DP IB Maths: AA HL

Your notes

4.1 Statistics Toolkit

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4.1.1 Sampling & Data Collection

Your notes

Types of Data

What are the different types of data?

- **Qualitative** data is data that is usually given in words not numbers to **describe** something
  - For example: the colour of a teacher's car
- **Quantitative** data is data that is given using numbers which **counts or measures** something
  - For example: the number of pets that a student has
- **Discrete** data is quantitative data that needs to be **counted**
  - Discrete data can only take **specific values** from a set of (usually finite) values
  - For example: the number of times a coin is flipped until a 'tails' is obtained
- **Continuous** data is quantitative data that needs to be **measured**
  - Continuous data can take **any value** within a range of infinite values
  - For example: the height of a student
- **Age** can be **discrete or continuous** depending on the context or how it is defined
  - If you mean how many years old a person is then this is discrete
  - If you mean how long a person has been alive then this is continuous

What is the difference between a population and a sample?

- The **population** refers to the **whole set** of things which you are interested in
  - For example: if a vet wanted to know how long a typical French bulldog slept for in a day then the population would be all the French bulldogs in the world
- A **sample** refers to a **subset of the population** which is used to collect data from
  - For example: the vet might take a sample of French bulldogs from different cities and record how long they sleep in a day
- A **sampling frame** is a **list** of all members of the **population**
  - For example: a list of employees' names within a company
- Using a **sample instead of a population**:
  - Is quicker and cheaper
  - Leads to less data needing to be analysed
  - Might not fully represent the population
  - Might introduce bias

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Sampling Techniques

What is a random sample and a biased sample?

- A **random sample** is where every member of the population has an equal chance of being included in the sample
- A **biased sample** is one from which misleading conclusions could be drawn about the population
  - **Random sampling** is an attempt to **minimise bias**

What sampling techniques do I need to know?

Simple random sampling

- **Simple random sampling** is where every group of members from the population has an **equal probability** of being selected for the sample
- To carry this out you would...
  - uniquely number every member of a population
  - randomly select n different numbers using a random number generator or a form of lottery (where numbers are selected randomly)
- **Effectiveness**:
  - Useful when you have a small population or want a small sample (such as children in a class)
  - It can be time-consuming if the sample or population is large
  - This can not be used if it is not possible to number or list all the members of the population (such as fish in a lake)

Systematic sampling

- **Systematic sampling** is where a sample is formed by choosing members of a population at regular intervals using a list
- To carry this out you would...
  - calculate the size of the interval $k = \frac{\text{size of population } (N)}{\text{size of sample } (n)}$
  - choose a random starting point between 1 and k
  - select every kth member after the first one
- **Effectiveness**:
  - Useful when there is a natural order (such as a list of names or a conveyor belt of items)
  - Quick and easy to use
  - This can not be used if it is not possible to number or list all the members of the population (such as penguins in Antarctica)

Stratified sampling

- **Stratified sampling** is where the population is divided into disjoint groups and then a random sample is taken from each group

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- The proportion of a group that is sampled is equal to the proportion of the population that belong to that group
- To carry this out you would...
  - Calculate the number of members sampled from each stratum
    - $\frac{\text{size of sample } (n)}{\text{size of population } (N)} \times \text{number of members in the group}$
  - Take a random sample from each group
- **Effectiveness**:
  - Useful when there are very different groups of members within a population
  - The sample will be representative of the population structure
  - The members selected from each stratum are chosen randomly
  - This can not be used if the population can not be split into groups or if the groups overlap

Quota sampling

- **Quota sampling** is where the population is split into groups (like stratified sampling) and members of the population are selected until each quota is filled
- To carry this out you would...
  - Calculate how many people you need from each group
  - Select members from each group until that quota is filled
    - The members do not have to be selected randomly
- **Effectiveness**:
  - Useful when collecting data by asking people who walk past you in a public place or when a sampling frame is not available
  - This can introduce bias as some members of the population might choose not to be included in the sample

Convenience sampling

- **Convenience sampling** is where a sample is formed using available members of the population who fit the criteria
- To carry this out you would...
  - Select members that are easiest to reach
- **Effectiveness**:
  - Useful when a list of the population is not possible
  - This is unlikely to be representative of the population structure
  - This is likely to produce biased results

What are the main criticisms of sampling techniques?

- Most sampling techniques can be improved by taking a larger sample
- Sampling can introduce bias - so you want to minimise the bias within a sample
  - To minimise bias the sample should be as close to random as possible
- A sample only gives information about those members

- Different samples may lead to different conclusions about the population

Your notes

Your notes

Worked example

Mike is a biologist studying mice in an open enclosure. He has access to approximately 540 field mice and 260 harvest mice. Mike wants to sample 10 mice and he wants the proportions of the two types of mice in his sample to reflect their respective proportions of the population.

a) Calculate the number of field mice and harvest mice that Mike should include in his sample.

