

IB • DP • Maths

 1 hour  11 questions

2.2 Quadratic Functions & Graphs

2.2.1 Quadratic Functions / 2.2.2 Factorising & Completing the Square / 2.2.3 Solving Quadratics / 2.2.4 Quadratic Inequalities / 2.2.5 Discriminants

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Total Marks

/81

1 (a) Consider $f(x) = x^2 + bx + c$. The graph of $y = f(x)$ has no real roots.

Show that $b^2 < 4c$ and explain why c must be a positive value.

(2 marks)

(b) The minimum point on the graph of $y = f(x)$ is $(3, 2)$.

Find the value of b and the value of c .

(2 marks)

(c) Sketch the graph of $y = f(x)$, clearly labelling the minimum point and any points where the graph intersects coordinate axes.

(3 marks)

2 Consider the function $f(x) = \log_b(6x - x^2)$, for $0 < x < 6$, where $b > 0$.

The equation $f(x) = 2$ has exactly one solution.

Find the value of b .

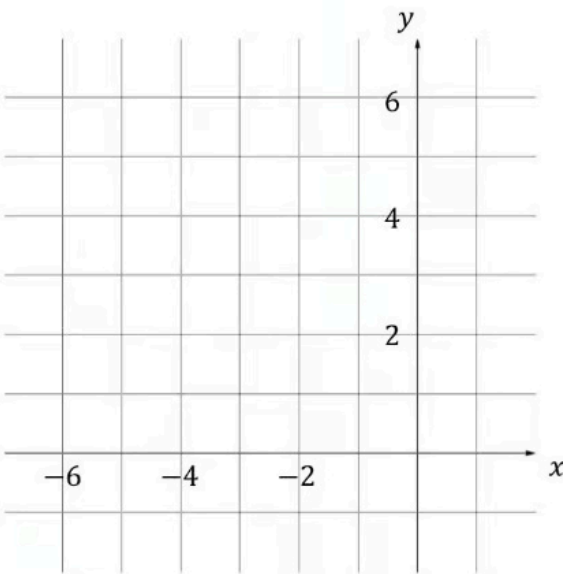
(6 marks)

- 3 Let $f(x) = 2k + \frac{9}{x}$, for $x \neq 0$, where k is a constant. The line $y = k - x$ does not intersect the graph of $y = f(x)$.

Find the possible values of k .

(6 marks)

- 4 (a)** The graph of a quadratic function has equation $y = \frac{1}{4}x^2 + bx + c$, where $b, c \in \mathbb{Z}$, and the axis of symmetry is $x = -4$.



Draw the axis of symmetry on the grid above.

(1 mark)

- (b)** The graph of the quadratic function intersects the x -axis at points $A(-6, 0)$ and B .
- Write down the coordinates of B .
 - Find the values of b and c .

(3 marks)

- (c)** (i) Mark and label A and B on the grid above.
- (ii) Write down the coordinates of the Vertex, V, and label it on the grid above.
- (iii) Write down the coordinates of the y -intercept, C, and label it on the grid above.
- (iv) Draw the graph of the quadratic function on the grid above.

(4 marks)

5 (a) Let $f(x) = 2kx^2 - (k + 10)x + \frac{5}{2} + \frac{5}{8}k$, for $x \in \mathbb{R}$, where $k \in \mathbb{Z}$.

