# SL IB Physics

Your notes

## Gravitational Fields

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# Newton's Law of Gravitation

## Newton's Law of Gravitation

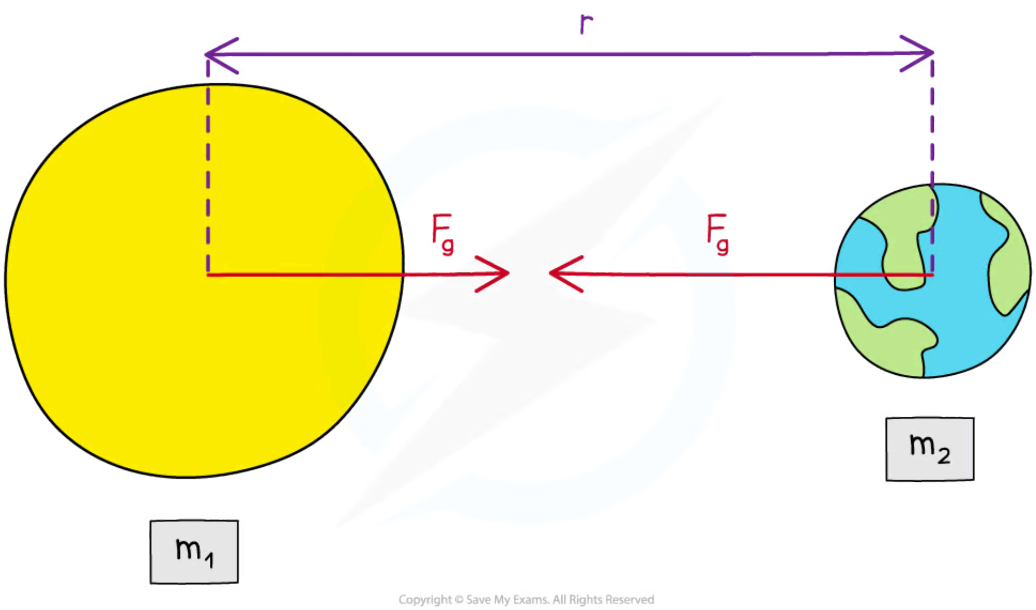
- The gravitational force between two bodies outside a uniform field, e.g. between the Earth and the Sun, is defined by Newton's Law of Gravitation
- Newton's Law of Gravitation states that:

  **The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation**

- All planets and stars are assumed to be point masses
- In equation form, this can be written as:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Where:
  - $F$ = gravitational force between two masses (N)
  - $G$ = Newton's Gravitational Constant
  - $m_1$ and $m_2$ = mass of body 1 and mass of body 2 (kg)
  - $r$ = distance between the centre of the two masses (m)




***The gravitational force between two masses outside a uniform field is defined by Newton's Law of Gravitation***

- Although planets are not point masses, their separation is much larger than their radius
  - Therefore, Newton's law of gravitation applies to planets orbiting the Sun

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- The $F \propto \frac{1}{r^2}$ relation is called the inverse square law
- This means that when a mass is twice as far away from another, its force due to gravity reduces by $(½)^2$ = ¼