Total number of mice

$540 + 260 = 800$

Field mice $\frac{540}{800} \times 10 = 6.75$

Fraction of field mice

Sample size

Harvest mice $\frac{260}{800} \times 10 = 3.25$

Fraction of harvest mice

Include 7 field mice and 3 harvest mice

b) Given that Mike does not have a list of all mice in the enclosure, state the name of this sampling method.

No list of population so can not be a random sample

Quota sampling

c) Suggest one way in which Mike could improve his sampling method.

Mark could improve his sampling method by increasing his sample size

Your notes

Reliability of Data

How can I decide if data is reliable?

- Data from a sample is reliable if similar results would be obtained from a different sample from the same population
- The sample should be **representative** of the population
- The sample should be **big enough**
  - Sampling a small proportion of a population is unlikely to be reliable

What can cause data to be unreliable?

- If the sample is **biased**
  - It is **not random**
- If **errors** are made when collecting data
  - Numbers could be recorded incorrectly, duplicated or missed out
- If the person collecting the data **favours some members** over others
  - They might seek out members who will lead to a desired outcome
  - They might exclude members if they would cause the sample to oppose the desired outcome
- If a significant proportion of **data is missing**
  - Some data may be unavailable
  - Some members might decide not to be part of the sample
    - This will mean the results are not necessarily representative of the population

4.1.2 Statistical Measures

Your notes

Mean, Mode, Median

What are the mean, mode and median?

- Mean, median and mode are **measures of central tendency**
  - They describe where the centre of the data is
- They are all types of **averages**
- In statistics it is important to be specific about which average you are referring to
- The **units** for the mean, mode and median are the **same** as the units for the data

How are the mean, mode, and median calculated for ungrouped data?

- The **mode** is the value that occurs **most often** in a data set
  - It is possible for there to be **more than one mode**
  - It is possible for there to be **no mode**
    - In this case **do not** say the mode is zero
- The **median** is the **middle** value when the data is in **order of size**
  - If there are two values in the middle then the median is the **midpoint** of the two values
- The **mean** is the **sum** of all the values **divided by the number of values**

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  - Where $\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$ is the sum of the *n* pieces of data
  - The mean can be represented by the symbol μ
- Your **GDC** can calculate these statistical measures if you input the data using the statistics mode

Your notes

Worked example

Find the mode, median and mode for the data set given below.

43	29	70	51	64	43

Mode is the most common

Mode = 43

Median is the middle when in order

29 43 43 51 64 70

$$\frac{43+51}{2} = 47$$

Median = 47

$$\text{Mean} = \frac{\sum x}{n}$$

$\sum x = 300$ and $n = 6$ $\qquad \frac{300}{6} = 50$

Mean = 50

Your notes

Quartiles & Range

What are quartiles?

- **Quartiles** are **measures of location**
- Quartiles divide a population or data set into **four equal sections**
  - The **lower quartile, Q_1** splits the lowest 25% from the highest 75%
  - The **median, Q_2** splits the lowest 50% from the highest 50%
  - The **upper quartile, Q_3** splits the lowest 75% from the highest 25%
- There are different methods for finding quartiles
  - Values obtained by hand and using technology may differ
- You will be expected to use your GDC to calculate the quartiles

What are the range and interquartile range?

- The **range** and **interquartile range** are both **measures of dispersion**
  - They describe how spread out the data is
- The **range** is the largest value of the data minus the smallest value of the data
- The **interquartile range** is the range of the central 50% of data
  - It is the upper quartile minus the lower quartile

$$\text{IQR} = Q_3 - Q_1$$

    - This is given in the **formula booklet**
- The **units** for the range and interquartile range are the **same** as the units for the data

Your notes

Worked example

Find the range and interquartile range for the data set given below.

43 29 70 51 64 43

Range = Maximum − Minimum

70 − 29

Range = 41

Find upper and lower quartiles using GDC

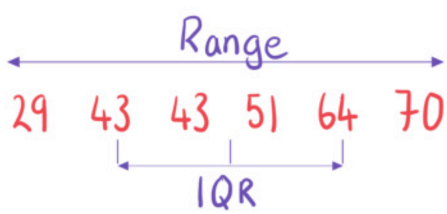
$Q_1 = 43$ and $Q_3 = 64$

$IQR = Q_3 - Q_1$

64 − 43

IQR = 21

By hand



Your notes

Standard Deviation & Variance

What are the standard deviation and variance?

