

# 6.1 Extended Questions (Paper 2, SL)

Medium (6 questions)	/93
Hard (6 questions)	/94
<b>Total Marks</b>	<b>/187</b>

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# Medium Questions

- 1 (a)** The number of seats a row has at a comedy festival follows a regular pattern where the first row has  $u_1$  seats and the number of seats in each successive row increases by  $d$  seats. In the fourth row there are 25 seats and in the sixteenth row there are 49 seats.

Write down an equation, in terms of  $u_1$  and  $d$ , for the number of seats

- (i) in the fourth row
- (ii) in the sixteenth row.

**(2 marks)**

- (b)** Find the value of  $u_1$  and the value of  $d$ .

**(2 marks)**

- (c)** The festival has 18 rows of seats in total.

Calculate the total number of seats.

**(3 marks)**

- (d)** The price for a seat in the first row is \$22 and the price decreases by 5% each successive row.

- (i) Find the row in which the price of a seat first falls below \$10.
- (ii) Find the total revenue the comedy festival generates if 19 tickets are sold for every row. Give your answer rounded to the nearest dollar.

**(8 marks)**

- 2 (a)** A study was conducted on 6 participants, measuring their body fat percentage (%) and their resting heart rate in beats per minute (*BPM*). The results are shown in the table below.

Body fat percentage ( $x$ )	22.0	14.2	15.5	12.6	29.8	10.1
Resting heart rate ( $y$ )	65	59	54	68	74	51

Use your graphic display calculator to find

- (i)  $\bar{x}$ , the mean body fat percentage
- (ii)  $\bar{y}$ , the mean resting heart rate
- (iii)  $r$ , the Pearson's product-moment correlation coefficient.

**(3 marks)**

- (b)** (i) Write down the equation of the regression line of  $y$  on  $x$  for this data, giving your answer in the form  $y = mx + c$  where  $m$  and  $c$  are constants to be found.
- (ii) Show that the point  $A(\bar{x}, \bar{y})$  lies on the regression line of  $y$  on  $x$ .

**(4 marks)**

- (c)** A seventh participant, John, has a resting heart rate of 60 *BPM*.

- (i) Use the regression line equation to estimate John's body fat percentage.
- (ii)

Justify whether it is valid to use the regression line of  $y$  on  $x$  to estimate John's body fat percentage.

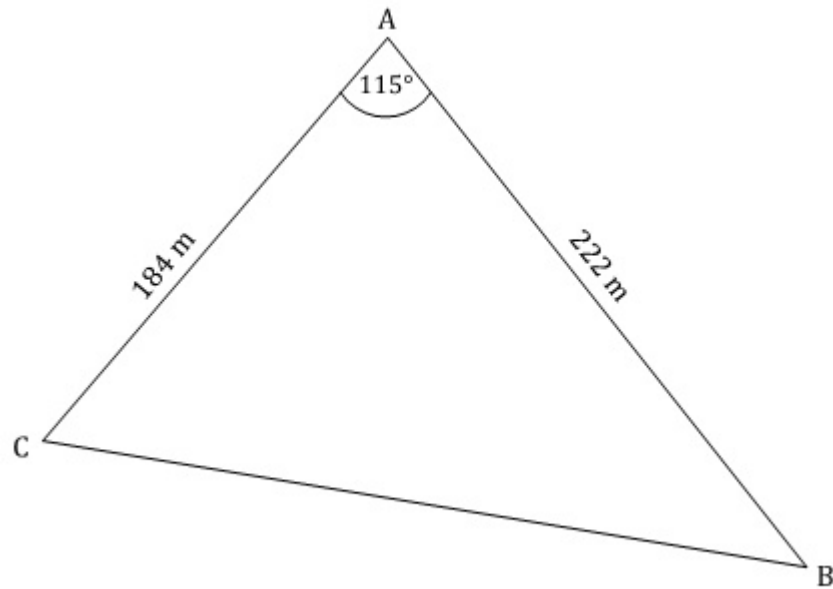
**(4 marks)**

**(d)** John's body fat percentage is 13.5%.

Calculate the percentage error in John's estimated body fat percentage from part (c).

**(2 marks)**

- 3 (a)** A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point **B** to point **C**, and the farmhouse is located at point **A**. **AB** and **AC** form the other two sides of the farm, and the distances from the farmhouse to points **B** and **C** are **222 m** and **184 m** respectively. Angle  $\widehat{CAB}$  is  $115^\circ$ , and points **A**, **B** and **C** lie in a horizontal plane.



