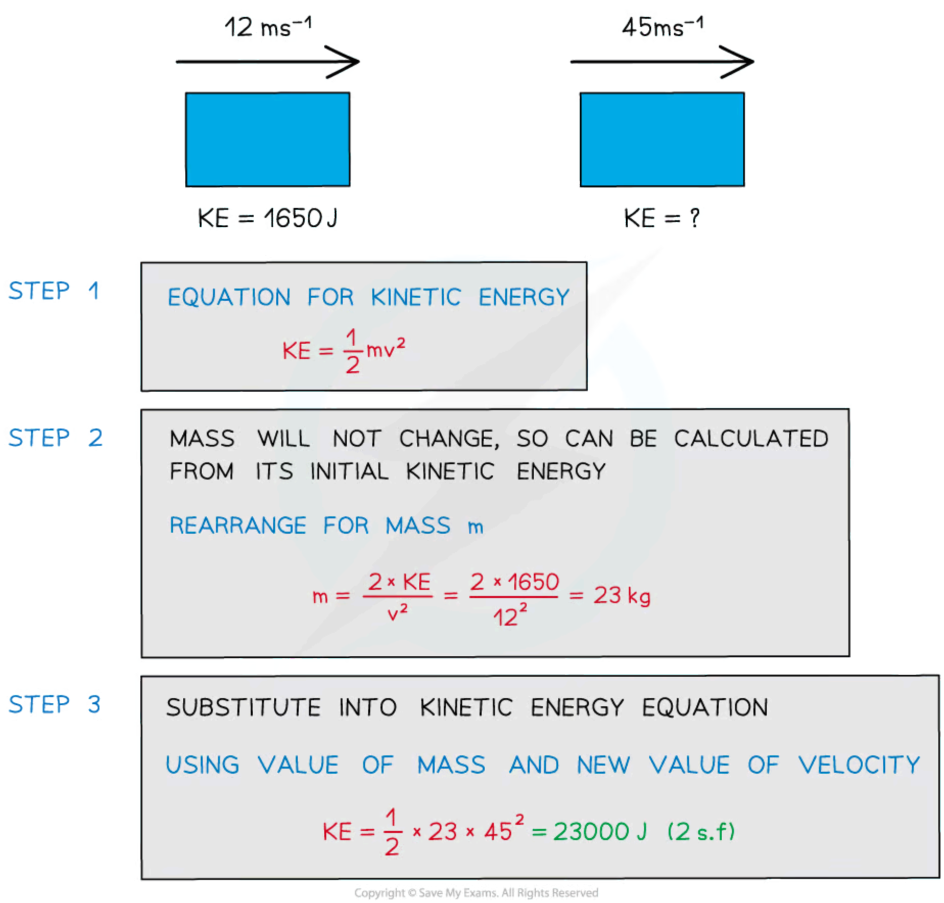
- This form is very useful in particle physics, when comparing the momentum and kinetic energy of a particle

Your notes

Worked example

A body travelling with a speed of 12 m s^{-1} has kinetic energy 1650 J. The speed of the body is increased to 45 m s^{-1}.

Estimate the body's new kinetic energy.



Examiner Tip

When using the kinetic energy equation, note that only the **speed** is **squared**, not the mass or the ½. If a question asks about the '**loss** of kinetic energy', remember **not** to include a negative sign since energy is a **scalar** quantity.

Both variations of the kinetic energy equation are given in your data booklet. You will most likely use $\frac{1}{2}mv^2$ in a mechanics question, and $\frac{p^2}{2m}$ in particle physics.

If you are not convinced these are in fact the same equation:

$$E_k = \frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{m^2v^2}{2m} = \frac{mv^2}{2} = \frac{1}{2}mv^2$$