- The **standard deviation** and **variance** are both **measures of dispersion**
  - They describe how spread out the data is in relation to the mean
- The **variance** is the **mean** of the **squares** of the **differences** between **the values and the mean**
  - Variance is denoted σ^2
- The **standard deviation** is the **square-root** of the **variance**
  - Standard deviation is denoted σ
- The **units** for the standard deviation are the **same** as the units for the data
- The **units** for the variance are the **square** of the units for the data

How are the standard deviation and variance calculated for ungrouped data?

- In the exam you will be expected to use the statistics function on your **GDC** to calculate the standard deviation and the variance
- Calculating the standard deviation and the variance by hand may deepen your understanding
- The formula for **variance** is $\sigma^2 = \frac{\sum_{i=1}^{k} f_i(x_i - \mu)^2}{n}$
  - This can be rewritten as

$$\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$$

- The formula for **standard deviation** is $\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i(x_i - \mu)^2}{n}}$
  - This can be rewritten as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2}$$

- You **do not** need to learn these formulae as you will use your GDC to calculate these

Your notes

Worked example

Find the variance and standard deviation for the data set given below.

43	29	70	51	64	43

Find variance and standard deviation using GDC

$\sigma_x^2 = 189.333...$ and $\sigma_x = 13.759...$

Variance = 189 (3sf)
Standard deviation = 13.8 (3sf)

By hand

$$\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^2$$

$\Sigma x^2 = 16136 \quad \bar{x} = 50 \quad n = 6$

$$\sigma^2 = \frac{16136}{6} - 50^2 = 189.333...$$

$$\sigma = \sqrt{189.333...} = 13.759...$$

4.1.3 Frequency Tables

Your notes

Ungrouped Data

How are frequency tables used for ungrouped data?

- Frequency tables can be used for ungrouped data when you have lots of the same values within a data set
  - They can be used to collect and present data easily
- If the value 4 has a frequency of 3 this means that there are three 4's in the data set

How are measures of central tendency calculated from frequency tables with ungrouped data?

- The **mode** is the value that has the **highest frequency**
- The **median** is the **middle** value
  - Use cumulative frequencies (running totals) to find the median
- The **mean** can be calculated by
  - Multiplying each value x_i by its frequency f_i
  - Summing to get $\Sigma f_i x_i$
  - Dividing by the total frequency $n = \Sigma f_i$
  - This is given in the formula booklet

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$$

  - Your **GDC** can calculate these statistical measures if you input the values and their frequencies using the statistics mode

How are measures of dispersion calculated from frequency tables with ungrouped data?

- The **range** is the largest value of the data minus the smallest value of the data
- The **interquartile range** is calculated by

$$\text{IQR} = Q_3 - Q_1$$

  - The **quartiles** can be found by using your GDC and inputting the values and their frequencies
- The **standard deviation** and **variance** can be calculated by hand using the formulae
  - **Variance**

$$\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$$

- **Standard deviation**

$$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2}$$

- You **do not need to learn** these formulae as you will be expected to use your GDC to find the standard deviation and variance
  - You may want to see these formulae to deepen your understanding

Your notes

Examiner Tip

- Always check whether your answers make sense when using your GDC
  - The value for a measure of central tendency should be within the range of data

Your notes

Worked example

The frequency table below gives information number of pets owned by 30 students in a class.

Number of pets	0	1	2	3
Frequency	11	5	8	6

Find

a) the mode.

Mode = value with highest frequency

Mode = 0

b) the median.

Median = middle value

n = 30 so median is midpoint of 15th and 16th

Number of pets	0	1	2	3
Cumulative frequency	11	16	24	30

Median = 1

c) the mean.

Your notes

Formula Booklet

Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ $\quad n = \sum_{i=1}^{k} f_i$

$$\bar{x} = \frac{\Sigma fx}{n} = \frac{11\times0 + 5\times1 + 8\times2 + 6\times3}{11+5+8+6} \quad \frac{39}{30}$$

Mean = 1.3

d) the standard deviation.

Use GDC $\sigma_x = 1.159...$

Standard deviation = 1.16 (3sf)

Your notes

Grouped Data

How are frequency tables used for grouped data?

- Frequency tables can be used for grouped data when you have lots of the same values within the same interval
  - Class intervals will be written using inequalities and without gaps
    - $10 \le x < 20$ and $20 \le x < 30$
  - If the class interval $10 \le x < 20$ has a frequency of 3 this means there are three values in that interval
    - You do not know the **exact data values** when you are given grouped data

How are measures of central tendency calculated from frequency tables with grouped data?

- The **modal class** is the class that has the **highest frequency**
  - This is for equal class intervals only
- The **median** is the **middle** value
  - The exact value can not be calculated but it can be estimated by using a **cumulative frequency graph**
- The **exact mean** can not be calculated as you do not have the raw data
- The **mean** can be **estimated** by
  - Identifying the mid-interval value (midpoint) x_i for each class
  - Multiplying each value by the class frequency f_i
  - Summing to get $\Sigma f_i x_i$
  - Dividing by the total frequency $n = \Sigma f_i$
  - This is given in the formula booklet

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$$

  - Your **GDC** can estimate the mean if you input the mid-interval values and the class frequencies using the statistics mode

How are measures of dispersion calculated from frequency tables with grouped data?