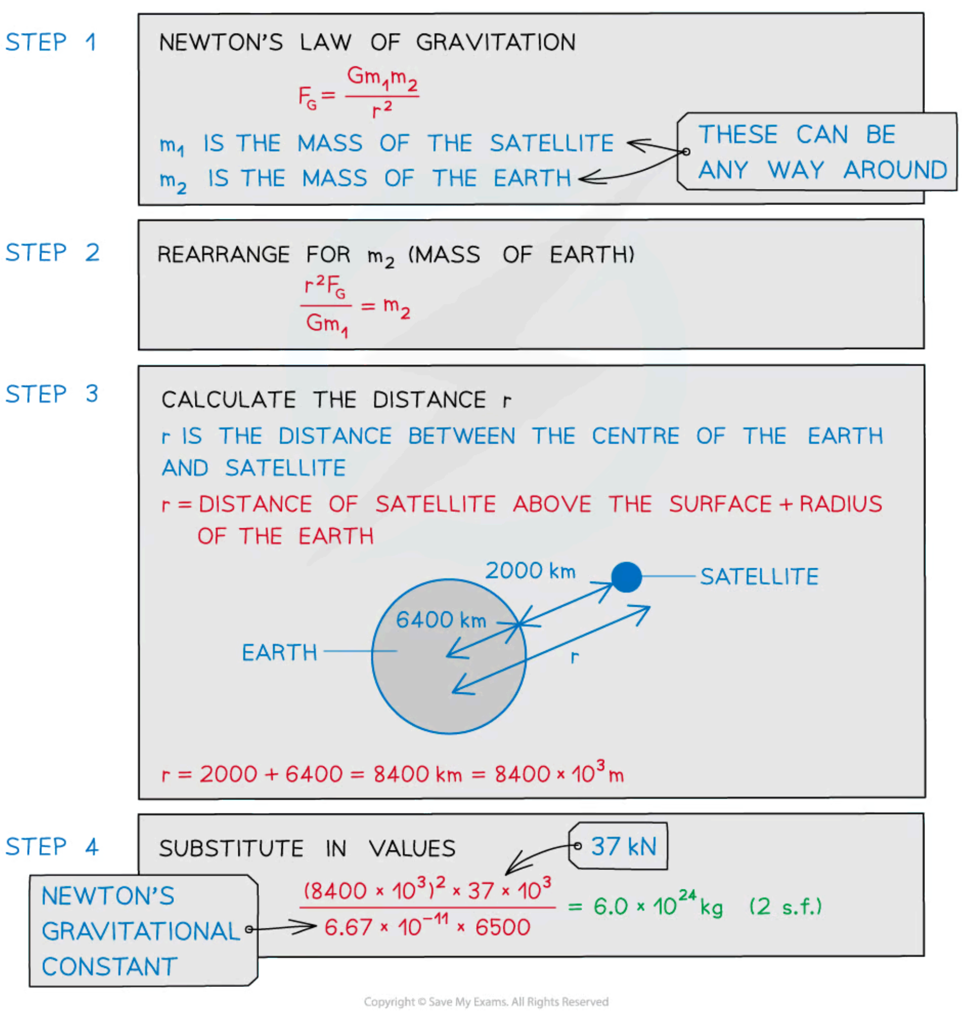
## Worked example

A satellite of mass 6500 kg orbits at 2000 km above the Earth's surface. The gravitational force between the Earth and the satellite is 37 kN.

Calculate the mass of the Earth.

Radius of the Earth = 6400 km

**Answer:**

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## Examiner Tip

A common mistake in exams is to forget to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured from the **centre** of the mass, which is from the **centre** of the planet.

Make sure to **square** the separation *r* in the equation!

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## Gravitational Field Strength

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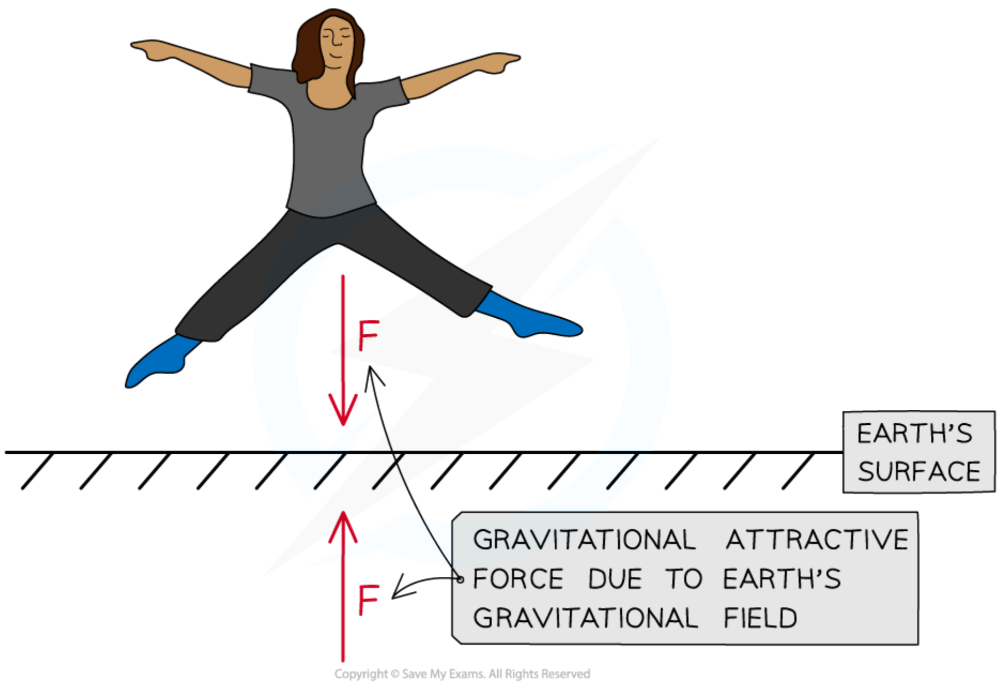
# Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
  - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- A gravitational field is defined as:

  **A region of space where a test mass experiences a force due to the gravitational attraction of another mass**

- The direction of the gravitational field is always towards the centre of the mass causing the field
  - Gravitational forces are **always** attractive
- Gravity has an infinite range, meaning it affects **all** objects in the universe
  - There is a **greater** gravitational force around objects with a **large mass** (such as planets)
  - There is a **smaller** gravitational force around objects with a **small mass** (almost negligible for atoms)




***The Earth's gravitational field produces an attractive force. The force of gravity is always attractive***

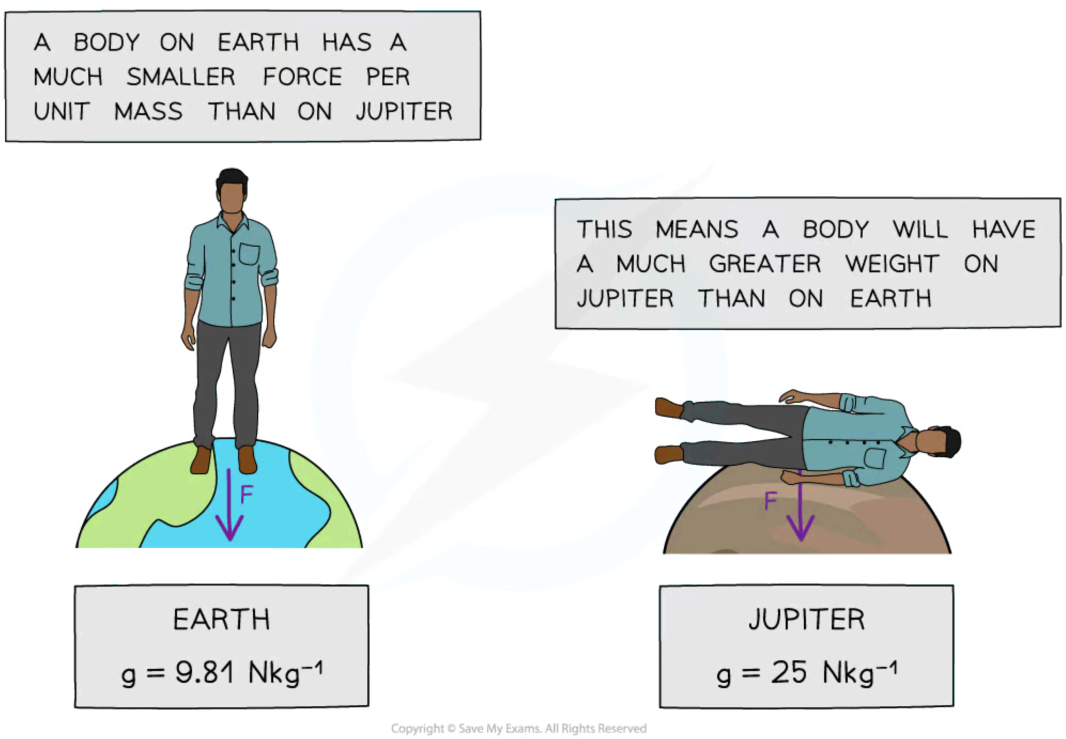
- The gravitational field strength at a point is defined as:

  **The force per unit mass experienced by a test mass at that point**

- This can be written in equation form as:

$$g = \frac{F}{m}$$

- Where:
  - $g$ = gravitational field strength (N kg$^{-1}$)
  - $F$ = force due to gravity, or weight (N)
  - $m$ = mass of test mass in the field (kg)
- This equation shows that:
  - On planets with a large value of $g$, the gravitational force per unit mass is **greater** than on planets with a smaller value of $g$
- An object's mass remains the **same** at all points in space
  - However, on planets such as Jupiter, the **weight** of an object will be greater than on a less massive planet, such as Earth
  - This means the gravitational force would be so high that humans, for example, would not be able to fully stand up




***A person's weight on Jupiter would be so large that a human would be unable to fully stand up***

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** $r$ (or diameter) of the planet
  - The **mass** $M$ (or density) of the planet
- This can be shown by equating the equation $F = mg$ with Newton's law of gravitation:

$$F = \frac{GMm}{r^2}$$

- Substituting the force $F$ with the gravitational force $mg$ leads to:

$$mg = \frac{GMm}{r^2}$$

- Cancelling the mass of the test mass $m$ leads to the equation:

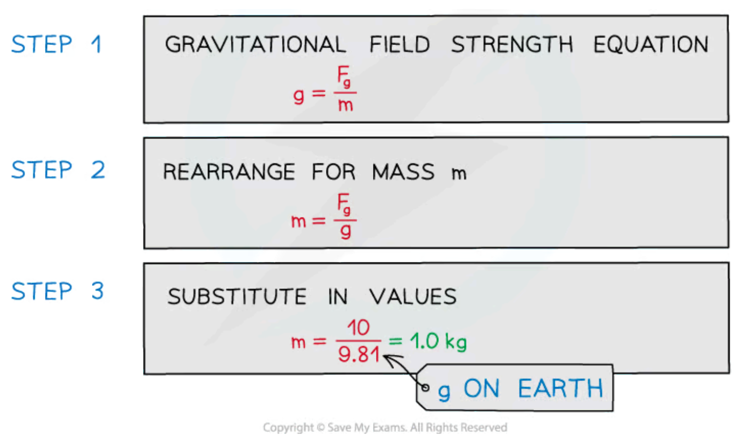
$$g = \frac{GM}{r^2}$$

- Where:
  - $G$ = Newton's Gravitational Constant
  - $M$ = *mass of the body causing the field (kg)*
  - $r$ = distance from the mass where you are calculating the field strength (m)
- This equation shows that:
  - The gravitational field strength $g$ depends only on the mass of the body $M$ causing the field
  - Hence, objects with any mass $m$ in that field will experience the **same gravitational field strength**
  - The gravitational field strength $g$ is **inversely proportional** to the **square** of the radial distance, $r^2$

## Worked example

Calculate the mass of an object with weight 10 N on Earth.

Answer:



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## Worked example

The mean density of the Moon is $\frac{3}{5}$ times the mean density of the Earth. The gravitational field strength on the Moon is $\frac{1}{6}$ the gravitational field strength on Earth.

Determine the ratio of the Moon's radius $r_M$ to the Earth's radius $r_E$.

**Answer:**

**Step 1: Write down the known quantities**

- $g_M$ = gravitational field strength on the Moon, $\rho_M$ = mean density of the Moon
- $g_E$ = gravitational field strength on the Earth, $\rho_E$ = mean density of the Earth

$$\rho_M = \frac{3}{5}\rho_E$$

$$g_M = \frac{1}{6}g_E$$

**Step 2: Write down the equations for the gravitational field strength, volume and density**

Gravitational field strength: $g = \frac{GM}{r^2}$

Volume of a sphere: $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad V \propto r^3$

Density: $\rho = \frac{M}{V} \quad \Rightarrow \quad M = \rho V = \frac{4}{3}\pi\rho r^3 \quad \Rightarrow \quad M \propto \rho r^3$

**Step 3: Substitute the relationship between M and r into the equation for g**

$$g \propto \rho\frac{(r^3)}{r^2} \quad \Rightarrow \quad g \propto \rho r$$

**Step 4: Find the ratio of the gravitational field strength**

$$g_M \propto \rho_M r_M$$

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$$g_E \propto \rho_E r_E$$

$$g_M = \frac{1}{6} g_E \quad \Rightarrow \quad \rho_M r_M = \frac{1}{6} \rho_E r_E$$

**Step 5: Substitute the ratio of the densities into the equation**

$$\left(\frac{3}{5} \rho_E\right) r_M = \frac{1}{6} \rho_E r_E$$

$$\frac{3}{5} r_M = \frac{1}{6} r_E$$

**Step 6: Calculate the ratio of the radii**

$$\frac{r_M}{r_E} = \frac{1}{6} \div \frac{3}{5} = \frac{5}{18} = 0.28$$

## Examiner Tip

There is a big difference between *g* and *G* (sometimes referred to as 'little g' and 'big G' respectively), *g* is the gravitational field strength and G is Newton's gravitational constant. Make sure not to use these interchangeably!

Remember the equation $density = \frac{mass}{volume}$, which may come in handy with some calculations.

The equation for the volume of common shapes is in your data booklet.