Calculate the distance along the motorway from **B** to **C**.

**(2 marks)**

- (b)** The cost of fencing in US dollars (*USD*) is \$ 89.99 per metre.

Calculate the total cost of fencing the whole perimeter of the farm. Give your answer to 2 decimal places.

**(2 marks)**

- (c)** Calculate the area of the farm.

(2 marks)

(d) Find the sizes of angles  $\widehat{ABC}$  and  $\widehat{ACB}$ .

(2 marks)

(e) Calculate the shortest distance from the farmhouse to the motorway.

(3 marks)

(f) A vertical signpost is located at point  $C$ , and the top of the signpost is designated as point  $D$ . The angle of elevation to the top of the signpost from point  $B$  is measured to be  $1.4^\circ$ .

Calculate the distance  $CD$ , the vertical height of the signpost.

(2 marks)

(g) Calculate the distance between the top of the signpost,  $D$ , and point  $A$ .

(2 marks)

- (h) The rate of growth of the grass on the farm,  $G$ , in inches per month, can be modelled by the function

$$G(T) = -0.015(T - 40)(T - 80)$$

where  $T$  is the temperature in degrees Fahrenheit.

Find the maximum rate of grass growth on the farm and the temperature required.

**(3 marks)**



- 4 (a)** The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

Calculate

- (i) the mean number of baskets scored by a player
- (ii) the standard deviation.

**(2 marks)**

- (b)** Find the median number of baskets scored.

**(1 mark)**

- (c)** Find the interquartile range.

**(2 marks)**

- (d)** Determine if a player who scored 8 baskets would be considered an outlier.

**(2 marks)**

- (e)** Two players are randomly chosen.

Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket

**(4 marks)**

- (f)** The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.
- (i) Calculate the probability that a player trains less than 6 hours a week.
  - (ii) Calculate the probability that a player trains less than 4 hours a week.
  - (iii) Calculate the expected number of players that train between 4 and 6 hours a week.

**(3 marks)**

- 5 (a)** Best Beans is a New Zealand-based company that sells baked beans packaged in cylindrical cans.

Given that their cans have a height of **15 cm** and a diameter **8 cm** of calculate

- (i) the volume of the can
- (ii) the surface area of the can.

**(4 marks)**

- (b)** Every month, Best Beans expects to sell  $x$  thousand cans of baked beans. It is known that

$$\frac{dP}{dx} = -2x + 472, \quad x \geq 0$$

where  $P$  is the monthly profit, in New Zealand dollars (NZD), from the sale of  $x$  thousand cans of baked beans. It is also known that Best Beans makes a profit of **2450 NZD** in a month where it sells **8000** cans of baked beans.

Find  $P(x)$ .

**(5 marks)**

(c) Find the least number of cans which must be sold each month in order to make a profit.

**(3 marks)**

(d) Find the monthly sales level that will maximise profit, and the expected profit at this level.

**(3 marks)**

(e) Best Beans wants to buy a new factory at a cost of 800 000 NZD. The CEO decides to invest 60% of the company's monthly profit into a savings account paying a nominal annual interest rate of 5.5%, **compounded monthly**.

Under the assumption that the company's monthly profit will attain its maximum value every month throughout the period, determine whether Best Beans will have saved enough to buy the factory by the end of two years.

**(4 marks)**

**6 (a)** 85 people are asked if they like juice ( $J$ ), tea ( $T$ ) and/or coffee ( $C$ ) for breakfast.

12 like all three

16 like coffee and tea

14 like coffee and juice

5 like juice only

27 like coffee only

14 like tea only

- (i) Draw a Venn diagram to represent the information provided.
- (ii) Write down the number of people who like coffee but not tea.

**(5 marks)**

**(b)** There are 31 people in total who like tea.

- (i) Calculate the number of people who like tea or juice.
- (ii) Find the number of people who like none of the drinks for breakfast.

**(4 marks)**

**(c)** A person is chosen at random from the 85 people.

Find the probability that this person

- (i) likes coffee
- (ii) likes coffee and tea but not juice
- (iii) does not like either tea or juice
- (iv) does not like coffee given that the person does not like tea.

**(5 marks)**

# Hard Questions

- 1 (a)** Sharon set up an experiment to investigate the relationship between the mass of a mouse and the time the mouse takes to complete a mini assault course. She conducted the experiment with six mice and recorded her results in the table below.