- The exact **range** can not be calculated as the largest and smallest values are unknown
- The **interquartile range** can be estimated by

$$\text{IQR} = Q_3 - Q_1$$

  - **Estimates** of the **quartiles** can be found by using a **cumulative frequency graph**
- The **standard deviation** and **variance** can be estimated using the mid-interval values x_i in the formulae
  - **Variance**

$$\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$$

- **Standard deviation**

$$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2}$$

- You **do not need to learn** these formulae as you will be expected to use your GDC to estimate the standard deviation and variance using the mid-interval values
  - You may want to use these formulae to deepen your understanding

Your notes

Examiner Tip

- As you can only estimate statistical measures from a grouped frequency table it is good practice to indicate that the values are not exact
  - You can do this by rounding values rather than leaving as surds and fractions
  - $\bar{x} = 0.333$ (3sf) rather than $\bar{x} = \frac{1}{3}$

Worked example

The table below shows the heights in cm of a group of 25 students.

Height, h	Frequency
$150 \le h < 155$	3
$155 \le h < 160$	5
$160 \le h < 165$	9
$165 \le h < 170$	7
$170 \le h < 175$	1

a) Write down the modal class.

Modal class = class with highest frequency

Modal class = $160 \le h < 165$

b) Write down the mid-interval value of the modal class.

Mid-interval value = $\frac{\text{Upper boundary} + \text{lower boundary}}{2}$

$\frac{160 + 165}{2}$

Mid-interval value = 162.5 cm

c) Calculate an estimate for the mean height.

Your notes

Your notes

Use mid-interval values to estimate the mean

Formula Booklet

Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ $\quad n = \sum_{i=1}^{k} f_i$

$$\bar{x} = \frac{3 \times 152.5 + 5 \times 157.5 + 9 \times 162.5 + 7 \times 167.5 + 1 \times 172.5}{3 + 5 + 9 + 7 + 1} = \frac{4052.5}{25}$$

Estimated mean = 162.1 cm

Your notes

4.1.4 Linear Transformations of Data

Linear Transformations of Data

Why are linear transformations of data used?

- Sometimes data might be very large or very small
- You can apply a **linear transformation** to the data to make the values more manageable
  - You may have heard this referred to as:
    - Effects of constant changes
    - Linear coding
- Linear transformations of data can **affect the statistical measures**

How is the mean affected by a linear transformation of data?

- Let $\bar{x}$ be the **mean** of some data
- If you **multiply each value** by a constant ***k*** then you will need to **multiply the mean by *k***
  - Mean is $k\bar{x}$
- If you **add or subtract** a constant ***a*** from all the **values** then you will need to **add or subtract** the constant ***a*** **to the mean**
  - Mean is $\bar{x} \pm a$

How is the variance and standard deviation affected by a linear transformation of data?

- Let σ^2 be the **variance** of some data
  - σ is the **standard deviation**
- If you **multiply** each value by a constant ***k*** then you will need to **multiply** the **variance by *k*²**
  - Variance is $k^2\sigma^2$
  - You will need to **multiply** the **standard deviation** by the **absolute value** of *k*
    - Standard deviation is $|k|\sigma$
  - If you **add or subtract** a constant ***a*** from all the **values** then the **variance** and the **standard deviation stay the same**
    - Variance is σ^2
    - Standard deviation is σ

Your notes

Examiner Tip

- If you forget these results in an exam then you can look in the HL section of the formula booklet to see them written in a more algebraic way
  - Linear transformation of a single variable

$$\mathrm{E}(aX+b) = a\mathrm{E}(X)+b$$

$$\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$$

  - where E(...) means the mean and Var(...) means the variance

Worked example

A teacher marks his students' tests. The raw mean score is 31 marks and the standard deviation is 5 marks. The teacher standardises the score by doubling the raw score and then adding 10.

a) Calculate the mean standardised score.

If data is multiplied by k then mean is multiplied by k
If k is added to data then k is added to the mean
$31 \times 2 + 10$
Mean of standardised scores = 72

b) Calculate the standard deviation of the standardised scores.

If data is multiplied by k then standard deviation is multiplied by $|k|$

If k is added to data then standard deviation is unchanged
5×2
Standard deviation of standardised scores = 10

4.1.5 Outliers

Your notes

Outliers

What are outliers?