Your notes

Your notes

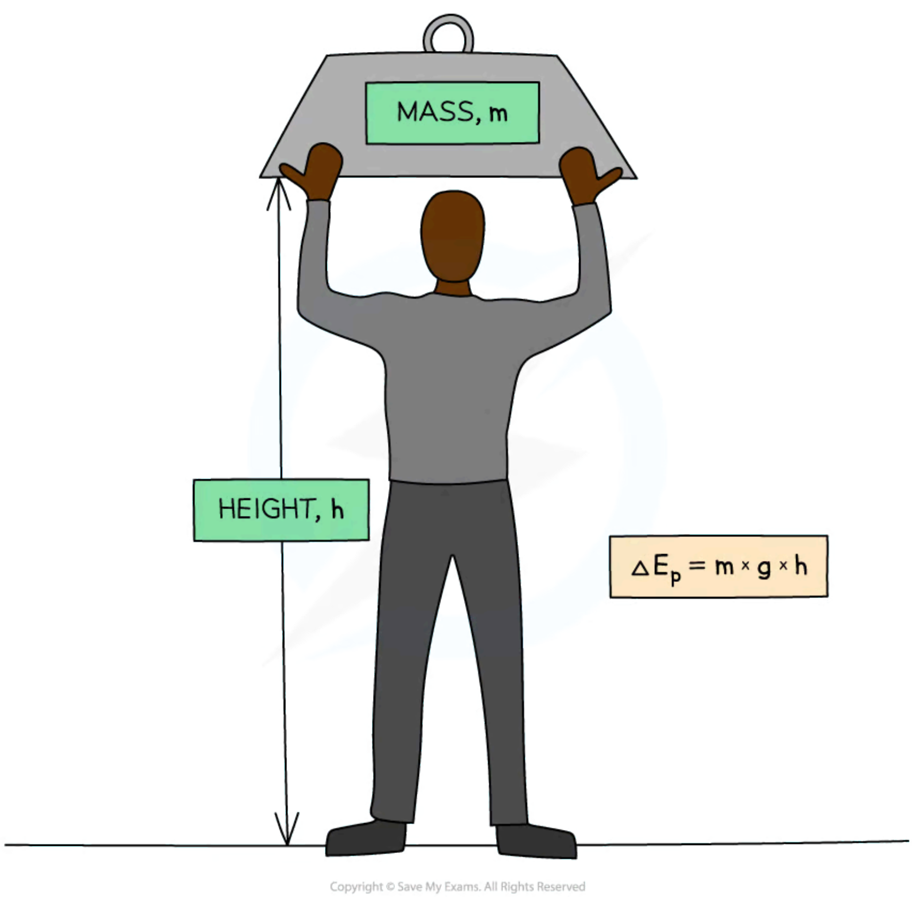
Gravitational Potential Energy

Gravitational Potential Energy

- Gravitational potential energy is the energy stored in a mass due to its position in a gravitational field
  - If a mass is **lifted** up, it will **gain** gravitational potential energy
  - If a mass **falls**, it will **lose** gravitational potential energy
- The equation for gravitational potential energy when **close** to the **surface** of the **Earth** is:

$$\Delta E_p = mg\Delta h$$

- Where:
  - ΔE_p = gravitational potential energy (J)
  - m = mass (kg)
  - g = gravitational field strength (9.8 N kg^{-1})
  - Δh = change in height (m)



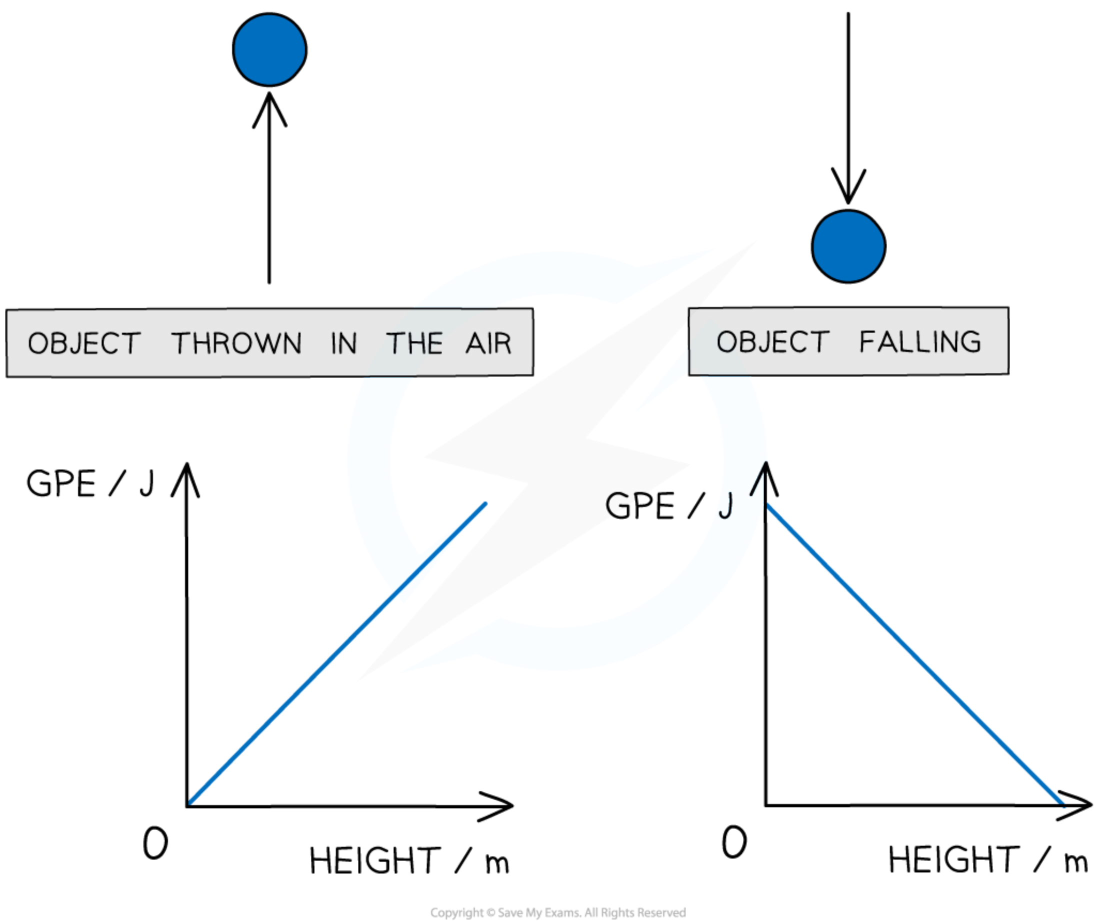
Your notes

Gravitational potential energy: The energy an object has when lifted up

- The potential energy on the Earth's surface at ground level is usually taken to be equal to zero
  - However, any position can be taken as zero if you are calculating the change in gravitational potential energy
- This equation is only relevant for energy changes in a **uniform gravitational field** (such as near the Earth's surface)
- A different potential energy is used in the gravitational fields topic, because the field is no longer uniform outside of the Earth's surface

Gravitational Potential Energy vs Height

- The two graphs below show how the gravitational potential energy changes with height for a ball being thrown up in the air and then falling down (ignoring air resistance)



Graphs showing the linear relationship between gravitational potential energy and height

- Since the graphs are straight lines, gravitational potential energy and height are said to have a **linear** relationship
  - These graphs would be identical for gravitational potential energy against time instead of height