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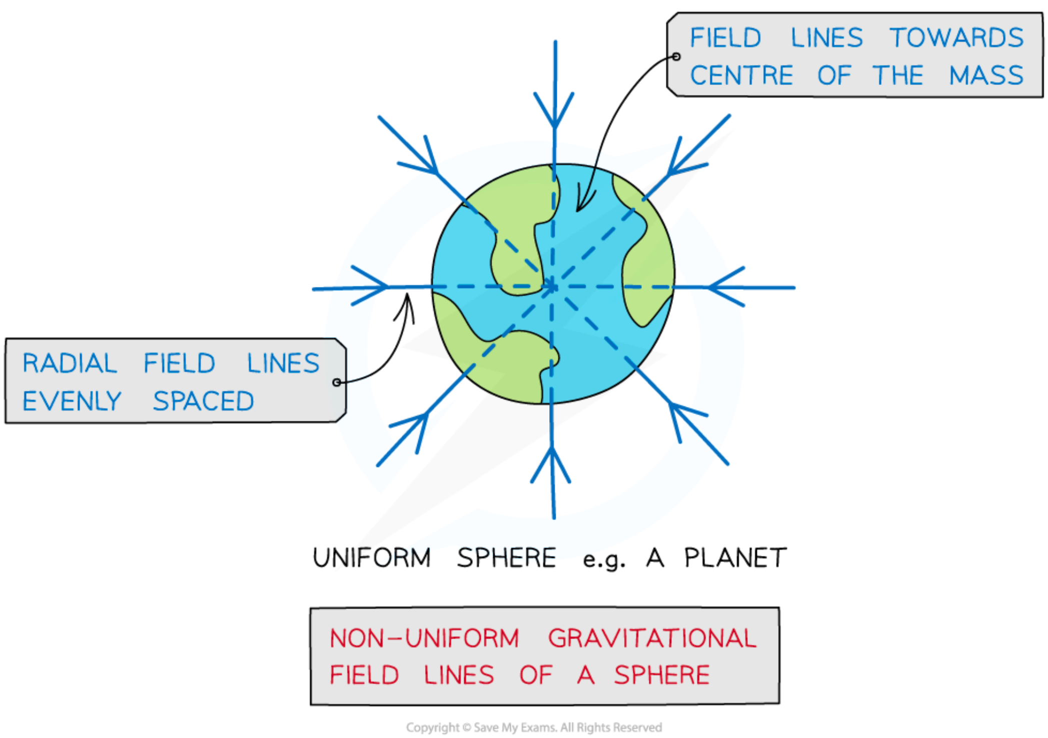
## Gravitational Field Lines

### Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as a point mass when:

  **A body covers a very large distance compared to its size, so, to study its motion, its size or dimensions can be neglected**

- An example of this is field lines around planets




***Gravitational field lines around a uniform sphere are identical to those on a point mass***

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength $g$ is different depending on how far an object is from the centre of mass of the sphere
- Newton's universal law of gravitation is extended to spherical masses of uniform density by assuming that their mass is concentrated at their centre i.e point masses

## Representing Gravitational Fields

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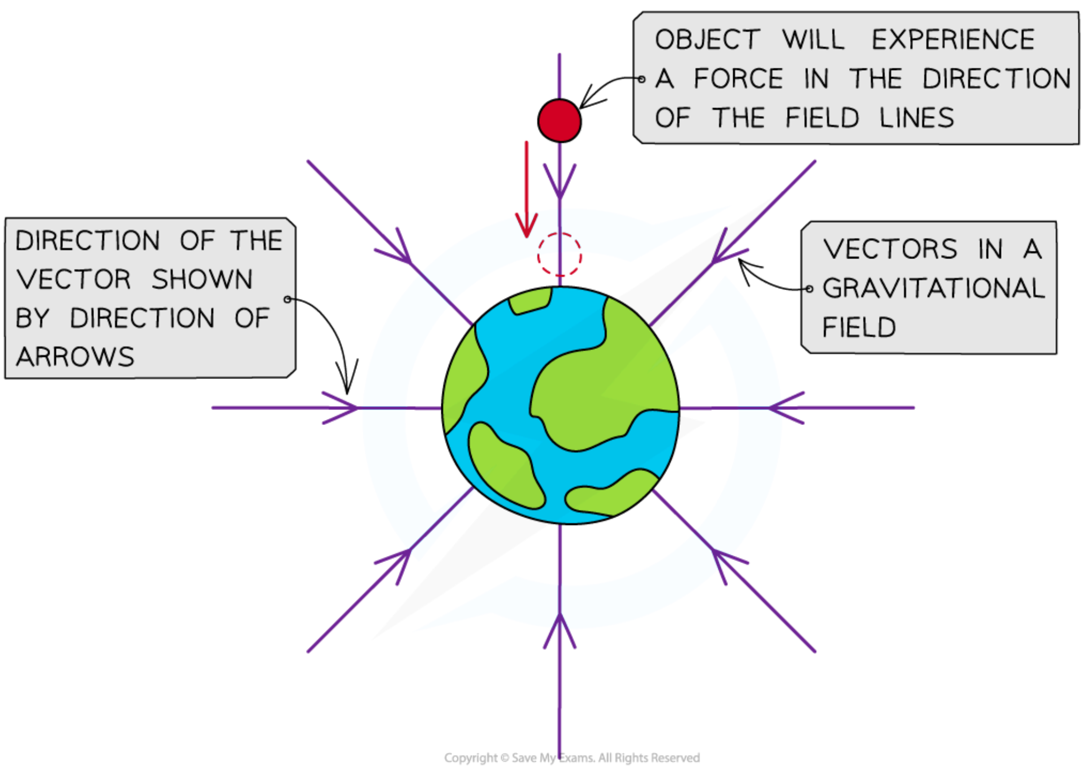
- Gravitational fields represent the **action** of gravitational forces between masses, the direction of these forces can be shown using vectors
  - The direction of the **vector** shows the direction of the **gravitational force** that would be exerted on a **mass** if it was placed at that position in the field
  - These vectors are known as **field lines** (or 'lines of force')
- The direction of a gravitational field is represented by gravitational field lines
  - Therefore, gravitational field lines also show the direction of **acceleration** of a mass placed in the field
- Gravitational field lines are always directed toward the centre of mass of a body
  - This is because gravitational forces are **attractive only** (they are never repulsive)
  - Therefore, masses **always** attract each other via the gravitational force
- The gravitational field around a point mass will be **radial** in shape and the field lines will always point towards the centre of mass