- Outliers are extreme data values that do not fit with the rest of the data
  - They are either a lot bigger or a lot smaller than the rest of the data
- Outliers are defined as values that are **more than 1.5 × IQR from the nearest quartile**
  - x is an outlier if $\boldsymbol{x < Q_1 - 1.5 \times \text{IQR}}$ or $\boldsymbol{x > Q_3 + 1.5 \times \text{IQR}}$
- Outliers can have a big effect on some statistical measures

Should I remove outliers?

- The decision to remove outliers will **depend on the context**
- Outliers **should be removed** if they are found to be **errors**
  - The data may have been recorded incorrectly
  - For example: The number 17 may have been recorded as 71 by mistake
- Outliers **should not be removed** if they are a **valid part of the sample**
  - The data may need to be checked to verify that it is not an error
  - For example: The annual salaries of employees of a business might appear to have an outlier but this could be the director's salary

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Worked example

The ages, in years, of a number of children attending a birthday party are given below.

2, 7, 5, 4, 8, 4, 6, 5, 5, 29, 2, 5, 13

a) Identify any outliers within the data set.

x is an outlier if $x < Q_1 - 1.5 \times IQR$ or $x > Q_3 + 1.5 \times IQR$

Using GDC

$Q_1 = 4$ and $Q_3 = 7.5$ $\therefore IQR = 3.5$

$Q_1 - 1.5 \times IQR = 4 - 1.5 \times 3.5 = -1.25$

$Q_3 + 1.5 \times IQR = 7.5 + 1.5 \times 3.5 = 12.75$

Outliers are 13 and 29

b) Suggest which value(s) should be removed. Justify your answer.

13 should not be removed as it is a valid age of a child.

29 should be removed as this is an age of an adult.

4.1.6 Univariate Data

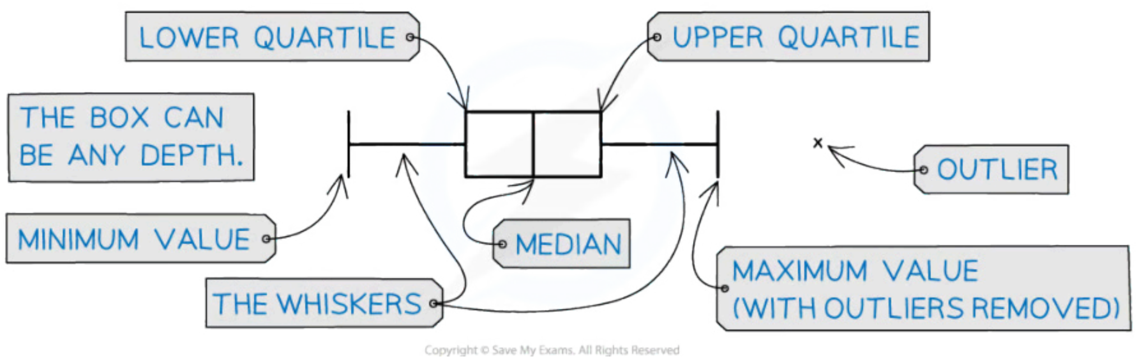
Your notes

Box Plots

Univariate data is data that is in **one variable.**

What is a box plot (box and whisker diagram)?

- A box plot is a graph that clearly shows key statistics from a data set
  - It shows the **median, quartiles, minimum** and **maximum values** and **outliers**
  - It does not show any other individual data items
- The middle 50% of the data will be represented by the box section of the graph and the lower and upper 25% of the data will be represented by each of the whiskers
- Any **outliers** are represented with a **cross** on the **outside of the whiskers**
  - If there is an outlier then the whisker will end at the value before the outlier
- Only one axis is used when graphing a box plot
- It is still important to make sure the axis has a clear, even scale and is labelled with units



What are box plots useful for?

- Box plots can clearly show the shape of the distribution
  - If a box plot is symmetrical about the median then the data could be **normally distributed**
- Box plots are often used for **comparing two sets of data**
  - Two box plots will be drawn next to each other using the same axis
  - They are useful for **comparing data** because it is easy to see the main shape of the distribution of the data from a box plot
    - You can easily compare the medians and interquartile ranges

Examiner Tip

- In an exam you can use your GDC to draw a box plot if you have the raw data
  - You calculator's box plot can also include outliers so this is a good way to check

Your notes

Worked example

The distances, in metres, travelled by 15 snails in a one-minute period are recorded and shown below:

0.5, 0.7, 1.0, 1.1, 1.2, 1.2, 1.2, 1.3, 1.4, 1.4, 1.4, 1.4, 1.5, 1.5, 1.5

a) i) Find the values of Q_1, Q_2 and Q_3.

ii) Find the interquartile range.

iii) Identify any outliers.