Your notes

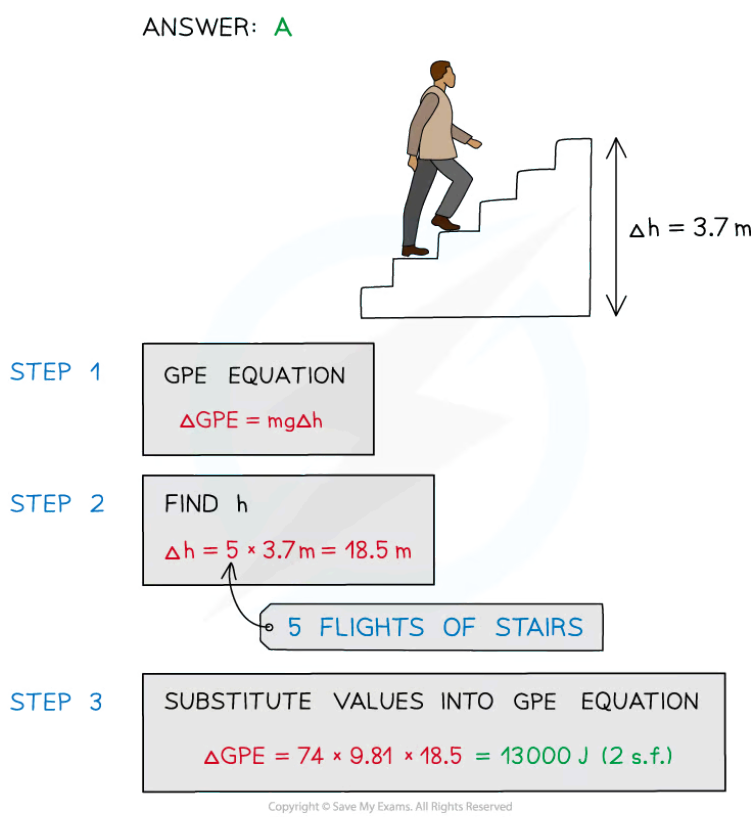
Worked example

To get to his apartment a man has to climb five flights of stairs.

The height of each flight is 3.7 m and the man has a mass of 74 kg.

What is the approximate change in the man's gravitational potential energy during the climb?

A. 13 000 J **B.** 2700 J **C.** 1500 J **D.** 12 500 J



Examiner Tip

Gravitational potential energy is often shortened to GPE for ease. In your equations, you should stick to the correct symbol, which is ΔE_p

Your notes

Your notes

Elastic Potential Energy

Elastic Potential Energy

- Elastic potential energy is defined as

  The energy stored within a material (e.g. in a spring) when it is stretched or compressed

- Therefore, for a material obeying Hooke's Law, elastic potential energy is equal to:

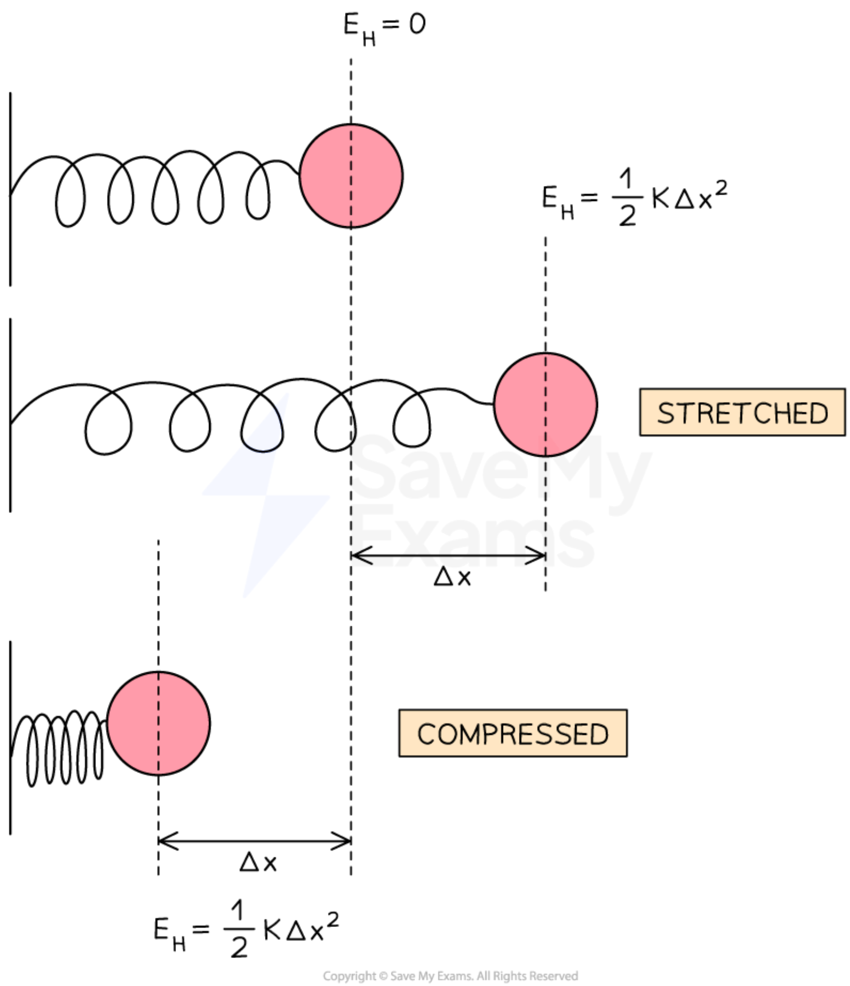
$$E_H = \frac{1}{2}k\Delta x^2$$

- Where:
  - k = spring constant of the spring (N m^{-1})
  - Δx = extension of the spring (m)
- This can also be written as:

$$E_H = \frac{1}{2}F\Delta x$$

- Where:
  - F = restoring force (N)
- This force is the same restoring force as in Hooke's law: $F = k\Delta x$