Mouse mass, $x$ (g)	19.9	18.3	21.1	19.8	17.5	16.3
Time, $y$ (seconds)	18.0	17.7	20.9	18.6	15.0	14.2

- (i) Calculate Pearson's product-moment correlation coefficient,  $r$ .
- (ii) Describe the relationship between the mass of the mice and the time taken to complete the mini assault course.

**(3 marks)**

- (b)** Write down the equation of the regression line of  $x$  on  $y$ , in the form  $y = mx + c$ .

**(2 marks)**

- (c)** Find the coordinates of the point  $M(\bar{x}, \bar{y})$ .

**(2 marks)**

- (d)** Show that the point  $M(\bar{x}, \bar{y})$  lies on the line of regression.

**(2 marks)**

- (e)** The mass of a seventh mouse is found to be 20.6 g.
- (i) Using your line of regression, estimate the time that the seventh mouse will take to complete the mini assault course.
  - (ii) Justify whether it is valid to use the line of regression to estimate the result for the seventh mouse.

**(4 marks)**

- (f)** In the actual experiment, it was found that the seventh mouse took 20.7 seconds to complete the mini assault course.

Calculate the percentage error in the estimated value.

**(2 marks)**



**2 (a)** Consider the functions  $f(x) = -x^3 - 2x^2 + 4$  and  $g(x) = 3(0.5^x) - 2$ .

Calculate  $g(-2)$ .

**(2 marks)**

**(b)** Find the value of  $x$  when  $f(x) = 5$ .

**(2 marks)**

**(c)** For  $g(x)$

- (i) write down the equation of the horizontal asymptote,
- (ii) write down the coordinates of the  $y$  intercept

**(2 marks)**

**(d)** Sketch the function  $y = g^{-1}(x)$ , labelling where the graph intersects with any axes.

**(3 marks)**

**(e)** Find the solutions of  $f(x) = g(x)$ .

(2 marks)

(f) The equation  $f(x) = k$  has exactly two solutions. Find the possible values of  $k$ .

(2 marks)

- 3 (a)** A charity for premature babies asks for people to knit baby hats for newborns. Becky finds that she already has 5 baby hats that she knitted for her own child and decides to knit some more. At the end of the first week she has managed to knit only 1 new hat, but as her speed increases she knits 2 more each week than she did the previous week.

Becky knits this way for 7 weeks in total.

- (i) Find the number of hats she will make in the seventh week.
- (ii) Find the total number of hats that she will have ready to donate by the end of the seventh week.

**(5 marks)**

- (b)** In week 1, the charity starts off with 25 volunteers. The number of volunteer knitters increases at a rate of 20% each week. Each knitter averages 6 new hats per week.

Determine the number of volunteers that the charity has knitting for them after 4 weeks.

**(3 marks)**

- (c)** Given that each hat requires 45 metres of yarn to make, find out how much yarn is used by the volunteers in the first 4 weeks.

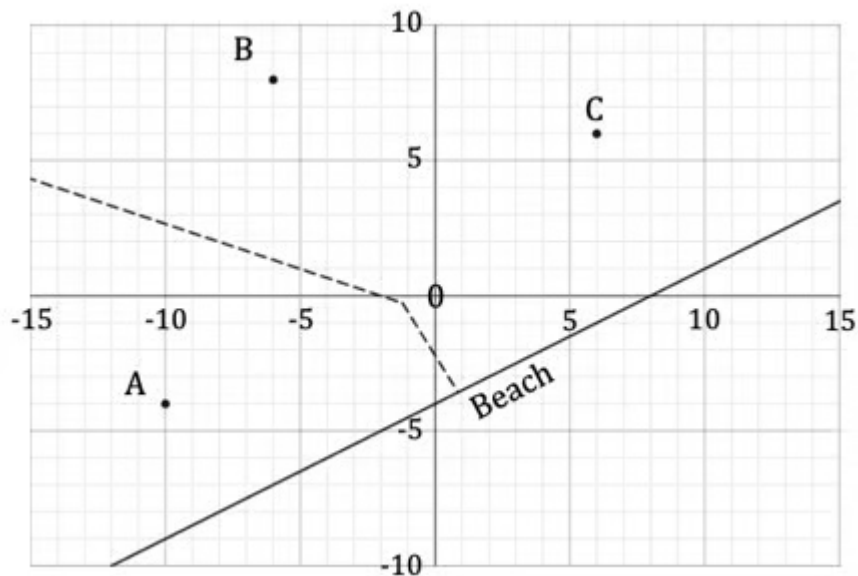
**(4 marks)**

- (d)** The charity sets a target of making 60, 000 hats, enough for each premature baby born in one year in the UK to receive one.