Using GDC

$Q_1 = 1.1$ m $\quad Q_2 = 1.3$ m $\quad Q_3 = 1.4$ m

$IQR = Q_3 - Q_1 = 1.4 - 1.1$

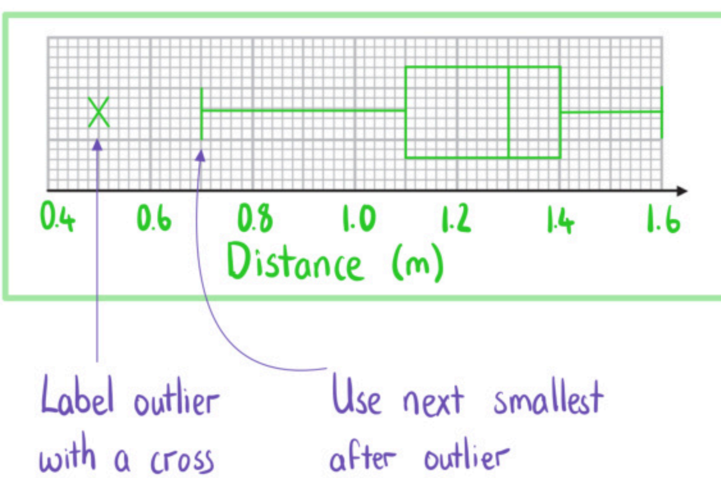
$IQR = 0.3$ m

$Q_1 - 1.5 \times IQR = 1.1 - 1.5 \times 0.3 = 0.65$

$Q_3 + 1.5 \times IQR = 1.4 + 1.5 \times 0.3 = 1.85$

0.5 m is an outlier

b) Draw a box plot for the data.



Your notes

Cumulative Frequency Graphs

What is cumulative frequency?

- The cumulative frequency of x is the running total of the frequencies for the values that are less than or equal to x
- For grouped data you use the upper boundary of a class interval to find the cumulative frequency of that class

What is a cumulative frequency graph?

- A cumulative frequency graph is used with data that has been organised into a **grouped frequency** table
- Some coordinates are plotted
  - The x-coordinates are the **upper boundaries** of the class intervals
  - The y-coordinates are the **cumulative frequencies** of that class interval
- The coordinates are then joined together by hand using a **smooth increasing curve**

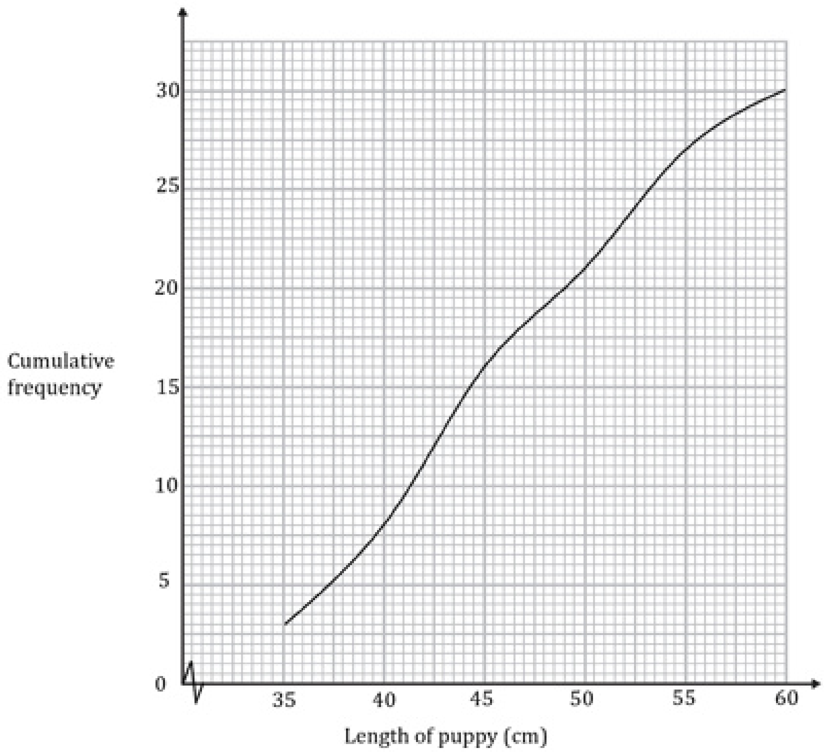
What are cumulative frequency graphs useful for?