***The direction of the gravitational field is shown by the vector field lines***

- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface

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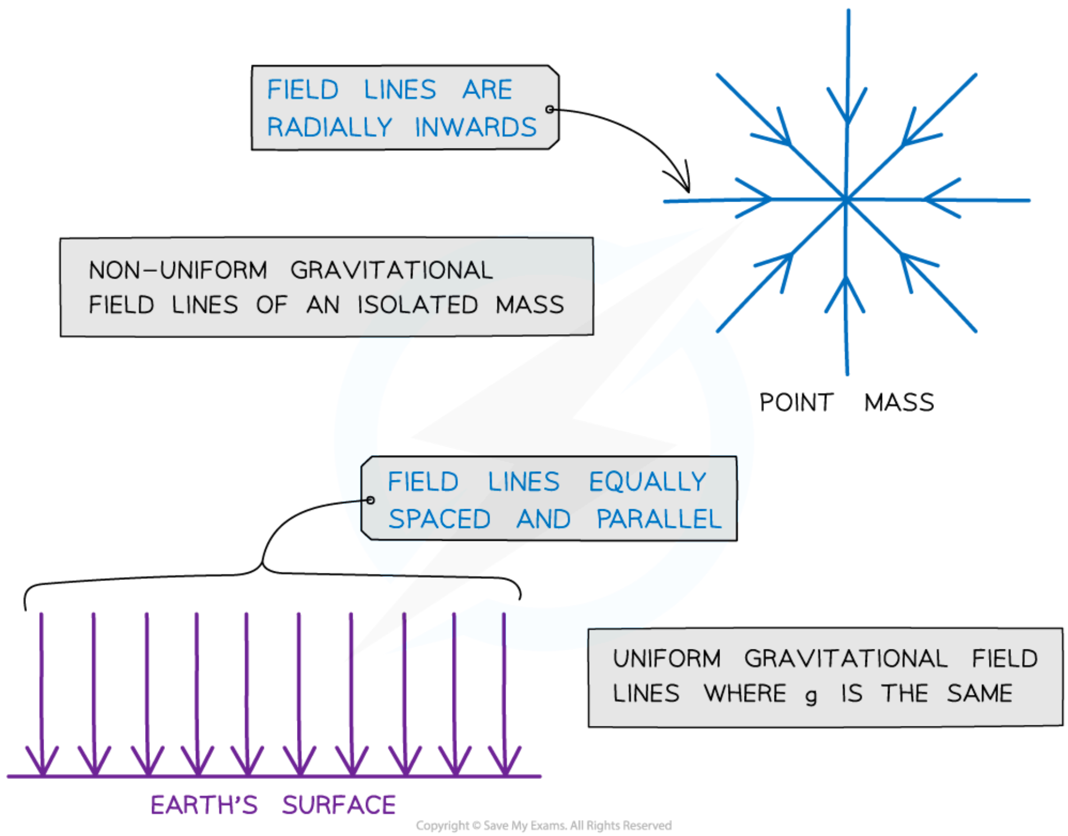


***Gravitational field lines for a point mass and a uniform gravitational field***

- Radial fields are considered **non-uniform fields**
  - The gravitational field strength $g$ is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
  - The gravitational field strength $g$ is the same throughout

## Examiner Tip

Always label the arrows on the field lines! Gravitational forces are attractive only. Remember:

- For a radial field: it is towards the centre of the sphere or point charge
- For a uniform field: towards the surface of the object e.g. Earth