Your notes



A spring that is stretched or compressed has elastic potential energy

- It is very dangerous if a wire under large stress suddenly breaks
- This is because the elastic potential energy of the strained wire is **converted** into kinetic energy

$$E_H = E_K$$

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2$$

$$v \propto \Delta x$$

- This equation shows
  - The greater the **extension** of a wire Δx the greater the **speed** v it will have when it breaks

Your notes

Worked example

A car's shock absorbers make a ride more comfortable by using a spring that absorbs energy when the car goes over a bump. One of these springs, with a spring constant of 50 kN m^{-1} is fixed next to a wheel and compressed a distance of 10 cm.

Calculate the energy stored by the compressed spring.

Answer:

Step 1: List the known values

- Spring constant, k = 50 kN m^{-1} = 50 × 10^{3} N m^{-1}
- Compression, x = 10 cm = 10 × 10^{-2} m

Step 2: Substitute the values into the elastic potential energy equation

$$E_H = \frac{1}{2} \times (50 \times 10^3) \times (10 \times 10^{-2})^2 = 250 \text{ J}$$

Conservation of Mechanical Energy

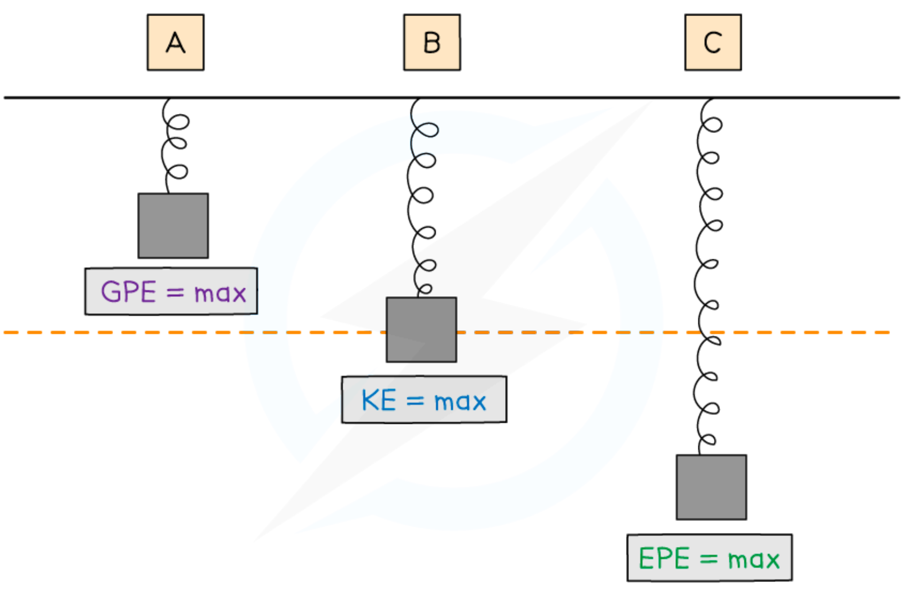
Your notes

Mechanical Energy

- Mechanical energy is the **sum** of kinetic energy, gravitational potential energy and elastic potential energy

$$\textbf{Mechanical energy} = E_k + \Delta E_p + E_H$$

- An example of a system that has mechanical energy is a spring and mass system
- The **change** in the total mechanical energy of a system should be interpreted in terms of the **work done** on the system by any non-conservative force
  - A non-conservative force is one that dissipates energy away from the system, such as friction
- When a vertical spring is extended and contracted, its energy is converted into other forms
- Although the total energy of the spring will remain constant, it will have changing amounts of:
  - **Elastic** potential energy (E_H or EPE)
  - **Kinetic** energy (E_k or KPE)
  - **Gravitational** potential energy (E_p or GPE)
- When a vertical mass is hanging on a spring and it moves up and down, its energy will convert between the three in various amounts



Position	GPE	KE	EPE
A	Maximum	Zero	Some
B	Some	Maximum	Some
C	Minimum	Zero	Maximum

- For a **horizontal** mass on a spring system, there is no gravitational potential energy to consider because this is constant
  - The spring would only convert between kinetic and elastic potential energy

Your notes

Conservation of Mechanical Energy

Your notes

- In the absence of frictional, resistive forces, the total **mechanical** energy of a system is **conserved**
  - This means the total kinetic, gravitational potential and elastic potential energy is the same throughout the motion of the system
  - Because the total energy of a system is always conserved
- There are many scenarios that involve the transfer of kinetic energy into gravitational potential, or vice versa
- Some examples are:
  - A swinging pendulum
  - Objects in freefall
  - Sports that involve falling, such as skiing and skydiving
- Using the principle of conservation of energy, and taking any drag forces as negligible:

Loss in gravitational potential energy = Gain in kinetic energy

- Another example is if a ball on a spring oscillates vertically
- In this case:

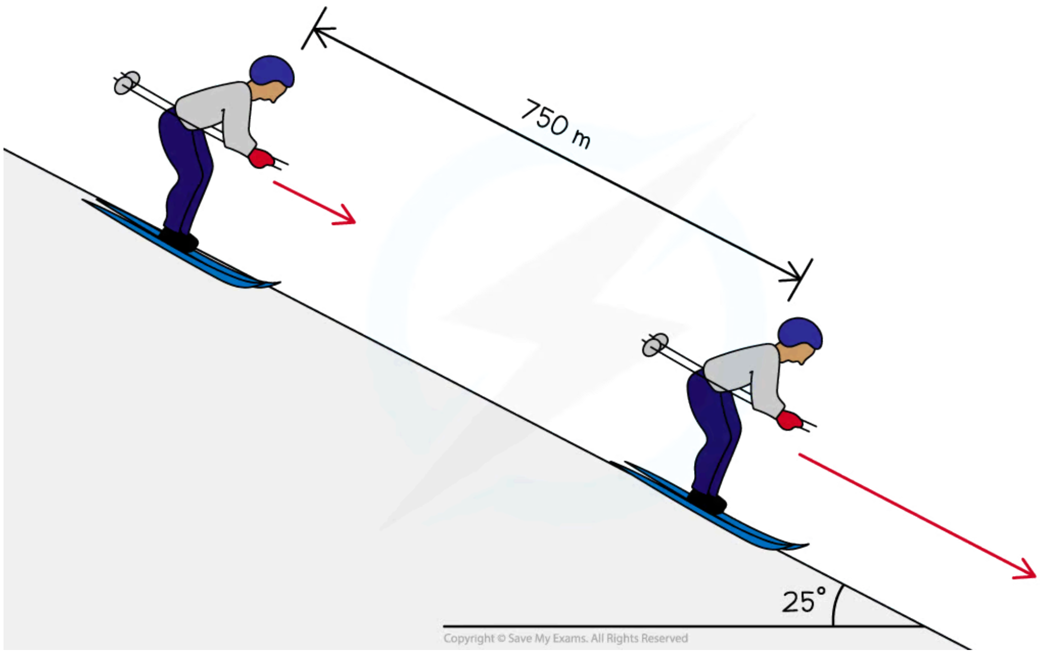
Loss in gravitational potential energy = Gain in elastic potential energy

- The change in energy is the **work done** on the system. The types of changes depend on the system

Your notes

Worked example

The diagram below shows a skier on a slope descending 750 m at an angle of 25° to the horizontal.

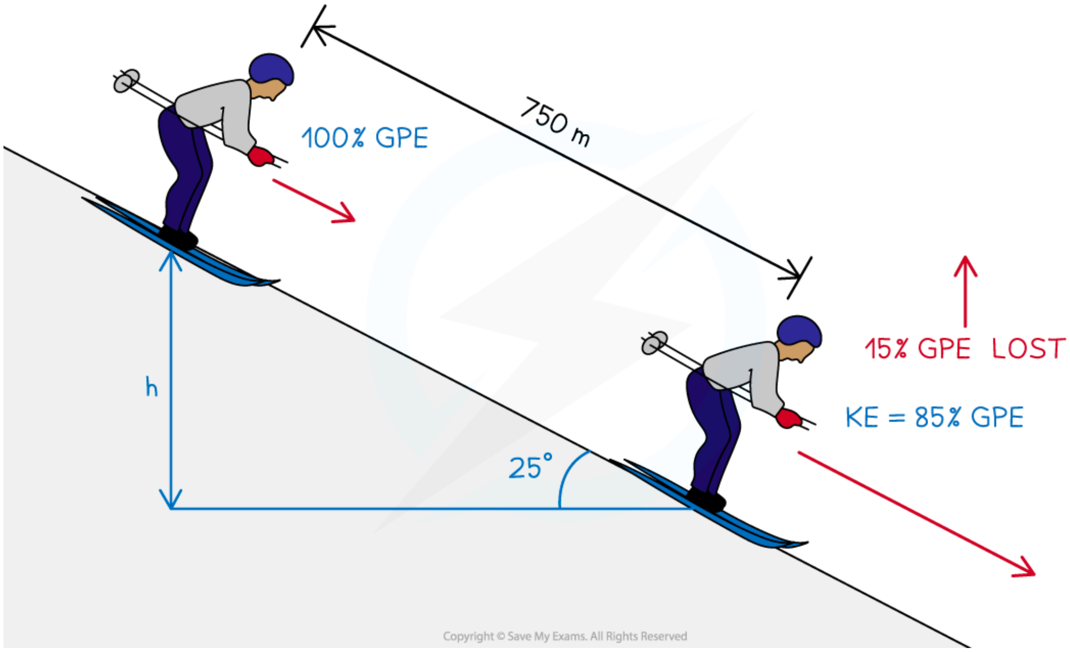


Calculate the final speed of the skier, assuming that he starts from rest and 15% of his initial gravitational potential energy is **not** transferred to kinetic energy.

Answer:

Step 1: Write down the known quantities

- Vertical height, h = 750 sin 25°
- $E_k = 0.85\ E_p$



Your notes

Step 2: Equate the equations for E_k and E_p

$$E_k = 0.85\,E_p$$

$$\tfrac{1}{2}mv^2 = 0.85 \times mgh$$

Step 3: Rearrange for final speed, *v*

$$\frac{1}{2}\cancel{m}v^2 = 0.85 \times \cancel{m}gh$$

$$v^2 = 0.85 \times 2gh$$

$$v = \sqrt{0.85 \times 2gh}$$

Step 4: Calculate the final speed, *v*

$$v = \sqrt{0.85 \times 2 \times 9.81 \times 750 \sin 25°} = 72.7$$

$$\text{Final speed, } v = 73 \text{ m s}^{-1}$$

Examiner Tip

Exam questions often ask about the 'work done' in the process. This means how much energy is **transferred**. You must consider **all** the energy changes in the system and remember that mechanical energy is always conserved. This is essentially the conservation of energy.