- They can be used to **estimate** statistical measures
  - Draw a **horizontal line** from the y-axis to the curve
    - For the median: draw the line at 50% of the total frequency
    - For the lower quartile: draw the line at 25% of the total frequency
    - For the upper quartile: draw the line at 75% of the total frequency
    - For the p^{th} percentile: draw the line at p% of the total frequency
  - Draw a **vertical line** down from the curve to the x-axis
  - This **x-value** is the relevant statistical measure
- They can used to estimate the number of values that are bigger/small than a given value
  - Draw a **vertical line** from the given value on the x-axis to the curve
  - Draw a **horizontal line** from the curve to the y-axis
  - This value is an estimate for how many values are less than or equal to the given value
    - To estimate the number that is greater than the value subtract this number from the total frequency
  - They can be used to **estimate** the **interquartile range** $\text{IQR} = Q_3 - Q_1$
  - They can be used to construct a **box plot** for grouped data

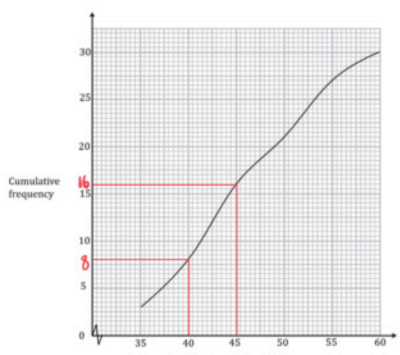
Your notes

Worked example

The cumulative frequency graph below shows the lengths in cm, l, of 30 puppies in a training group.



a) Given that the interval $40 \leq l < 45$ was used when collecting data, find the frequency of this class.

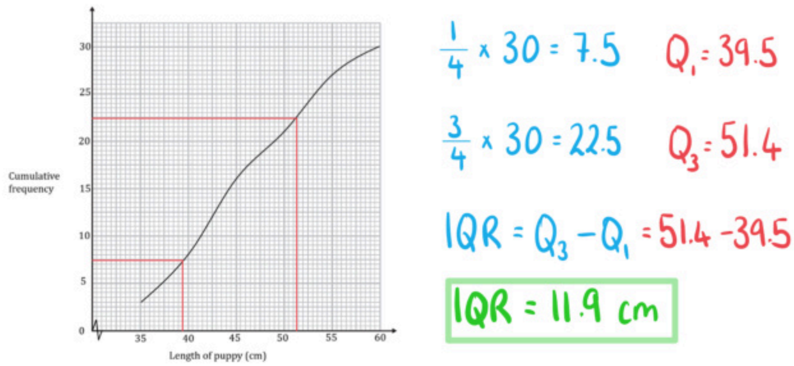


16 - 8

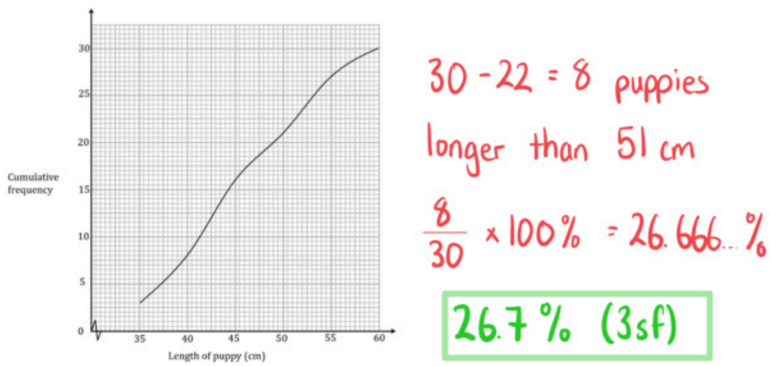
Frequency = 8

b) Use the graph to find an estimate for the interquartile range of the lengths.

Your notes



c) Estimate the percentage of puppies with length more than 51 cm.



Your notes

Histograms

What is a (frequency) histogram?

- A frequency histogram clearly shows the frequency of class intervals
  - The classes will have **equal class intervals**
  - The **frequency** will be on the y-axis
  - The bar for a class interval will begin at the lower boundary and end at the upper boundary
- A frequency histogram is **similar to a bar chart**
  - A **bar chart** is used for **qualitative or discrete data** and **has gaps** between the bars
  - A **frequency histogram** is used for **continuous data** and **has no gaps** between bars

What are (frequency) histograms useful for?

- They show the **modal class** clearly
- They show the shape of the distribution
  - It is important the class intervals are of equal width
- They can show whether the variable can be modelled by a **normal distribution**
  - If the shape is symmetrical and bell-shaped

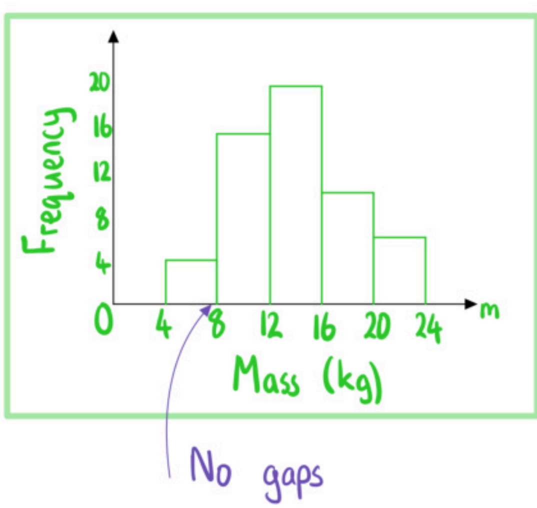
Your notes

Worked example

The table below and its corresponding histogram show the mass, in kg, of some new born bottlenose dolphins.