Determine the first week in which this target is achieved.

**(3 marks)**

- 4 (a) An area of sea alongside an island is infested with three man-eating sharks. Each shark has its own base located at the points marked **A**, **B** and **C** on the grid below. Each unit on the grid represents 1 km.



The sharks are very territorial and only hunt in their own section of the sea, the boundaries between which have naturally evolved so that they are equidistant from each shark's base. The boundaries between sharks **A** and **B** as well as sharks **A** and **C** are shown but the boundary between shark **B** and **C** is not indicated on the grid.

- (i) Find the midpoint between **B** and **C**.
- (ii) Find the gradient of the line connecting **B** and **C**.
- (iii) Hence, find the equation of boundary line between sharks **B** and **C**.
- (iv) Draw the boundary on the grid.

**(5 marks)**

**(b)** A person takes a boat out on the water and decides to go for a swim between the shark bases.

- (i) Find the equation of the boundary line between **A** and **B**.
- (ii) Hence, or otherwise, find the coordinates of the safest point in the water, furthest from each of the shark bases, in which the person should swim.

**(5 marks)**

**(c)** Calculate the distance between the swimmer and the nearest shark when the swimmer is at the point identified in part (b)(ii).

**(2 marks)**

**(d)** After the deadly shark attack on the swimmer, it is decided to install a shark protection barrier parallel to the beach line. The barrier is built along the line with equation

$$y = \frac{1}{2}x - 2.$$

Calculate the area of the sea in the region shown in the diagram that it is now safe to swim in.

(4 marks)

- 5 (a)** An IB student, Fred, decides to survey students in his school to find out their preferred area of study out of the arts, humanities and sciences. He also wants to know what their preferred part of the CAS program is out of creativity, activity and services. Fred uses a list of all the students in the school and selects every 5<sup>th</sup> student to survey. The data is shown in the following table.

	Creativity	Activity	Service	Total
Arts	19	15	7	41
Humanities	9	22	11	42
Sciences	16	8	15	39
Total	44	45	33	122

State the type of sampling that Fred uses.

**(1 mark)**

- (b)** A student is picked at random from the data.

Calculate the probability that the student picked

- (i) prefers to study the arts,
- (ii) prefers to complete service or creative options in their free time, given that their favourite area of study is the sciences.

**(4 marks)**

- (c)** Given that there are 612 students in Fred's school, find the expected number of students who prefer to study humanities or to take part in activity CAS options in their free time.



**(3 marks)**

- (d)** Fred performs a  $\chi^2$  test on the data at a significance level of 5%. The critical value for this test is 9.488.

Write down

- (i)  $H_0$ , the null hypothesis,
- (ii)  $H_1$ , the alternative hypothesis,
- (iii) the number of degrees of freedom.

**(3 marks)**

- (e)** Find

- (i) the chi squared statistic,
- (ii) the  $p$ -value.

**(3 marks)**

- (f)** Find

- (i) the chi squared statistic,
- (ii) the  $p$ -value.

**(2 marks)**

**(g)** State whether you would reject the null hypothesis. Give a reason for your answer.

**(2 marks)**

- 6 (a)** Chun-hee is creating some packaging in the shape of a square based pyramid where the base has length  $x$  cm and the perpendicular height of the pyramid is  $h$  cm. Chun-hee wants to keep the distance from the apex of the pyramid to the midpoint of the base edge fixed at 7 cm.

Write down an equation for the volume,  $V$ , of the packaging in terms of  $x$  and  $h$ .

**(1 mark)**

- (b)** Show that  $V$  can be expressed by  $\frac{196}{3}h - \frac{4}{3}h^3$ .

**(3 marks)**

- (c)** Find  $\frac{dV}{dh}$ .

**(2 marks)**

- (d)** Find the value of  $h$  for which the volume of the pyramid is maximised.

**(2 marks)**

- (e)** Find the value of  $x$  when the volume of the pyramid is maximised.

**(2 marks)**

- (f)** Chun-hee decides to make the packaging using the dimensions required to maximise the volume. The material for the packaging costs 4 KRW /  $\text{cm}^2$ .

Calculate the number of units that Chun-hee can make given that she has 90,000 KRW.

**(4 marks)**

- (g)** Chun-hee takes out a 3 year loan for 90,000 KRW at a nominal annual interest rate of 2.3% compounded monthly. Repayments are made at the end of each month.

Find the value of the repayments that Chun-hee must make to pay off the loan.

**(3 marks)**