Your notes

Your notes

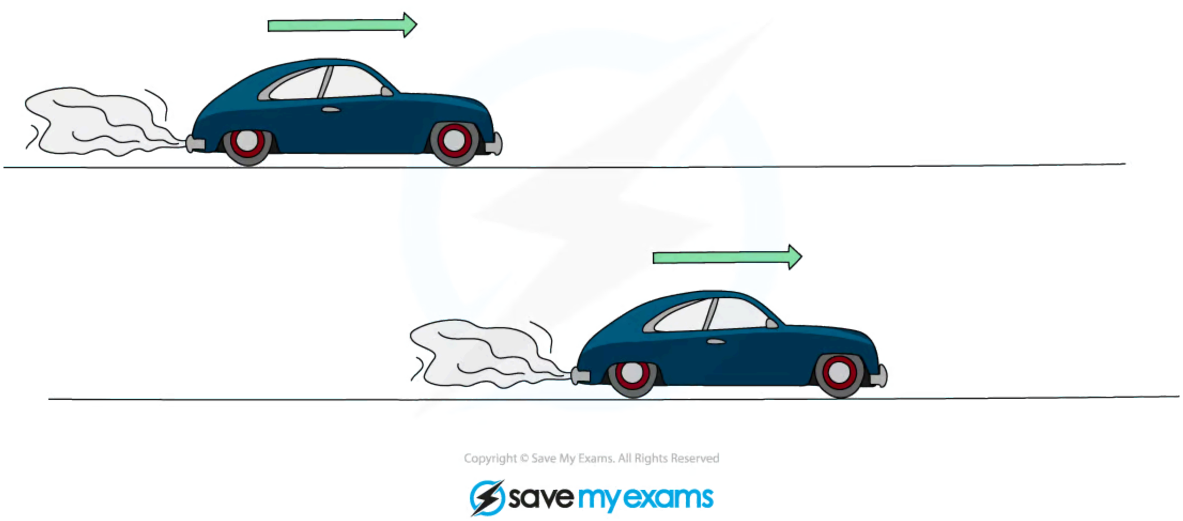
Energy & Power

Energy & Power

- The power of a mechanical process is the **rate at which energy is transferred**
- This energy transferred is the **work done**
- Therefore, power is:

The rate of work done (energy transfer)

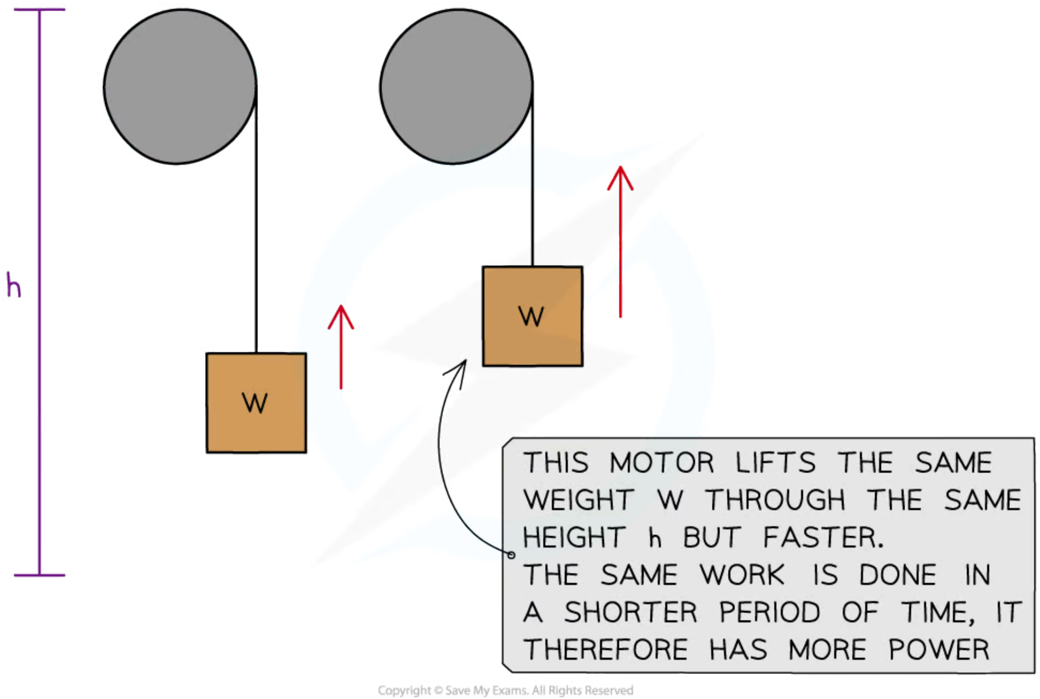
- **Time** is an important consideration when it comes to **power**
- Two cars transfer the **same amount of energy**, or do the **same amount of work** to accelerate over a distance
- If one car has **more power**, it will transfer that energy, or do that work, in a **shorter amount of time**



Two cars accelerate to the same final speed, but the one with the most power will reach that speed sooner

- Two electric motors:
  - lift the same weight
  - by the same height
  - but one motor lifts it **faster** than the other
- The motor that lifts the weight faster has more **power**

Your notes



Two motors with different powers

- Power can be calculated using the equation:

$$P = \frac{\Delta W}{\Delta t} = Fv$$

- Where:
  - P = power (W)
  - ΔW = change in work done (J)
  - Δt = time interval (s)
  - F = force (N)
  - v = velocity (m s^{-1})