Mass, m kg	Frequency
$4 \le m < 8$	4
$8 \le m < 12$	15
$12 \le m < 16$	19
$16 \le m < 20$	10
$20 \le m < 24$	6

a) Draw a frequency histogram to represent the data.



b) Write down the modal class.

Modal class = class with highest frequency

Modal class : $12 \le m < 16$

4.1.7 Interpreting Data

Your notes

Interpreting Data

How do I interpret statistical measures?

- The **mode** is useful for **qualitative data**
  - It is not as useful for quantitative data as there is not always a unique mode
- The **mean includes all values**
  - It is affected by outliers
  - A smaller/larger mean is preferable depending on the scenario
    - A smaller mean time for completing a puzzle is better
    - A bigger mean score on a test is better
- The **median is not affected by outliers**
  - It does not use all the values
- The r**ange gives the full spread** of the all of the data
  - It is affected by outliers
- The **interquartile range gives the spread of the middle 50%** about the median and is not affected by outliers
  - It does not use all the values
  - A bigger IQR means the data is more spread out about the median
  - A smaller IQR means the data is more centred about the median
- The **standard deviation** and **variance** use all the values to give a measure of the **average spread** of the data about the mean
  - They are affected by outliers
  - A bigger standard deviation means the data is more spread out about the mean
  - A smaller standard deviation means the data is more centred about the mean

How do I choose which diagram to use to represent data?

- **Box plots**
  - Can be used with ungrouped **univariate** data
  - Shows the range, interquartile range and quartiles clearly
  - Very useful for comparing data patterns quickly
- **Cumulative frequency graphs**
  - Can be used with continuous grouped univariate data
  - Shows the running total of the frequencies that fall below the upper bound of each class
- **Histograms**
  - Can be used with continuous grouped univariate data
  - Used with equal class intervals
  - Shows the frequencies of the group
- **Scatter diagrams**
  - Can be used with ungrouped **bivariate** data
  - Shows the graphical relationship between the variables

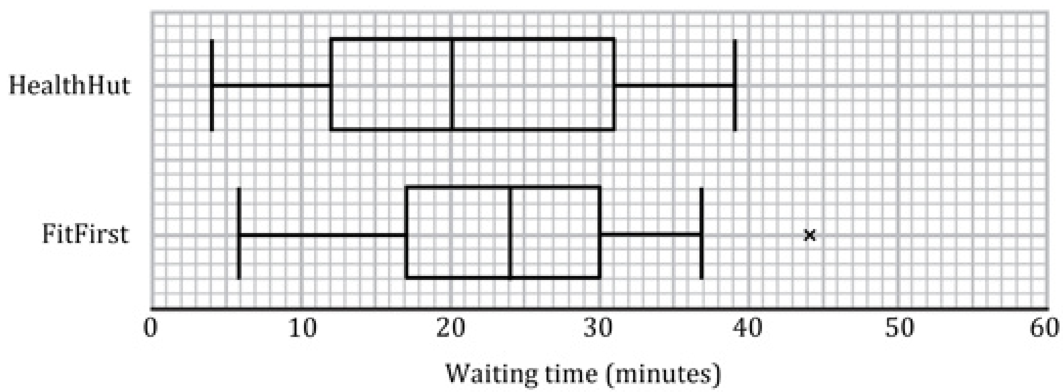
How do I compare two or more data sets?

- Compare a **measure of central tendency**
  - If the data **contains outliers – use the median**
  - If the data is **roughly symmetrical – use the mean**
- Compare a **measure of dispersion**
  - If the data **contains outliers – use the interquartile range**
  - If the data is **roughly symmetrical – use the standard deviation**
- Consider whether it is better to have a smaller or bigger average
  - This will depend on the context
    - A smaller average time for completing a puzzle is better
    - A bigger average score on a test is better
- Consider whether it is better to have a smaller or bigger spread
  - Usually a smaller spread means it is more consistent
- Always relate the **comparisons to the context** and consider reasons
  - Consider the **sampling technique** and the **data collection** method

Your notes

Worked example

The box plots below show the waiting times for the two doctor surgeries, HealthHut and FitFirst.



Compare the two distributions of waiting times in context.

Compare :

- a measure of central tendency
- a measure of dispersion

HealthHut's median waiting time is smaller than FitFirst's (20 < 24). On average patients get seen quicker at HealthHut.

FitFirst's interquartile range is smaller than HealthHut's (13 < 19). There is less variability of waiting times at FitFirst.