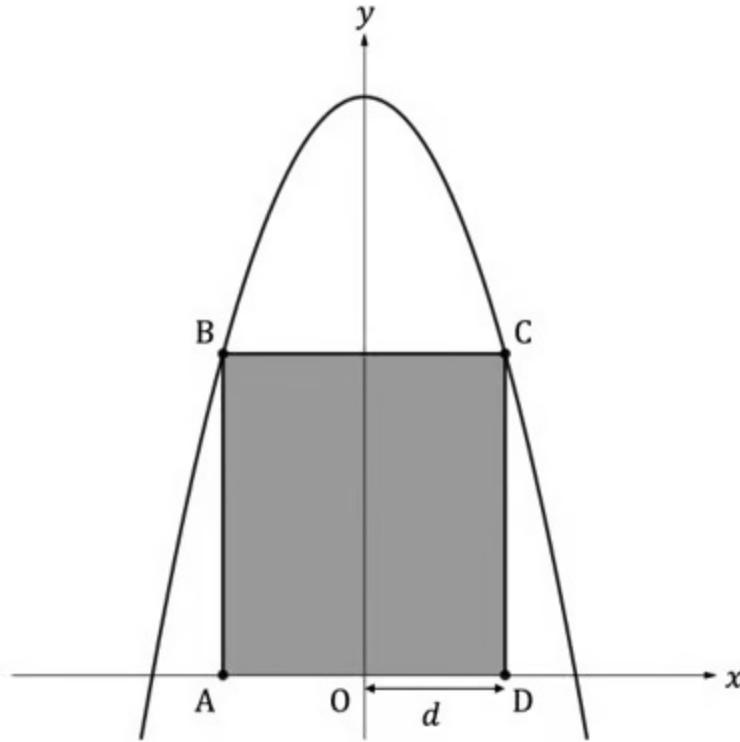
Show that the discriminant of f is $100 - 4k^2$.

(3 marks)

(b) Find the values of k such that the graph of has two **equal** roots.

(3 marks)

6 (a) Let $f(x) = 9 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f . Let $OD = d$.



Find an expression in terms of d for the area of the rectangle ABCD.

(3 marks)

(b) The coordinates of A are $(-2, 0)$.

Find the area of ABCD.

(2 marks)

(c) Let $g(x) = (x - 3)^2 + k$, for $x \in \mathbb{R}$, where k is a constant.

Given that the graphs of f and g intersect exactly once, find the value of k .

(5 marks)

7 (a) A tunnel is being constructed and its opening can be modelled by the quadratic function

$$h(x) = ax(b - x), \quad x \geq 0,$$

where h is the height of the tunnel, in metres, and x is the width of the tunnel, in metres.

It is given that $h(10) = 10$ and $h(20) = 15$.

Find the values of a and b .

(3 marks)

(b) The height required for a lane of traffic is 5 m and each lane requires a width of 2.8 m.

Find the number of lanes of traffic the tunnel can fit.

(4 marks)

- 8 (a)** A company sells 55 cars per month for a sale price of \$2000, whilst incurring costs for supplies, production and delivery of \$890 per car. Reliable market research shows that for each increase (or decrease) of the sale price by \$50 the company will sell 5 cars less (or more) and vice versa.

Find an expression for the total profit, P , in terms of the sale price, x .

(3 marks)

- (b)** Find the values of x when $P(x) = 0$ and explain their significance in the context of the question.

(2 marks)

- (c)** Calculate

- (i) the maximum monthly profit, giving your answer to the nearest dollar.
- (ii) the sale price needed to generate the maximum monthly profit.
- (iii) the number of cars sold to generate the maximum monthly profit.

(3 marks)

- 9 (a)** A company sells L litres of water per month and their total monthly profit, P , can be modelled by the function

$$P(x) = (x - 0.45) \times N(x),$$

where x is the sale price of each litre sold, in dollars, at and N is the linear function for the number of litres the company can sell per month at each given sale price.

In the context of the question, explain the significance of the 0.45.

(1 mark)

- (b)** It is given that $N(0.5) = 400$ and $N(1.25) = 250$.

Write down the function of N , in the form $N(x) = mx + c$, where m and c are constants.

(2 marks)

- (c)** Find the values of x when $P(x) = 0$ and explain their significance in the context of the question.

(2 marks)

(d) Calculate

- (i) the maximum monthly profit.
- (ii) the sale price needed to generate the maximum monthly profit.
- (iii) the number of litres sold to generate the maximum monthly profit.

(3 marks)

- 10** Let $f(x) = -(x - a)^2 + b$ and $g(x) = 4(x - c)^2 + d$, where $a, b, c, d \in \mathbb{R}$. The vertex of the graph of f is at $(2k, 8k^2)$ and the vertex of the graph of g is at $(k, 2k)$, where $0 < k < 1$. The graphs of f and g intersect at exactly one point. Find the value of k .

(7 marks)

- 11** Consider $f(x) = r(x - s)(x + 3)$. The graph of f has an axis of symmetry at $x = -1$ and y -intercept at $(0, -6)$. The line $y = mx - 14$ is a tangent to the graph of f .

Find the possible values of m .

(8 marks)