- The equation with F and v is only relevant where a **constant force** moves a body at **constant velocity**
  - Power is required in order to produce an acceleration
- The force must be applied in the **same** direction as the velocity

- Power is also used in electricity
- Appliances are given a power rating, for example, 1000 W

- The power ratings indicate the amount of energy transferred per second to the appliance

Your notes

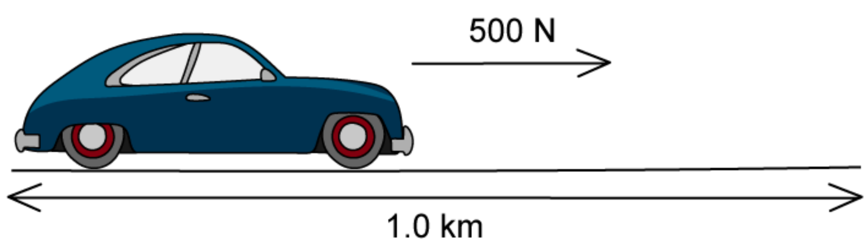
The Watt

- Power is measured in **watts (W)**
- The watt, *W*, is commonly used as the unit power (and radiant flux)
  - It is defined as **1 W = 1 J s^{-1}**
- The SI unit for energy is kg m^2 s^{-3}
- One watt is defined as:

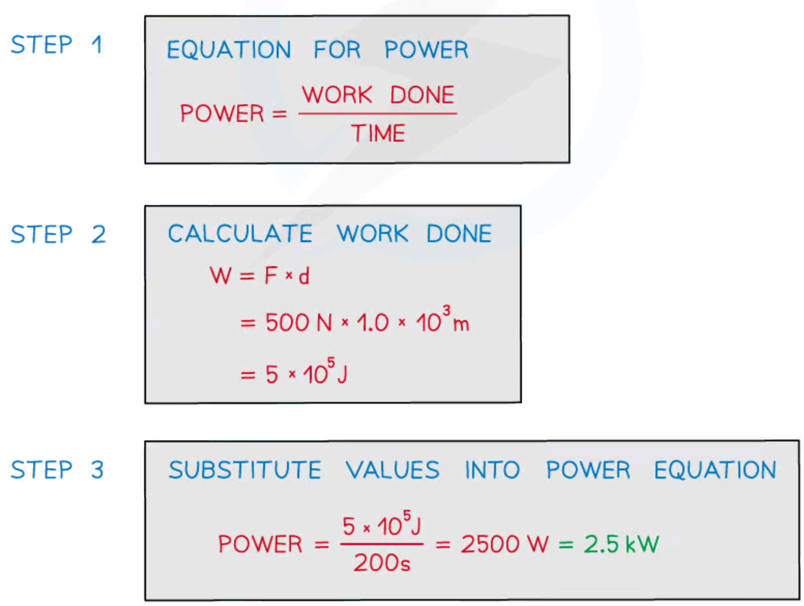
A transfer of 1 joule of energy in 1 second

Worked example

A car engine exerts the following force for 1.0 km in 200 s.



Determine what is the average power developed by the engine.

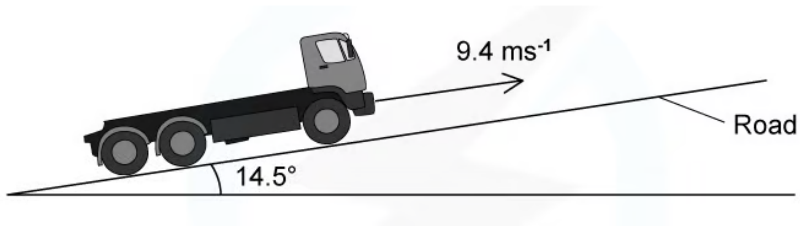


Your notes

Your notes

Worked example

A lorry moves up a road that is inclined at 14.5° to the horizontal.



The lorry has a mass of 3500 kg and is travelling at a constant speed of 9.4 m s^{-1}. The force due to air resistance is negligible.

Calculate the useful power from the engine to move the lorry up the road.

Answer:

Step 1: List the known quantities

- Angle of slope, $\theta = 14.5°$
- Mass, $m = 3500 \text{ kg}$
- Speed, $v = 9.4 \text{ m s}^{-1}$

Step 2: Write out the equation for the power of a constant force at a constant speed

$$P = Fv$$

Step 3: Calculate the constant force

- The force needed to move the lorry up the slope is that which overcomes the component of the weight force pulling it down the slope

$$F = mg\sin\theta$$

$$F = 3500 \times 9.81 \times \sin(14.5)$$

$$F = 8596.8 \text{ N}$$

Step 4: Determine the power

$$P = 8596.8 \times 9.4$$

$$P = 80\,810 \text{ W} = 81\,000 \text{ W (2 s.f.)}$$

Examiner Tip

The force represented in exam questions will often be a **drag** force. Whilst this is in the **opposite** direction to its velocity, remember the force needed to calculate the **power** is equal to (or above) this drag force to **overcome** it therefore you equate it to that value.