# Kepler's Laws of Planetary Motion

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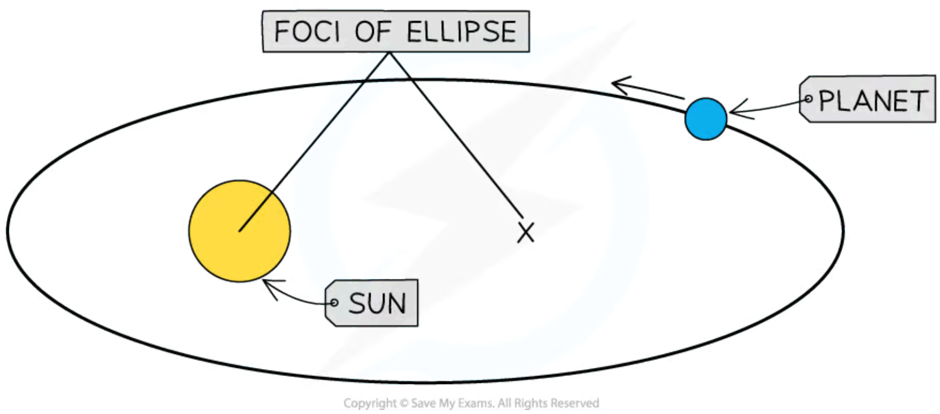
## Kepler's Laws of Planetary Motion

### Kepler's First Law

- Kepler's First Law describes the **shape** of planetary orbits
- It states:

**The orbit of a planet is an ellipse, with the Sun at one of the two foci**



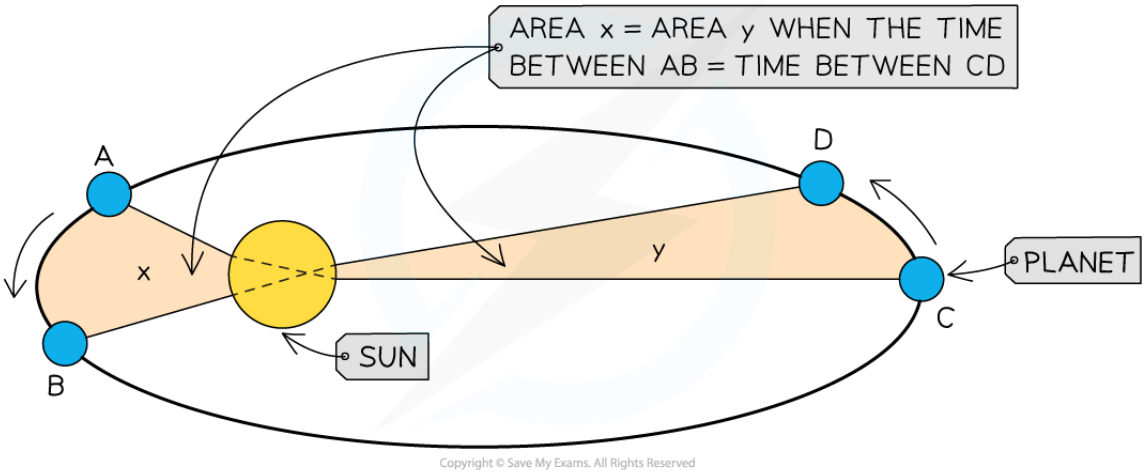

***The orbit of all planets are elliptical, and with the Sun at one focus***

- An ellipse is just a 'squashed' circle
  - Some planets, like Pluto, have highly elliptical orbits around the Sun
  - Other planets, like Earth, have near circular orbits around the Sun

### Kepler's Second Law

- Kepler's Second Law describes the **motion** of all planets around the Sun
- It states:

**A line segment joining the Sun to a planet sweeps out equal areas in equal time intervals**

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- The consequence of Kepler's Second Law is that planets move **faster** nearer the Sun and **slower** further away from it

## Kepler's Third Law

- Kepler's Third Law states

  **For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit**

- This law describes the relationship between the **time** of an orbit and its **radius**

$$T^2 \propto r^3$$

- Where:
  - $T$ = orbital time period (s)
  - $r$ = mean orbital radius (m)

## Time Period & Orbital Radius Relation

- Since a planet or a satellite is travelling in circular motion when in order, its orbital time period $T$ to travel the circumference of the orbit $2\pi r$, the linear speed $v$ is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed $v$ from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

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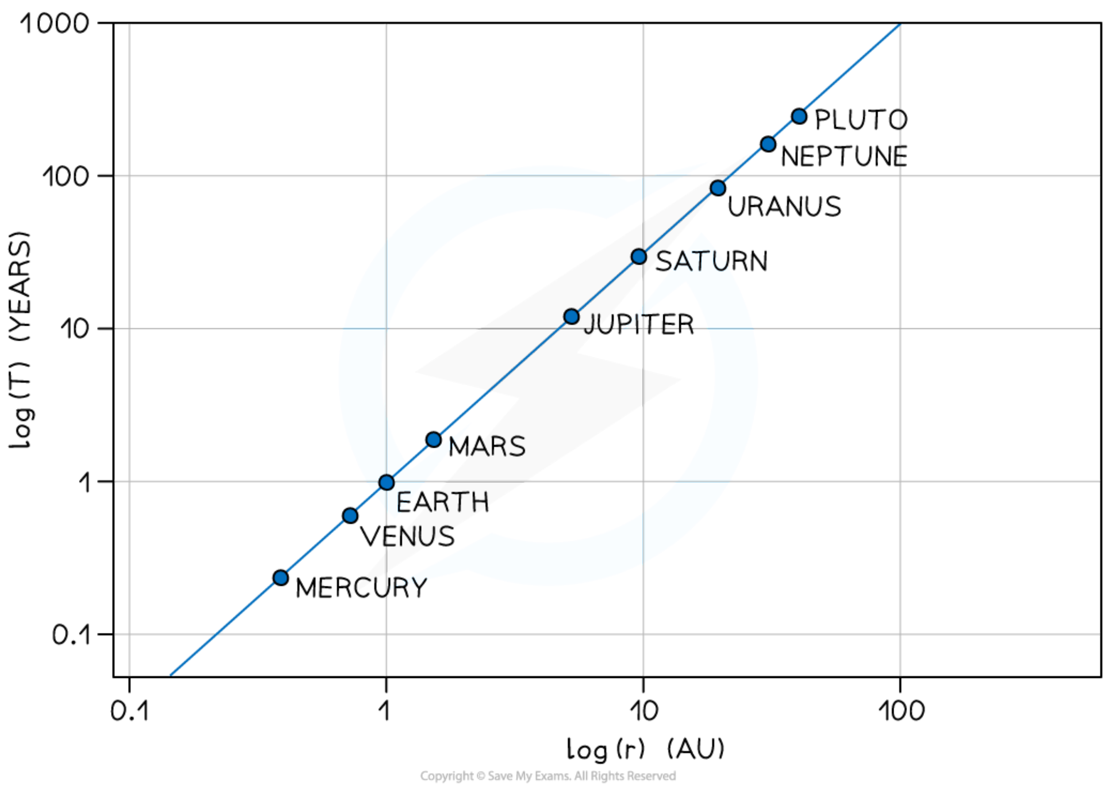
- Squaring out the brackets and rearranging for $T^2$ gives the equation relating the time period $T$ and orbital radius $r$:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

- Where:
  - $T$ = time period of the orbit (s)
  - $r$ = orbital radius (m)
  - $G$ = Gravitational Constant
  - $M$ = mass of the object being orbited (kg)
- The relationship between $T$ and $r$ can be shown using a logarithmic plot

$$T^2 \propto r^3 \quad \Rightarrow \quad 2 \log T \propto 3 \log r$$

- The graph of log $T$ in years against log $r$ in AU (astronomical units) for the planets in our solar system is a straight-line graph:




***The logarithmic graph of log T against log r gives a straight line***

- The graph does not go through the origin since it has a negative y-intercept
  - Only the graph of log $T$ and log $r$ will produce a straight-line graph, a graph of T vs r would not

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## Worked example

Planets A and B orbit the same star.

Planet A is located an average distance *r* from the star. Planet B is located an average distance 6*r* from the star

What is $\frac{orbital\ period\ of\ planet\ A}{orbital\ period\ of\ planet\ B}$?

**A.** $\frac{1}{\sqrt[3]{6}}$ **B.** $\frac{1}{\sqrt{6}}$ **C.** $\frac{1}{\sqrt[3]{6^2}}$ **D.** $\frac{1}{\sqrt{6^3}}$

**Answer:** **D**

- Kepler's third law states $T^2 \propto r^3$
- The orbital period of planet A: $T_A \propto \sqrt{r^3}$
- The orbital period of planet B: $T_B \propto \sqrt{(6r)^3}$
- Therefore the ratio is equal to:

$$\frac{T_A}{T_B} = \frac{\sqrt{r^3}}{\sqrt{(6r)^3}} = \frac{1}{\sqrt{6^3}}$$

## Examiner Tip

You are expected to be able to describe Kepler's Laws of Motion, so make sure you are familiar with how they are worded.