Your notes

Your notes

Efficiency Formula

Efficiency Formula

- The efficiency of a system is a measure of how successfully energy is transferred in a system
- Efficiency is defined as:

The ratio of the useful power or energy transfer output from a system to its total power or energy transfer input

- If a system has **high** efficiency, this means most of the energy transferred is **useful**
- If a system has **low** efficiency, this means most of the energy transferred is **wasted**

- Determining which type of energy is useful or wasted depends on the **system**
  - When energy is transferred from the thermal store of a kettle's heating element to the thermal store of the water, this is **useful** energy
  - When energy is transferred to the plastic or metal casing of the kettle and to the surrounding air, this energy is **wasted**

- Efficiency is represented as a fraction, and can be calculated using the equation:

$$\eta = \frac{E\,(output)}{E\,(input)} = \frac{P\,(output)}{P\,(input)}$$

- Where:
  - η = efficiency (the greek letter "eta")
  - E = energy (J)
  - P = power (W)
- To turn this equation into a percentage, just × 100 %
- It can also be written in words as:

$$\eta = \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$$

Your notes

Worked example

An electric motor has an efficiency of 35 %. It lifts a 7.2 kg load through a height of 5 m in 3 s.

Calculate the power of the motor.

Answer:

Step 1: Write down the efficiency equation (as a percentage)

$$\eta = \frac{useful\ power\ out}{useful\ power\ in} \times 100\%$$

Step 2: Rearrange equation for the useful power in

$$useful\ power\ in = \frac{userful\ power\ out \times 100\%}{\eta}$$

Step 3: Calculate the power output

- The power output is equal to energy ÷ time
- The electric motor transferred electric energy into gravitational potential energy to lift the load

Gravitational potential energy = mgh = 7.2 × 9.81 × 5 = 353.16 J

$$\text{Power} = \frac{353.16}{3} = 117.72\ \text{W}$$

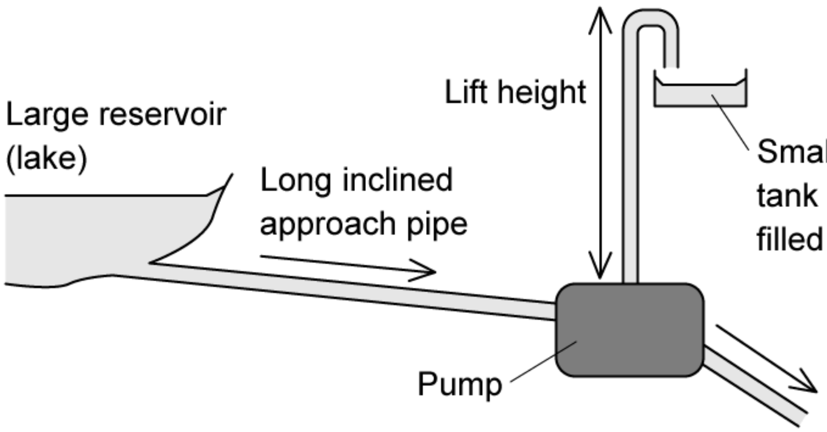
Step 4: Substitute values into power input equation

$$useful\ power\ in = \frac{117.72 \times 100}{35} = 336\ W$$

Your notes

Worked example

The diagram shows a pump called a hydraulic ram.



In one such pump, the long approach pipe holds 700 kg of water. A valve shuts when the speed of this water reaches 3.5 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by height of 12m.

The efficiency of the pump is 20%.

Which mass of water could be lifted 12 m?

A. 6.2 kg **B.** 4.6 kg **C.** 7.3 kg **D.** 0.24 kg

Your notes

ANSWER: C

THE KINETIC ENERGY OF THE WATER IS CONVVERTED TO GRAVITATIONAL POTENTIAL ENERGY WHEN LIFTED BY 12m

$$KE = GPE$$

$$\frac{1}{2}mv^2 = mgh$$

SINCE EFFICIENCY IS 20% ONLY 20% OF THE KINETIC ENERGY WILL BE CONVERTED.

$$0.2 \times \frac{1}{2}mv^2 = mgh$$

$$0.2 \times \frac{1}{2} \times 700 \times (3.5)^2 = m \times 9.81 \times 12$$

$$857.5 = m \times 117.72$$

$$\frac{857.5}{117.72} = m$$

$$m = 7.3 \text{ kg} \ (2 \text{ s.f.})$$

- The pump transfers energy from the **kinetic store** to the **gravitational potential store** of the water
- Since its efficiency is 20%, the kinetic energy can be multiplied by 0.2 since only 20% of the **kinetic energy** will be transferred (**not** 20% of the gravitational potential energy)

Examiner Tip

Efficiency can be in a ratio (between 0 and 1) or percentage format (between 0% and 100%).

If the question asks for efficiency as a ratio, give your answer as a **fraction** or **decimal**. If the answer is required as a percentage, remember to multiply the ratio by **100** to convert it: if the ratio = 0.25, percentage = 0.25 × 100 = 25 %.

Remember that efficiency has **no units**. It is a ratio with both the numerator and denominator with the same units.

Your notes

Energy Density

Energy Density

- A fuel is anything that can be burned to produce heat, which can be used for an engine to work
- The energy that an amount of fuel can provide is an important consideration for the modern world
  - When this is compared by **volume** of fuel, it is known as **energy density**
- Energy density is a measure of the amount of **energy per unit volume** of a fuel
  - Energy density is measured in J m^{-3}
- Different fuels contain different amounts of energy, which make them suitable for certain uses e.g. petrol for running vehicles
- Some examples are:

Energy Density Table

Fuel	Energy density / $MJ\ L^{-1}$
coal	38
liquid hydrogen	9
methane (natural gas)	0.3
diesel	39
biodiesel	33
vegetable oil	30
wood	3

- 1 L (litre) is 0.001 m^3
- This means that we can get **more** energy per unit volume of coal than we can wood
- Fuels are chosen for specific uses based on a number of factors, including energy density, safety of use and pollutants released in combustion